A real-time SPC inventory replenishment system to improve supply chain performances

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Abstract
Inventory replenishment rules contribute significantly to the bullwhip effect and inventory instability in supply chains. Smoothing replenishment rules have been suggested as a mitigation solution for the bullwhip effect but dampening the bullwhip effect might increase inventory instability. This paper evaluates a real-time inventory replenishment system denoted as SPC that utilizes a control chart approach to counteract the bullwhip effect whilst achieving competitive inventory stability. The SPC employs two control charts integrated with a set of decision rules to estimate the expected demand and adjust the inventory position, respectively. The first control chart works as a forecasting mechanism and the second control chart is devoted to control the inventory position variation whilst allowing order smoothing. A simulation analysis has been conducted to evaluate and compare SPC with a generalized (R, S) policy in a four-echelon supply chain, under various operational settings in terms of demand process, lead-time, and information sharing. The results show that SPC is superior to the traditional (R, S) and comparable to the smoothing one in terms of bullwhip effect, inventory variance, and service level. Further managerial implications have been obtained from the results.

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1. Introduction

Inventory control is a major activity for operating supply chains in which each partner attempts to decide how much and when to order for maintaining a high service level. There is a large body of inventory-related theory that optimizes the inventory costs within simple inventory systems but when these optimal policies are used together in a supply chain system, they create the bullwhip effect (Fig. 1), that is, order variability is amplified as moving up the supply chain (Disney & Grubbström, 2004; Lee, Padmanabhan, & Whang, 1997a, 1997b). Bullwhip effect can cause stock outs, low service level, and extra transportation and capacity costs in supply chains. The bullwhip effect has been observed in many real cases such as Campbell Soup’s (Fisher, Hammond, Obermeyer, & Raman, 1997), HP and P&G (Lee et al., 1997a), a clothing supply chain (Disney & Towill, 2003), Glosuch (McCullen & Towill, 2000), and fast moving consumer goods (Zotteri, 2012).

Previous research has shown the importance of selecting/designing the appropriate ordering policies integrated with the accurate forecasting methods in order to mitigate the bullwhip effect (Jaipuria & Mahapatra, 2014; Wright & Yuan, 2008). Furthermore, other researchers have developed smoothing replenishment rules to avoid/eliminate the bullwhip effect with modifying the structure of the periodic review (R, S) policy, commonly used in practice, by incorporating smoothing controllers (Dejonckheere, Disney, Lambrecht, & Towill, 2003, 2004). In traditional (R, S), the order is generated to recover the entire gaps between the target and current levels of net inventory (safety stock) and supply line inventory while in smoothing (R, S) only a fraction of each gap is recovered, where the target levels are dynamically updated with demand forecast every review period. However, dampening the bullwhip effect might increase inventory instability causing low service level (Disney & Lambrecht, 2008; Jaipuria & Mahapatra, 2014). Thus, a replenishment rule not only affects order variability amplification which contributes to the upstream partners’ costs but also affects the inventory variance which determines the partner’s ability to meet a desired service level. Therefore, inventory replenishment systems should be designed to avoid the bullwhip effect without affecting the inventory stability.

The available smoothing replenishment rules in literature are mainly based on the periodic review (R, S) and their rationale is
to restrict the over/under-reaction to short-run fluctuations in demand (Ciancimino, Cannella, Brucoleri, & Framinan, 2012). Recently, some researchers have employed control charts to develop inventory control system that not only improve inventory performance but also can counteract the bullwhip effect through order smoothing (Costantino, Di Gravio, Shaban, & Tronci, 2014a, 2014b; Lee & Wu, 2006). Table 1 represents the trend in integrating control charts to inventory control along with the scope of study, performance measures, and supply chain structure of each study. Most of these studies other than Costantino et al. (2014a, 2014b) have been focusing on improving the inventory performance measures in simple inventory systems. Costantino et al. (2014a, 2014b) have alternatively employed control charts to handle supply chain dynamics, showing a superior ordering and inventory stability compared to the standard (R, S) in a multi-echelon supply chain. However, they have evaluated their model only under normal demand, without investigating its sensitivity to other demand processes, or investigating its sensitivity to other important operational factors such as lead-time. This research extends and extensively evaluates this novel inventory system in a multi-echelon supply chain under various operational conditions.

This research mainly attempts to formulate and evaluate a real-time inventory replenishment system with smoothing capability that relies on a control chart approach to be used in dynamic and complex environments like multi-echelon supply chains. The inventory replenishment system denoted as SPC utilizes two control charts integrated with a set of decision rules to estimate the expected demand and adjust the inventory position, respectively. The first control chart denoted as ‘demand control chart’ works as an improved forecasting mechanism to dynamically estimate the expected demand every review period without over/under-reacting to demand changes. The second control chart is employed to adjust the inventory position and to control order smoothing with restricting the over/under-reaction to inventory position variation. The replenishment order is determined in each period as the sum of the expected demand and a fraction of the amount needed to recover the inventory position to enhance order smoothing. Therefore, SPC has two dimensions for order smoothing. Similar to (R, S), the SPC approach can be very suitable to environments that replenish inventory frequently (daily, weekly, monthly) such as in retailing that needs regular repeating schedules of inventory replenishments (Disney, Farasyn, Lambrecht, Towill, & de Velde, 2006). This kind of inventory control system is preferred in competitive markets with high variability, where tracing the time series of orders and demand gives more information on future trends than applying traditional and static forecasting and inventory planning system that often over/under-react to market demand changes. Dynamic retailing, as for consumer goods, fashion industry or high tech portable devices, where customer profile of requirements may harshly vary and different products from different suppliers can easily substitute each other, is the main field of application.

A simulation approach is adopted to evaluate the effectiveness of the SPC policy in a four-echelon supply chain. An exhaustive comparison is conducted between the SPC policy and a generalized order-up-to (R, S) policy in terms mainly of bullwhip effect, inventory variance and service level. The sensitivity of both policies to different demand processes, lead-time, and information sharing is evaluated. The simulation results show that SPC outperforms the traditional (R, S) and is comparable to the smoothing one where SPC can eliminate the bullwhip effect whilst achieving acceptable inventory performance, under various operational settings. The results have provided further insights for supply chain managers on how to control instability propagation in supply chains.

The paper is organized as follows. Section 2 presents the related literature review. Section 3 describes the formulation of the SPC inventory replenishment system. Section 4 describes the supply chain model, generalized (R, S) policy, performance measures, and simulation model validation. Sections 5 and 6 present simulation results and sensitivity analysis. The discussion and implications are provided in Section 7, and the conclusions are summarized in Section 8.

2. Related work

The replenishment orders variability often increases as one moves up the supply chain, causing severe problems across the supply chain. Lee et al. (1997a, 1997b) identified five fundamental causes of the bullwhip effect: demand signal processing, lead-time, order batching, price fluctuations and rationing and shortage gaming. Of our particular interest is the demand signal processing in which forecasting methods and replenishment rules are integrated to regulate the replenishment orders and inventory levels. Extensive studies have quantified the impact of the different bullwhip effect causes using three modeling approaches: statistical modeling (Chen, Drezner, Ryan, & Simchi-Levi, 2000; Chen, Ryan, & Simchi-Levi, 2000; Cho & Lee, 2013), control theoretic (Dejonckheere et al., 2003, Dejonckheere, Disney, Lambrecht, & Towill, 2004; Hoberg, Bradley, & Thonemann, 2007) and simulation modeling (Chatfield, Kim, Harrison, & Hayya, 2004; Ciancimino et al., 2012; Costantino et al., 2014a, 2014b, Costantino, Di Gravio, Shaban, & Tronci, 2014c). These studies have shown that the bullwhip effect can be mitigated with selecting the proper forecasting method and ordering policy (Chatfield et al., 2004; Chen et al., 2000; Chen & Ryan et al., 2000; Jaipuria & Mahapatra, 2014; Li, Disney, & Gaalman, 2014; Zhang, 2004), reducing the lead-time (Chen et al., 2000; Chen & Ryan et al., 2000; Ciancimino et al., 2012; Zhang, 2004) and reducing the uncertainty in supply chains through increasing the collaboration and information visibility (Ciancimino et al., 2012; Costantino, Di Gravio, Shaban, & Tronci, 2014d; Dejonckheere et al., 2004).

Inventory replenishment policies have been recognized as a major cause of the bullwhip effect and thus it has received a signif-
The utilization of control charts in inventory control.

Hoberg et al., 2007; Jakšič & Rusjan, 2008). The majority of the bull-whip effect studies have been considering the periodic review order-up-to (R, S) policy because of its popularity in practice as it is known to minimize inventory costs (Chandra & Grabis, 2005; Ciancimino et al., 2012; Costantino, 2014d, Costantino, Di Gravio, Shaban, & Tronci, 2013a, Costantino, Di Gravio, Shaban, & Tronci, 2013b; Dejonckheere et al., 2003; Hoberg et al., 2007; Li et al., 2014). In this policy, at the end of each review period, the order quantity is generated to recover the entire gap between target and available levels of inventory position (net inventory + supply line inventory). Dejonckheere et al. (2003, 2004) proved through a control theoretic approach that the bullwhip effect is guaranteed in the (R, S) irrespective of the forecasting method used and without making any assumptions about the demand process. Many studies have quantified the bullwhip effect in supply chain models employ the (R, S) under different forecasting methods, and various operational conditions to provide useful insights for bullwhip effect mitigation (Costantino et al., 2014d; Dejonckheere et al., 2003, 2004; Hoberg et al., 2007; Li et al., 2014).

For avoiding the bullwhip effect, extensive research has been focusing on investigating smoothing replenishment rules that are modified from the (R, S) policy with adding proportional controllers to regulate the reaction to demand changes (Chen & Disney, 2007; Dejonckheere et al., 2003, 2004; Disney & Lambrecht, 2008). The available smoothing rules are a natural extension of the Inventory and Order Based Production Control System (IOBPCS) and Automatic Pipeline, Inventory and Order Based Production Control System (APIOBPCS) (Dejonckheere et al., 2003, 2004). The proportional controllers of a periodic review (R, S) are smoothing terms for the gaps between target and current levels of net inventory (safety stock) and supply line inventory. Previous research have shown that the proper tuning of the proportional controllers (smoothing parameters) can eliminate the bullwhip effect as can be found in Boute, Disney, Lambrecht, and Van Houdt (2007, 2009), Ciancimino et al. (2012), Disney and Towill (2003), Disney, Towill, and Van de Velde (2004, 2006), Hosoda and Disney (2006a, 2006b). However, dampening the bullwhip effect largely may increase the inventory variance that affects customer service level (Ciancimino et al., 2012; Disney et al., 2006). Some recent studies have shown that employing the smoothing ordering policies in collaborative supply chains leads to balance the trade-off between bullwhip effect and inventory variance by improving both the operational performance and customer service level (Ciancimino et al., 2012; Dejonckheere et al., 2004; Dominguez, Cannella, & Flamman, 2014).

Statistical Process Control (SPC), which is used to detect unstable changes in processes, has recently been employed to develop inventory replenishment policies for swiftly changing environments. The first pioneer work can be attributed to Watts, Hahn, and Sohn (1994) who attempted to employ control charts for monitoring the performance of a reorder-point inventory system through monitoring stock-outs, demand and inventory turnover to identify the causes of system malfunctions. Pfohl, Cullmann, and Stölzle (1999) developed a real-time inventory decision support system by employing the control charts for inventory level and demand along with a series of decision rules to determine the time and quantity to order. Cheng and Chou (2008) extended the work of Pfohl et al. (1999) by introducing the ARMA control chart for the demand and the individual control chart with western electric rules for the inventory level. Lee and Wu (2006) compared traditional replenishment policies and SPC based replenishment policy and concluded that the later policy can reduce inventory variability compared to the traditional methods. Kurano, McKay, and Black (2014) developed a dynamic inventory policy based on cooperative supplier monitoring with control charts to mitigate disruption risk from the supply side.

The majority of the above SPC inventory models produce replenishment orders without differentiating between forecasting and inventory control. They also do not allow order smoothing, have ignored the impact of lead-time, and have been evaluated in simple supply chain models based on inventory performance measures without considering the bullwhip effect measures (see, Table 1). This research attempts to fill some of these gaps by extending the work of Costantino et al. (2014a, 2014b) who have proposed novel inventory control systems to improve supply chain dynamics (bullwhip effect and inventory stability) in which control charts are employed to estimate expected demand (forecasting) and control inventory position variation, respectively. The smoothing in this policy is realized with restricting the reaction to inventory position variation through a target smoothing zone around the centerline of the inventory position control chart. The replenishment order is determined in each period as the sum of the expected demand and the entire gap between the current inventory position and target smoothing zone. They have evaluated this inventory control system in a multi echelon supply chain with non-zero lead-time through simulation modeling finding a superior performance to the traditional and smoothing (R, S) under normal demand process. This research extends the work of Costantino et al. (2014a, 2014b) by improving mainly the decision rules integrated with the second control chart through incorporating a smoothing parameter (similar to the proportional control in the smoothing (R, S)) to fractionally recover the inventory position gap. The modified SPC inventory replenishment system has a very flexible structure with two dimensions for order smoothing and can also be turned into a generalized (R, S) policy with this new added smoothing parameter as a proportional controller. It will also be evaluated and compared to a generalized (R, S) under various operational conditions in terms of demand process, smoothing level, lead-time, and information sharing. This evaluation also contributes to the current understanding of the impact of these factors on the supply chain performances.

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### Table 1

The utilization of control charts in inventory control.
3. SPC Inventory replenishment system formulation

The inventory replenishment system denoted as SPC integrates two control charts for demand forecasting and inventory position control, respectively (Costantino et al., 2014a, 2014b). The working mechanism of SPC is depicted in Fig. 2. The first control chart is devoted to monitor the variation of the demand/incoming order over time to make the proper changes in the expected demand whenever a considerable demand change is present; without over/under-reacting to demand changes. The demand control chart is integrated with a set of decision rules in order to estimate the expected demand under different out-of-control situations.

The second control chart is employed for monitoring and controlling the inventory position variation with the purpose of controlling the sensitivity of the ordering process to the inventory position variations to allow order smoothing. The inventory position, at the end of a review period, is defined as the sum of the inventory on hand (items immediately available to meet demand), the inventory in the supply line (items ordered but not yet arrived due to the lead time), minus the backlog (demand that could not be fulfilled and still has to be delivered). This control chart is employed to identify whether the inventory position variation is in-control or not, according to a set of decision rules. Furthermore, it is used to enhance order smoothing by restricting the reaction to inventory position variation.

3.1. Demand forecast

A typical control chart consists of three basic elements: a centerline that represents the average of the process variable, and lower and upper control limits (Montgomery, 2008). If a process variable (e.g., customer demand process) is in-control, then it is expected that 99.73% of the demand data points will be within the lower and upper control limits. The control limits of the demand control chart can be calculated as follows in Eqs. (1)–(3) (Costantino et al., 2014a, 2014b).

\[
UCL_{d,t.i} = CL_{d,t} + 3\sigma_{d,t}^i \\
CL_{d,t} = \frac{1}{\bar{W}_t} \sum_{t=-\infty}^{t} IO_t^i \\
LCL_{d,t} = CL_{d,t} - 3\sigma_{d,t}^i
\]

The \(UCL_{d,t,i}^i\) stands for the centerline of the demand control chart at time \(t\) and is calculated based on the average of the last 12 data points of the demand/incoming order data. The \(CL_{d,t}^i\) represents the lower control limit and equals the difference between \(CL_{d,t}^i\) and \(3\sigma_{d,t}^i\), where \(\sigma_{d,t}^i\) stands for the demand/incoming order standard deviation over the \(Wi\) time length. Similarly, the upper control limit \((UCL_{d,t}^i)\) equals the sum of \(CL_{d,t}^i\) and \(3\sigma_{d,t}^i\).

The demand decision rules are based on the status of the last observations of incoming order on the control chart (Costantino et al., 2014a, 2014b). Specifically, if the demand control chart signals that the demand is in-control and no change in its level, then the expected demand should be considered equal to the centerline of the control chart. Otherwise, if the demand is out-of-control, then the expected demand should be altered based on the corresponding decision rule.

**Demand Rule 1:** At echelon \(i\), if \(q_i\) points of the last consecutive \(N_i\) data points of incoming order are above a defined forecast smoothing zone between \(CL_{d,t}^i - K_d^i\sigma_{d,t}^i\) and \(CL_{d,t}^i + K_d^i\sigma_{d,t}^i\), then, the expected demand \((ED_{d,t}^i)\) should be based on the Maximum of the average of the last \(MC\) data points of incoming order and \(CL_{d,t}^i\) as shown in Eq. (4).

\[
ED_{d,t}^i = \max \left\{ \frac{1}{MC} \sum_{t=t-MC}^{t} IO_t^i, CL_{d,t}^i \right\}
\]

**Demand Rule 2:** At echelon \(i\), if \(q_i\) points of the last consecutive \(N_i\) data points of incoming order are below the forecast smoothing zone, then \(ED_{d,t}^i\) should be based on the Minimum of the average of the last \(MC\) data points and \(CL_{d,t}^i\), as shown in Eq. (5).

\[
ED_{d,t}^i = \min \left\{ \frac{1}{MC} \sum_{t=t-MC}^{t} IO_t^i, CL_{d,t}^i \right\}
\]

**Demand Rule 3:** If the above condition is not satisfied, then, \(ED_{d,t}^i\) should be equal to the centerline of the demand control chart as represented in Eq. (6).

\[
ED_{d,t}^i = CL_{d,t}^i = \frac{1}{\bar{W}_t} \sum_{t=-\infty}^{t} IO_t^i
\]

When the demand or the incoming order to echelon \(i\) is zero \((IO_t^i = 0)\), then, the order quantity of echelon \(i\) should be set to zero \((i.e., \delta_{d,t}^i = 0)\).

3.2. Inventory position control

The inventory position is a linear combination of the normal distribution; accordingly it will be normally distributed (Disney & Grubbström, 2004). The following method is used to calculate the limits of the inventory position control chart (Eqs. (7)–(9)) (Costantino et al., 2014b).

\[
UCL_{ip,t}^i = CL_{ip,t}^i + 3\sigma_{ip,t}^i
\]

\[
CL_{ip,t}^i = L_t ED_{d,t} + SS_t
\]

\[
LCL_{ip,t}^i = CL_{ip,t}^i - 3\sigma_{ip,t}^i
\]

The centerline of the inventory position control chart \((CL_{ip,t}^i)\) is dynamically updated in each time period based on the expected demand with the demand control chart. It is calculated by multiplying \(ED_{d,t}^i\) by the delivery lead-time \((\bar{L}_t)\), and the product is added to a safety stock component \(SS_t\). Following the literature, we extend the lead-time with \(K_t\) to account for the safety stock as follows:

\[
CL_{ip,t}^i = (\bar{L}_t + K_t) ED_{d,t}^i
\]

\(K_t\) is the safety stock component and \(SS_t\) is the safety stock level. In this study, we set \(\delta_{ip,t}^i = \delta_{d,t}^i\) and for simplicity we set \(\delta_{ip,t}^i = \delta_{d,t}^i\) (Costantino et al., 2014a, 2014b).

The inventory position decision rules depend only on the last observation of inventory position \((IP_{t}^i)\) which combines the net inventory \((N_{t}^i)\) and supply line inventory \((SL_{t}^i)\). Instead of relying on a single point for the target inventory position as in (R, S), we define a dynamic target smoothing zone (range); and thus, if \(IP_{t}^i\) is within this range, then, there is no need for inventory adjustment/otherwise, inventory adjustment/balance should be determined. By controlling this range, order smoothing can be realized and thus bullwhip effect can be controlled. The decision rules are summarized as follows.

**Inventory Rule 1:** At echelon \(i\), if the last observation on the inventory position control chart, \(IP_{t}^i\), is above the upper limit of a target smoothing zone \(IP_{t}^i > CL_{ip,t}^i + K_t\delta_{ip,t}^i\), then, a negative inventory balance \((\text{Inve. Balance})\) should be determined as shown in equation (10). The parameter \(Cn\) is the main difference with Costantino et al. (2014a, 2014b) where it represents a smoothing
term to only recover a fraction of the gap between \( IP_i \) and the target smoothing zone and thus order smoothing is enhanced to mitigate the bullwhip effect.

\[
Invb_i^t = \frac{1}{Cn} \left[ CL_{tp} + K_p \sigma_{tp} - IP_i^t \right]
\]  

(10)

Inventory Rule 2: If \( IP_i^t \) is beyond the lower limit of the target smoothing zone, then, a positive inventory balance should be determined as shown in Eq. (11).

\[
Invb_i^t = \frac{1}{Cn} \left[ CL_{tp} - K_p \sigma_{tp} - IP_i^t \right]
\]  

(11)

Inventory Rule 3: If \( IP_i^t \) is within the smoothing zone, then, there is no need for inventory balance, i.e., \( Invb_i^t = 0 \).

Setting \( K_p = 0 \) turns the SPC inventory policy into a generalized order-up-to policy with the smoothing parameter \( Cn \):

\[
Invb_i^t = \left[ CL_{tp} - IP_i^t \right] / Cn
\]

The governing rules of the generalized order-up-to policy is discussed in the next section.

The final order to be placed at the end of time \( t \) equals the sum of the expected demand and the inventory balance determined by the above decision rules as shown in Eq. (12).

\[
O_i^t = \max \left\{ ED_i^t + Invb_i^t, 0 \right\}
\]  

(12)

4. Supply chain modeling

4.1. Supply chain structure and assumptions

To evaluate SPC, we utilize a four-echelon supply chain consisting of a customer, a retailer, a wholesaler, a distributor, a factory, and an external supplier (see Fig. 3). This structure is known as the Beer Game model and has been utilized in many similar investigations (Costantino et al., 2013a, 2014d). Fig. 3 depicts a representation of the supply chain model in which the retailer receives the customer demand and places orders with the wholesaler, and so on up the supply chain to the factory, which places its orders with an external supplier.

The supply chain is modeled with the following assumptions that are common in the relevant literature (Ciancimino et al., 2012; Costantino et al., 2013a; Costantino et al., 2014d; Wright & Yuan, 2008):

- The factory and the supplier have unlimited capacity.
- The stocking capacity at the different supply chain echelons is unlimited.
- The unfulfilled orders due to out of stock situations at either echelon will be backlogged and to be satisfied as soon as inventory becomes available.
- The transportation capacity between the adjacent echelons is unlimited.
- The ordering lead-time \( \left( L_o^t \right) \) and the delivery lead-time \( \left( L_d^t \right) \) are assumed to be deterministic and fixed across the supply chain.
- The non-negativity condition is considered for the demand and replenishment orders.

The state variables at each echelon \( i \) are updated in each period \( t \) based on the following sequence of steps:

Step 1: Receive the amount of shipment \( SR_{t-1}^{i-1} \) released by echelon \( i + 1 \) at time \( t - L_o^t \);

Step 2: Update the amount in supply line:

\[ SL_i^t = SL_{i-1}^t + O_{i-1}^t - SR_{t-1}^{i-1}, \]

where \( SL_{i-1}^t \) and \( O_{i-1}^t \) stand for the amount in supply line at time \( t - 1 \) and the order released by echelon \( i \) at time \( t - 1 \), respectively;

Step 3: Update the amount to ship to echelon \( i - 1 \):

\[ SR_i^t = \min \left\{ IO_i^t + B_{i-1}^t + SR_{t-1}^{i-1}, \right\} \]

where \( IO_i^t \) is incoming order, \( B_{i-1}^t \) is initial backlog, and \( SL_{i-1}^t \) is initial inventory level;

Step 4: Update the current inventory level:

\[ L_i^t = L_{i-1}^t + SR_{t-1}^{i-1} - SR_i^t; \]

Step 5: Update the backlog level:

\[ B_i^t = B_{i-1}^t + IO_i^t - SR_i^t; \]
4.2. Generalized order-up-to policy

The order-up-to, known also as (R, S), is common in the literature of supply chain dynamics because of its popularity in practice (Disney & Lambrecht, 2008). A generalized variant of this policy is used here to initiate the SPC inventory system, and to be used as a benchmark policy. In this policy, at the end of each review period (R), an order $O_t$ is placed whenever the inventory position $I^t$ is lower than a specific target level $S_t$ (Chen et al., 2000; Chen & Ryan et al., 2000). The order-up-to policy can be represented mathematically as follows in Eqs. (13)–(15).

\[
O_t = \text{Max}\left\{ \left( S_t - I^t \right), 0 \right\}
\]

\[
S_t = L D^t_i + S S^t_i
\]

\[
ED^t_i = \frac{1}{MA} \sum_{t-M+1}^{t} IO^t_i
\]

The target inventory position $S_t$ is dynamically updated in each time period based on the expected demand over the total lead-time (review period and delivery lead-time). The moving average forecasting technique is employed to estimate the expected demand ($ED^t_i$) motivated by its popularity in research and in practice (Disney & Lambrecht, 2008). We have considered the safety stock that is required to account for demand variation by extending the lead-time by $K_t$ (Costantino et al., 2013a; Dejonckheere et al., 2004) as shown in Eq. (16).

\[
S_t = (L + K_t) ED^t_i
\]

In the order-up-to policy, the order can be divided into: a demand forecast, a net inventory error term and a supply line inventory error term, but both the errors are completely taken into account in the placed order (see Eq. (17)). Sterman (1989) showed that decision makers in the beer game mimicked a generalized order-up-to replenishment rule. Some authors have proposed some ways that can increase the flexibility of (R, S) to allow order smoothing in which the decision maker does not recover the entire deficit between the order-up-to level (OUT) level and the inventory position in one time period. Eq. (18) represents a generalized order-up-to policy which can allow order smoothing with setting $Tn > 1$ and $Ts > 1$.

\[
O_t = \left( L^d_i + R + K_t \right) ED^t_i - NI^t_i - SL^t_i
\]

\[
O_t = ED^t_i + L^d_i ED^t_i + K_t ED^t_i - NI^t_i - SL^t_i
\]

\[
O_t = ED^t_i + L^d_t ED^t_i - SL^t_i + K_t ED^t_i - NI^t_i
\]

Step 6: Update the net inventory level: $N_i^t = I^t - B^*_i$.

Step 7: Update the inventory position: $I^t = I^t_i + SL^t_i - B^*_i$. 

4.3. Performance measurement system

The performance of the supply chain under the ordering policies will be evaluated by quantifying the bullwhip effect, inventory stability and total variance measures.

4.3.1. Bullwhip effect measures

The bullwhip effect expresses the amplification of demand variability across the supply chain and can be quantified by measuring bullwhip effect ratio ($BWE_i$) shown in Eq. (20) (Chatfield et al., 2004). The $BWE_i$ represents the ratio of orders variance at echelon $i$ relative to the demand variance where $BWE_i > 1$ results in bullwhip effect; $BWE_i < 1$ results in order smoothing; and $BWE_i = 1$ results in a “pass-on-orders” (Disney & Lambrecht, 2008).

\[
BWE_i = \frac{\sigma^2_{d}}{\sigma^2_{d} / \mu_d}
\]
The variance ratio ($\text{InvR}_i$) represents the ratio of net inventory variance ($\text{InvR}_i^2$) to the customer demand variance as shown in Eq. (21) (Costantino et al., 2014d; Disney & Towill, 2003).

\[
\text{InvR}_i = \frac{\sigma_{\text{Inv}}^2}{\sigma_{\text{D}}^2}
\]  

The average service level (ASL) quantifies the percentage of items delivered immediately by echelon $i$ to satisfy the order of echelon $i - 1$ (Zipkin, 2000). Service level (fill rate, $\text{Slt}_i$) is computed over the effective delivery time (i.e., $\text{IO}_i^e > 0$) as shown in Eq. (22).

\[
\text{ASL}_i = \frac{\text{Slt}_i}{\text{IO}_i^e}
\]  

### Table 2

<table>
<thead>
<tr>
<th>$T_n$</th>
<th>Disney et al. (2006)</th>
<th>SPC with $K_{ip} = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\text{BWE}$</td>
<td>$\text{InvR}$</td>
</tr>
<tr>
<td>0.6</td>
<td>5</td>
<td>3.8</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>1.61803</td>
<td>0.4472</td>
<td>3.1708</td>
</tr>
<tr>
<td>2</td>
<td>0.3333</td>
<td>3.3333</td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
<td>3.8</td>
</tr>
<tr>
<td>4</td>
<td>0.1429</td>
<td>4.2857</td>
</tr>
<tr>
<td>6</td>
<td>0.0909</td>
<td>5.2727</td>
</tr>
</tbody>
</table>

### Fig. 5

The ordering and inventory behavior under ($R, S$) and SPC when $\rho = 0$.

### Fig. 6

The effect of $K_{ip}$ and $K_{id}$ on the bullwhip effect.

---

1. (a) Bullwhip effect ratio ($\rho = 0$)
2. (b) Bullwhip effect ratio ($\rho = 0.3$)
\[ \text{TSV}_i = \left\{ \begin{array}{ll} \frac{\text{SR}_i^t - B_{i-1}^t}{\text{B}_{i-1}^t} \times 100 & \text{if } \text{SR}_i^t - B_{i-1}^t > 0 \\ 0 & \text{if } \text{SR}_i^t - B_{i-1}^t \leq 0 \end{array} \right. \]  

(22)

4.3.3. Total variance measures

The bullwhip effect increases the upstream echelons' costs and the inventory variance ratio increases the local inventory costs. Thus, it is worth estimating Total Stage Variance (TSV) as BWE plus InvR, (see, Eq. (23)) considering both factors equally important at

\[ \text{ASL}_i = \sum_{j=1}^{n} \frac{\text{TSV}_j}{\text{InvR}_j} \]
each echelon (Disney & Lambrecht, 2008). Similarly, the Total Supply Chain Variance (TSCV) can be estimated as shown in Eq. (24) where \( m \) stands for the number of echelons.

\[
TSCV = \sum_{i=1}^{m} TSV_i
\]

4.4. Simulation modeling and validation

4.4.1. Simulation set-up

A simulation model has been built considering the above supply chain model, using SIMUL8. To conduct the simulation experiments, the simulation model is run for 10 replications of 2400 periods each (Costantino et al., 2014a, 2014b). Each simulation run consists of four stages (see, Fig. 4), the first stage is a warm-up per-
iod for the generalized order-up-to policy, and the second stage is the effective simulation run, then, another warm-period for the SPC policy followed by an effective simulation. Both warm-up periods have the same length of 200 periods, and both effective simulation runs have the same length of 1000 periods. This simulation set-up is fixed for all the following experiments unless something else is mentioned.

4.4.2. Simulation model validation

To validate the simulation model, we compare our simulation results with the closed form expressions obtained by Disney et al. (2006) for bullwhip effect (see, Eq. (25)) and inventory variance ratio (see, Eq. (26)) under different smoothing levels. These expressions have been derived by Disney et al. (2006) for a single echelon supply chain when the demand is normally distributed and the forecasting is based on the mean demand \( \mu_d \), and the operational parameters are set to: \( L_d = 2 \), and \( K_i = 0 \). The SPC parameters are selected to behave as an \((R, S)\): \( CL_{at} = \mu_d = 30 \), \( \sigma_d = 3 \), \( K_{ip} = 0 \) and \( ED_{it} = CL_{at} \). We evaluate the bullwhip effect and inventory variance ratios under different values of \( T_n \).

The simulation model was adjusted to a single-echelon model by setting a large initial inventory level at the wholesaler (assuming unlimited inventory).

Table 2 shows that the simulation model is valid. The results also confirm the trade-off between the bullwhip effect and inventory variance ratio where increasing the smoothing level leads to dampening the bullwhip effect whilst increasing the inventory instability. The minimum total stage variance is realized when \( T_n = 1.61803 \) which represents the golden section value (see bold values in Table 2), calculated by optimizing this measure with regards to \( T_n \) (Disney et al., 2006).

5. Evaluating the SPC in a multi-echelon supply chain

The SPC is initially evaluated and compared with the generalized \((R, S)\) policy in the four-echelon supply chain model considering an autoregressive demand process.

![Fig. 10. A comparison between SPC and smoothing (R, S) in terms of BWE.](image)

![Fig. 11. A comparison between SPC and smoothing (R, S) in terms of InvR and ASL.](image)
5.1. Autoregressive demand model

Lee et al. (2000) reported that the AR(1) demand process was found to match the sales patterns of 150 SKUs in a supermarket. The AR demand observations can be generated from the demand generator in Eq. (27) (Disney & Grubbström, 2004; Hussain, Shome, & Lee, 2012):

\[ D_{t+1} = \mu_d + \epsilon_1 + \rho (D_t - \mu_d) \]

where \( \mu_d \) represents the mean of the demand process, which should be set relatively high to \( 4\sigma_d \), \( \epsilon_1 \) represents white noise of normal distribution with \( \mu = 0 \) and \( \sigma^2 \), \( \rho \) is an autoregressive coefficient, where \( -1 < \rho < 1 \), and \( D_{t+1} \) is the AR demand at time \( t \). The AR demand variance equals \( \sigma^2 = \sigma^2 / (1 - \rho^2) \), where \( \sigma^2 = \sigma^2 \) when the demand is identically and independently distributed (i.i.d), i.e., \( \rho = 0 \). For \( -1 < \rho < 0 \), the process is negatively correlated and exhibits period-to-period oscillatory behavior. For \( 0 < \rho < 1 \), the demand process is positively correlated which is reflected by a meandering sequence of observations (Disney & Grubbström, 2004).

5.2. Comparing SPC with traditional (R, S) policy

We first compare the SPC with the traditional (R, S) policy under different settings of the policies’ forecasting and inventory control parameters. The sensitivity of moving average to demand changes can be controlled through only changing MA while the sensitivity of the SPC forecasting mechanism can be controlled through \( MC, K_d, W, q_i, N_t \). The forecasting parameter \( K_d \) can be used for forecast smoothing without increasing the average age of data (MC) as in the moving average. By doing that, SPC can be protected from frequent reaction to demand noise that drives the bullwhip effect, without affecting its responsiveness to serious demand changes. Therefore, the SPC is evaluated under two levels of \( K_d \) (\( K_d = 1 \) and \( K_d = 2 \)) combined with different values of \( K_{t} \) (inventory smoothing parameter), and compared with (R, S) under two levels of MA (\( MA = MC = 15 \) and \( MA = W = 50 \)). We repeat these comparisons under two different values of \( \rho \) with considering

![Figure 12](image-url) A comparison between SPC and smoothing (R, S) in terms of TSV and TSCV.

### Table 4

Forecasting under TSCM and IESCM.

<table>
<thead>
<tr>
<th>Forecasting method</th>
<th>Scenario #1 (TSCM)</th>
<th>Scenario #2 (IESCM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moving average</td>
<td>( ED_t = \frac{1}{MMA} \sum_{i=E_{t-MMA}}^{i=E_{t-1}} IO_i )</td>
<td>( ED_t = \frac{1}{MMA} \sum_{i=E_{t-MMA}}^{i=E_{t-1}} IO_i )</td>
</tr>
<tr>
<td>SPC Forecasting system</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Demand control chart</td>
<td>( UCL_{d,t} = CL_{d,t} + K_d\sigma_{d,t} )</td>
<td>( UCL_{d,t} = CL_{d,t} + K_d\sigma_{d,t} )</td>
</tr>
<tr>
<td></td>
<td>( CL_{d,t} = \frac{1}{W} \sum_{i=E_{t-W}}^{i=E_{t-1}} IO_i )</td>
<td>( CL_{d,t} = \frac{1}{W} \sum_{i=E_{t-W}}^{i=E_{t-1}} IO_i )</td>
</tr>
<tr>
<td></td>
<td>( LCL_{d,t} = CL_{d,t} - K_d\sigma_{d,t} )</td>
<td>( LCL_{d,t} = CL_{d,t} - K_d\sigma_{d,t} )</td>
</tr>
<tr>
<td>Demand decision rules (1–3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( ED_t = \max \left{ \frac{1}{MMA} \sum_{i=E_{t-MMA}}^{i=E_{t-1}} IO_i, CL_{d,t} \right} )</td>
<td>( ED_t = \max \left{ \frac{1}{MMA} \sum_{i=E_{t-MMA}}^{i=E_{t-1}} IO_i, CL_{d,t} \right} )</td>
<td></td>
</tr>
<tr>
<td>( ED_t = \min \left{ \frac{1}{MMA} \sum_{i=E_{t-MMA}}^{i=E_{t-1}} IO_i, CL_{d,t} \right} )</td>
<td>( ED_t = \min \left{ \frac{1}{MMA} \sum_{i=E_{t-MMA}}^{i=E_{t-1}} IO_i, CL_{d,t} \right} )</td>
<td></td>
</tr>
<tr>
<td>( ED_t = CL_{d,t} + \frac{1}{W} \sum_{i=E_{t-W}}^{i=E_{t-1}} IO_i )</td>
<td>( ED_t = CL_{d,t} + \frac{1}{W} \sum_{i=E_{t-W}}^{i=E_{t-1}} IO_i )</td>
<td></td>
</tr>
</tbody>
</table>
the following parameters settings: $C_n = T_n = 1$, $W_i = 50$, $MC = 15$, $K_i = 1$ and $L_d = 2$, fixed for the comparison.

The results of a single simulation run exhibiting the variation of order rate and inventory over time when shifting from (R, S) to SPC are presented in Fig. 5. The results show mainly the impact of the forecasting of each policy on the ordering and inventory stability at the factory which is the most exposed partner to the variability amplification (Chatfield et al., 2004). The SPC shows a smoother ordering and inventory variation than (R, S) with moving average.

### 5.2.1. Bullwhip effect analysis

The bullwhip effect results presented in Fig. 6 show that SPC is superior to (R, S), under the different forecasting settings. For i.i.d demand, the bullwhip effect ratio increases from 1.70 at the retailer to 12.05 at the factory under (R, S) with $MA = 15$ while increases from 1.21 at the retailer to 2.20 at the factory under SPC with $K_d = 1$ and $K_{ip} = 0$. This proves the effectiveness of the SPC forecasting mechanism in comparison to the moving average (see also, Fig. 5). However, when $MA = 50$ and $K_d = 2$, both policies generate

### Table 5

<table>
<thead>
<tr>
<th>Performance measure</th>
<th>Ordering policy</th>
<th>$T_n = C_n = 1$</th>
<th>$T_n = C_n = 1.618$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TSCM</td>
<td>SPC</td>
<td>TSCM</td>
</tr>
<tr>
<td></td>
<td>(R, S)</td>
<td>(R, S)</td>
<td>(R, S)</td>
</tr>
<tr>
<td>BWE</td>
<td>Retailer</td>
<td>1.20</td>
<td>1.20</td>
</tr>
<tr>
<td></td>
<td>Wholesaler</td>
<td>1.44</td>
<td>1.23</td>
</tr>
<tr>
<td></td>
<td>Distributor</td>
<td>1.74</td>
<td>1.24</td>
</tr>
<tr>
<td></td>
<td>Factory</td>
<td>2.13</td>
<td>1.25</td>
</tr>
<tr>
<td>InvR</td>
<td>Retailer</td>
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<td>3.23</td>
</tr>
<tr>
<td></td>
<td>Wholesaler</td>
<td>3.85</td>
<td>3.60</td>
</tr>
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<td></td>
<td>Distributor</td>
<td>4.62</td>
<td>3.83</td>
</tr>
<tr>
<td></td>
<td>Factory</td>
<td>5.60</td>
<td>3.91</td>
</tr>
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<td>ASL</td>
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<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td></td>
<td>Wholesaler</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td></td>
<td>Distributor</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td></td>
<td>Factory</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>TSV</td>
<td>Retailer</td>
<td>4.43</td>
<td>4.43</td>
</tr>
<tr>
<td></td>
<td>Wholesaler</td>
<td>5.29</td>
<td>4.83</td>
</tr>
<tr>
<td></td>
<td>Distributor</td>
<td>6.36</td>
<td>5.08</td>
</tr>
<tr>
<td></td>
<td>Factory</td>
<td>7.73</td>
<td>5.16</td>
</tr>
<tr>
<td>TSCV</td>
<td>23.81</td>
<td>19.49</td>
<td>13.98</td>
</tr>
</tbody>
</table>

### Table 6

<table>
<thead>
<tr>
<th>Performance measure</th>
<th>Ordering policy/lead-time</th>
<th>$T_n = C_n = 1$</th>
<th>$T_n = C_n = 1.618$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$L_d = 1$</td>
<td>$L_d = 3$</td>
<td>$L_d = 6$</td>
</tr>
<tr>
<td>BWE</td>
<td>Retailer</td>
<td>1.15</td>
<td>1.24</td>
</tr>
<tr>
<td></td>
<td>Wholesaler</td>
<td>1.32</td>
<td>1.55</td>
</tr>
<tr>
<td></td>
<td>Distributor</td>
<td>1.53</td>
<td>1.97</td>
</tr>
<tr>
<td></td>
<td>Factory</td>
<td>1.79</td>
<td>2.51</td>
</tr>
<tr>
<td>InvR</td>
<td>Retailer</td>
<td>2.14</td>
<td>4.34</td>
</tr>
<tr>
<td></td>
<td>Wholesaler</td>
<td>2.44</td>
<td>5.49</td>
</tr>
<tr>
<td></td>
<td>Distributor</td>
<td>2.81</td>
<td>6.79</td>
</tr>
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<td></td>
<td>Factory</td>
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<td>8.64</td>
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<td>ASL</td>
<td>Retailer</td>
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<td>100.0</td>
</tr>
<tr>
<td></td>
<td>Wholesaler</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td></td>
<td>Distributor</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td></td>
<td>Factory</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>TSV</td>
<td>Retailer</td>
<td>3.29</td>
<td>5.58</td>
</tr>
<tr>
<td></td>
<td>Wholesaler</td>
<td>3.76</td>
<td>6.95</td>
</tr>
<tr>
<td></td>
<td>Distributor</td>
<td>4.34</td>
<td>8.75</td>
</tr>
<tr>
<td></td>
<td>Factory</td>
<td>5.04</td>
<td>11.16</td>
</tr>
<tr>
<td>TSCV</td>
<td>16.43</td>
<td>32.44</td>
<td>70.87</td>
</tr>
</tbody>
</table>
the same bullwhip effect. The bullwhip effect is still present when 0 = 0 (BWE > 1) but as 0 increases the bullwhip effect is reduced and eliminated (BWE < 1) regardless of the value of 0. For example, increasing 0 from 0 to 1 decreases the bullwhip effect, from 1.20 to 0.45 at the retailer and from 2.13 to 0.39 at the factory. The same conclusions can be inferred when 0 = 0.3. Thus, the SPC can eliminate the bullwhip effect and achieve ordering stability especially at the most upstream echelons.

5.2.2. Inventory variance ratio measures

The inventory variance ratio results are presented in Fig. 7 and show again that SPC is superior to the (R, S). The SPC shows a lower inventory variance ratio across the supply chain compared to 0 = 1. It can be further seen that increasing 0 leads to reduce the inventory variance ratio across the supply chain. For i.i.d demand, the inventory variance ratio increases from 3.23 at the retailer to 5.60 at the factory under SPC with 0 = 0 and 0 = 2 while increases from 3.36 at the retailer to 3.74 at the factory under SPC with 0 = 0.5 and 0 = 2. This increase in inventory variance (with 4% at the retailer) is coupled with 46% decrease in the retailer’s bullwhip effect (see, Fig. 6).

The average service level, although not presented here, has also been obtained under the different ordering policies for 0 = 0 and 0 = 0.3. Both policies are successful to achieve acceptable service level across the supply chain (~100%). However, the (R, S) with MA = 15 shows a lower service level at the upstream echelons compared to SPC (with 0 = 1) which indicates the effectiveness of the forecasting mechanism of SPC.

5.2.3. Systemic measures

The total stage variance (TSVi) results are presented in Fig. 8 showing that SPC outperforms (R, S) regardless of the value of 0, except at the retailer that has higher TSVi under SPC with 0 = 2 and 0 > 0.25 in comparison to (R, S) with MA = 50 when 0 = 0.3. However, by inspecting the total supply chain variance (TSCV) presented in Fig. 9, it can be found that SPC is superior compared to (R, S) regardless of the value of 0.

The analysis of the above results reveals that SPC is superior to the standard (R, S) policy in terms of bullwhip effect, inventory performance, and total variance measures. Thus, the SPC can achieve a higher ordering and inventory stability in the supply chain compared to (R, S). Furthermore, the forecasting mechanism of SPC has shown superior performance to moving average and therefore it could be employed with (R, S) instead of the moving average method.

5.3. The sensitivity of the SPC to the demand parameter

To further understand the behavior of SPC, we evaluate SPC under different values of 0 ranges between -0.6 and 0.6 (with 0 = 30, 0 = 3). We consider the following settings based on the results of the previous section: 0 = 2, MA = 50, 0 = 1, Wi = 50, Tn = Cn = 1, MC = 15, 0 = 0.5 and 0 = 2. The results are summarized in Table 3 and show that SPC outperforms (R, S) in terms of all performance measures, across all values of 0. However, the InvRi and TSVi, at the retailer becomes higher under SPC in comparison to (R, S) at higher levels of 0. This can be attributed to the effect of order smoothing with 0 > 0, therefore, when the inventory cost of the downstream echelons is significant and the demand is highly positively correlated, this problem can be solved by setting 0 = 0 so that SPC is turned into traditional (R, S) but integrated with the improved forecasting mechanism of SPC (see also, Fig. 7(a) and (b)). In general, the interaction between the autoregressive parameter and the smoothing level require further investigation that is out of the scope of this study.

5.4. Comparing SPC with smoothing (R, S) policy

The above comparisons have been conducted between SPC and standard (R, S) which is commonly used in practice. Several authors have attempted to increase the flexibility of (R, S) ordering rule to allow order smoothing (as we explained above) and thus we attempt in this section to compare SPC with optimized smoothing (R, S) under a normal demand with 0 = 30 and 0 = 30 which is suitable for the optimized smoothing (R, S) policy (Disney et al., 2006). In particular, the SPC is evaluated under two levels of 0 ( 0 = 1 and 0 = 2) combined with various levels of 0 and compared with the (R, S) under two smoothing levels (Tn = 1 and Tn = 1.61803) and combined with two levels of MA (MA = 15 and MA = 50). The parameters of the smoothing (R, S) in the second comparison are set to achieve optimum performance in terms of total stage variance by selecting a large value for MA and recommended value for Tn: Tn = 1.61803 and MA = 50 (see, Table 2). The other parameters are set to: Wi = 50 and MC = 15, and 0 = 1 and 0 = 2.

The BWE results in Fig. 10 show that both replenishment policies are successful to eliminate the bullwhip effect across the supply chain as the smoothing level increases where (R, S) has a single smoothing parameter (Tn) and SPC has two dimensions for order smoothing (Cn and 0). It can be further seen that the smoothing (R, S) with MA = 15 shows BWE > 1 at the upstream echelons that can be improved with larger MA proving the importance of selecting the proper forecasting parameter to improve supply chain performances. This performance proves again the effectiveness of the SPC forecasting mechanism compared to the moving average where SPC is successful to eliminate the bullwhip effect under the different values of 0 even when 0 = 0 and Cn = Tn = 1.61803, and it can achieve higher order smoothing by increasing 0; however, this may have a significant impact on InvRi, ASLi, and TSVi at
the retailer (see, Figs. 11 and 12). Although the retailer is influenced by increasing the smoothing level, the supply chain stability will be improved as shown in Fig. 12(b). Beside proving that SPC with its two smoothing dimensions is comparable to the optimized smoothing (R, S), the results further confirm the findings in the literature regarding the value of smoothing replenishment rules in supply chains (Ciancimino et al., 2012; Dejonckheere et al., 2004; Dominguez et al., 2014).

5.5. Evaluating SPC in information-enriched supply chain

Previous research has shown that although information sharing can reduce the bullwhip effect, smoothing replenishment rules are also needed in such configurations to reduce/eliminate the bullwhip effect especially at the upstream echelons (Dejonckheere et al., 2004; Dominguez et al., 2014). We compare SPC with (R, S) under two information sharing scenarios:

Fig. 14. A comparison between SPC and (R, S) in terms of bullwhip effect under seasonal demand.
1. Traditional supply chain model (TSCM), used for the above analysis, in which the retailer is the only partner that has access to the customer demand \( (I_0^{ret} = D_t) \) while the other partners receive the downstream echelon's orders \( (I_0^i) \).

2. Information-enriched supply chain model (IESCM) in which all partners have access to the customer demand in the real-time (Dejonckheere et al., 2004). The shared demand \( (I_0^{ret}) \) will be used in both \((R, S)\) and SPC to make the forecasting at each echelon instead of the incoming orders as indicated in Table 4.

---

**Fig. 15.** A comparison between SPC and \((R, S)\) in terms of inventory variance ratio under seasonal demand.
In this evaluation, each information sharing scenario is evaluated under two smoothing levels: $Tn = Cn = 1$ and $Tn = Cn = 1.618$, with the following settings: $L_i^1 = 2$, $MA = 50$, $K_i = 1$, $Wi = 50$, $MC = 15$. $K_i^p = 0.5$ and $K_i^p = 2$. The customer demand follows the normal distribution with $\mu_d = 30$ and $\sigma_d = 30$. The results in Table 5 show that all performance measures are improved under both policies when shifting from TSCM to IESCM. In all cases, the SPC is superior to (R, S) confirming that SPC is superior in both TSCM and IESCM. The results also confirm the findings in the related literature that smoothing replenishment rules are essential to improve ordering and inventory stability even in IESCM (Dominguez et al., 2014).

5.6. The sensitivity of the SPC policy to lead-time

The interaction of ordering policies with lead-time influences considerably the supply chain performances (Chen et al., 2000; Ciancimino et al., 2012; Lee et al., 1997a, 1997b). We investigate the sensitivity of the SPC policy to the lead-time in comparison to the generalized (R, S) under two smoothing levels. We assume a normal demand with $\mu_d = 30$ and $\sigma_d = 30$, and considering these settings: $MA = 50$, $K_i = 1$, $Wi = 50$, $MC = 15$, $K_i^p = 0.5$ and $K_i^p = 2$.

The results in Table 6 show that both SPC and (R, S) are sensitive to lead-time. It can be observed that longer lead-times results in higher $BWE_i$, higher $InvR_i$ and lower $ASL_i$ across the supply chain, influencing considerably the upstream echelons. However, the SPC shows a lower sensitivity to lead-time than (R, S) and this sensitivity decreases as the smoothing level increases as can also be concluded from the total supply chain variance. Therefore, SPC is recommended for supply chains with longer lead-times. In general, smoothing replenishment rules are recommended for supply chains with longer lead-times.

6. Further analysis

The above results have been obtained under normal and autoregressive demand. In this section, we evaluate the SPC under a seasonal demand that can be generated with the formula in Eq. (28) which consists of constant demand ($base$), trend component ($slope$), seasonal components ($season$ and $SeasonCycle$), and noise component ($\sigma_c$) (Zhao & Xie, 2002).

$$D_t = base + slope \times t + season \times \sin \left( \frac{2\pi \times season \times t}{SeasonCycle} \right) + N(0, \sigma^2)$$  \hspace{1cm} (28)

We compare the SPC with the traditional (R, S) under $season = 5$ and $season = 10$ combined with $SeasonCycle = 7$ and 14 with $base = 30$ and $\sigma_c = 3$. The SPC is evaluated under two levels of $K_i^p$ each combined with different levels of $K_i^p$, and compared with (R, S) under two levels of $MA$. The other parameters are set to: $L_i^1 = 2$, $K_i = 1$, $Wi = 30$, $Tn = Cn = 1$, and $MC = 15$. The results of a single simulation run (under specific settings) in terms of order rate and inventory level variation are presented in Fig. 13 showing higher stability under SPC.

The bullwhip effect and inventory variance ratio under the different ordering policies are presented in Figs. 14 and 15, respectively. The results show that SPC is superior to (R, S) and that both policies produce lower variability amplification when their sensitivity to demand changes is decreased. Again, increasing $K_i^p$ leads to lower $BWE_i$, lower $InvR_i$, and improved $ASL_i$ across the supply chain (Table 7) especially when the seasonal cycle is short. For longer seasonal cycles, the $InvR_i$ at the downstream echelons is higher under the SPC compared to the (R, S) or SPC with $K_i^p = 0$. The total supply chain variance presented in Fig. 16 shows the superiority of SPC.

6.1. The impact of information sharing and smoothing

In this section, the SPC is compared to (R, S) under two smoothing levels with and without information sharing. The demand is generated with $base = 30$, $\sigma_c = 3$, $season = 10$ and $SeasonCycle = 14$. The other parameters are set to: $L_i^1 = 2$, $MC = 30$, $K_i = 1$, $Wi = 30$, $Tn = Cn$, $MC = 15$, $K_i^p = 2$ and $K_i^p = 1$. The results in Table 8 show again that the performances of both SPC and (R, S) are improved considerably in IESCM. The SPC is superior to (R, S) in IESCM under different smoothing levels. In general, the results confirm that order smoothing through either SPC or (R, S) has a significant value in seasonal supply chains. This is a new
7. Discussion and implications

Demand signal processing which encompasses inventory replenishment policies integrated with forecasting contribute significantly to the bullwhip effect and inventory instability in supply chains. Smoothing replenishment rules have been adopted as a mitigation/avoidance approach for the bullwhip effect. The available smoothing replenishment rules are mainly modified from the periodic review (R, S) by incorporating smoothing parameters to regulate the reaction to demand changes. This study has formulated and extensively evaluated a novel real-time inventory replenishment system that relies on two control charts for demand forecast and inventory control whilst providing the capability of order smoothing. The first control chart acts as a forecasting mechanism to estimate the expected demand without over/under-reacting to demand changes. The second control chart is employed to adjust the inventory position and to control order smoothing through restricting the reaction to and recovery of inventory position variation. The extensive evaluation in this study helped us to understand the dynamic behavior of the SPC in multi-echelon supply chains, and

Fig. 16. A comparison between SPC and (R, S) in terms of TSCV under seasonal demand.
to compare its performance to the widely applied \((R, S)\) ordering policy under various operational settings. The results have shown that the SPC outperforms the periodic review \((R, S)\) and is comparable to the smoothing \((R, S)\) in terms of bullwhip effect, inventory variance ratio, average service level, total stage variance, and total supply chain variance. The SPC forecasting mechanism has also shown a superior performance to the moving average method and thus it can be employed with other ordering policies. The results obtained from the extensive evaluation in this research have also provided further important managerial implications.

It has been proven that increasing the smoothing level leads to improve the operational performance of supply chains but it may increase the inventory variance at the downstream echelon. This trade-off can be solved by providing incentive mechanism to share the benefits with the downstream echelons in order to encourage them to use smoothing ordering policies like SPC since the operational stability starts at the retailer as the above results have shown. The results confirm that order smoothing has a significant value in seasonal supply chains and when the demand is autoregressive. However, further investigations are needed to understand the interaction between the different demand parameters and smoothing level.

The results have confirmed the findings in literature that forecasting has a significant contribution to the supply chain stability (Dejonckheere et al., 2004; Lee et al., 1997a, 1997b). It has been found that improved forecasting can mitigate the bullwhip effect while bullwhip