Formal Languages and Automata Exam

Faculty of Computers & Information
Department: Computer Science Grade: Third
Course code: CSC 344 Total Mark: 80
Date: 23/1/2010 Time: 3 hours

Answer the following questions:

1- a) Consider the language \( S^* \), where \( S = \{ab \ ba\} \). Write out all the words in \( S^* \) that have seven or fewer letters. Can any word in this language contain the substrings \( aaa \) or \( bbb \)? Why?

b) Does the two following two regular expressions define the same language: 

\[ (a+b)^* a(a+b)^* a(a+b)^* \] 
\[ b^* ab^* a(a+b)^* \] 

Investigate your answer.

c) Construct a regular expression defining each of the following languages over the alphabet \( \Sigma = \{a, b\} \):

- The language of all strings of \( a \)'s and \( b \)'s in which either the strings are all \( b \)'s or else there is an \( a \) followed by some \( b \)'s.
- Language of all strings of \( a \)'s and \( b \)'s that at some point contain a double letter.
- All words that contain at least one of the strings \( s_1, s_2, s_3 \) or \( s_4 \).

2- a) Let \( \Sigma = \{a, b\} \), draw deterministic finite automata (DFA) that recognize, exactly:

- Language \( ( (a+b)^*(aa+bb)(a+b)^* ) \)
- All words with \( b \) as the third letter and reject all other words.
- All words that have different first and last letters.

b) Draw the corresponding NFA to the following DFA

(See the next page)
c) If \( L \) is the language recognized by the DFA in the figure, Draw DFA to recognize \( L' \), where \( L' \) is the complement of \( L \).

3- a) State pumping lemma.
   
b) Use pumping lemma to prove that \( L = \{a^n b^n, n \geq 0\} \) is nonregular.
   
c) Define the language \( \text{EQUAL} \) to have only the words with the same total number of \( a \)'s and \( b \)'s. Does \( \text{EQUAL} \) a nonregular language? Prove your answer.

4) a) Define what we mean by a decidable problem.
   
b) Is there an effective procedure to determine the finiteness of a language of finite automate? State it, if any.
   
c) Write the context free grammar that generates \( L = \{a^n b^n, n \geq 0\} \)

5- a) Define \( \text{PALINDROMEX} \) as the language of all words of the form \( s \text{X} \text{reverse}(s) \). Construct deterministic pushdown automata that accept this language.
   
b) Define \( \text{ODDPALINDROME} \) to be the language of all strings of \( a \)'s and \( b \)'s that are palindromes and have an odd number of letters. Construct nondeterministic pushdown automata that accept this language.
1- a) Consider the language $S^*$, where $S = \{ab \ ba\}$. Write out all the words in $S^*$ that have seven or fewer letters. Can any word in this language contain the substrings $aaa$ or $bbb$? Why?

**Solution:**
- The words of length less than 7 are:
  - $\lambda$: contains no factors
  - $ab$, $ba$: contains one factor
  - $abab$, $abba$, $baab$, $baba$: contains 2 factors
  - $ababab$, $ababba$, $abbaab$, $bababa$, $babaab$, $baabba$, $abbaba$: contains 3 factors
  All words contain 4 or more factors have length $\geq 8$.
- The language words can't contain neither $aaa$ nor $bbb$. The reason is the blocks that build the words are only 2. These 2 blocks start with $a$ and end with $b$ or vice versa. So, the result of combination yields only $ab$, $ba$, $aa$, or $bb$.

b) Does the two following two regular expressions define the same language: $(a+b)^*a(a+b)^*a(a+b)^*$ and $b^*ab^*a(a+b)^*$? Investigate your answer.

**Solution:**
Yes. Both of them denote all the words with at least two $a$'s.

c) Construct a regular expression defining each of the following languages over the alphabet $\Sigma = \{a \ b\}$:
- The language of all strings of $a$'s and $b$'s in which either the strings are all $b$'s or else there is an $a$ followed by some $b$'s.
- The language of all strings of \( a \)'s and \( b \)'s that at some point contain a double letter.

- All words that contain at least one of the strings \( s_1, s_2, s_3 \) or \( s_4 \).

**Solution:**

- \( b^* + ab^* \) or equivalently \( (\Lambda + a)b^* \)
- \( (a+b)^* (aa+bb)(a+b)^* \)
- \( (a+b)^* (s_1+s_2+s_3+s_4)(a+b)^* \)
2- a) Let $\Sigma = \{a, b\}$, draw deterministic finite automata that recognize -exactly- the following:
- Language $(a+b)^* (aa+bb)(a+b)^*$

**Solution:**

- All words with $b$ as the third letter and reject all other words.

**Solution:**

- All words that have different first and last letters.

**Solution:**
b) Draw the corresponding NFA to the following DFA

![DFA Diagram]

Solution:

OR

![NFA Diagram]

OR

![NFA Diagram]

c) If L is the language recognized by the DFA in the figure, Draw DFA to recognize
L’, where L’ is the complement of L.

Solution:
3- a) State pumping lemma.

**Solution:**

Let L be any regular language that has infinitely many words. Then there exist some three strings x, y, z (where y is not the null string) such that all the strings of the form \( xy^nz \in L, n = 1, 2, 3, \ldots \)

b) Use pumping lemma to prove that \( L = \{ a^nb^n, \ n \geq 0 \} \) is nonregular.

**Solution:**

Assume the contrary, i.e. L is regular.
Hence, by pumping lemma, \( \exists x, y, z \) such that \( xyz, xyyz \in L \)
We differentiate among 3 cases of y:
- y constitute of only a's:
  
  We have \( xyz \) contains the same number of a's and b's
  
  So, \( xyyz \) contains number of a's bigger than number of b's
  
  So, \( xyyz \not\in L \)
  
  But \( xyyz \in L \), which is contradiction.

- y constitute of only b's:
  
  Same argument as above.

- y constitutes of equal number of a's and b's:
  
  To be \( xyz \in L \), y must equals to \( a^pb^p \), for some \( p \)
  
  So \( yy = a^pb^p a^pb^p \)
  
  Now \( xyyz = x a^pb^p a^pb^p z \not\in L \), which is contradiction.

The contradictions show that L is nonregular.
c) Define the language \text{EQUAL} to have only the words with the same total number of \text{a}'s and \text{b}'s. Does \text{EQUAL} a nonregular language? Prove your answer.

\textbf{Solution:}

\text{EQUAL} is nonregular because \( \{ a^n b^n \} = a^* b^* \cap \text{EQUAL} \)

We know that \( a^* b^* \) is a regular language and the set of regular language is closed under intersection. Hence \text{EQUAL} can't be regular.
4) a) define what we mean by a decidable problem.

**Solution:**

An effective solution to a problem that has a yes or no answer is called a decision procedure. A problem that has a decision procedure is called decidable.

b) Is there an effective procedure to determine the finiteness of a language of finite automata? State it, if any.

**Solution:**

Yes.

- Assume that the finite automaton has N states
- Test all words of length p on the machine, where N ≤ p < 2N
- If at least one of these words is accepted, then the language is infinite.
- Else if all these words are consumed at nonfinal states, then the language is finite.

c) Write the context free grammar that generates $L = \{a^n b^n, n \geq 0\}$

**Solution:**

$G = (\{S\}, \{a, b\}, \{S \rightarrow aSb\}, S)$
5- a) Define PALINDROMEX as the language of all words of the form $sX$\text{reverse}(s).

Construct deterministic pushdown automata that accept this language.

**Solution:**

Any deterministic pushdown automata equivalent to the following is sufficient:
b) Define ODDPALINDROME to be the language of all strings of a's and b's that are palindromes and have an odd number of letters. Construct nondeterministic pushdown automata that accept this language.

**Solution:**
Any nondeterministic pushdown automata equivalent to the following is sufficient: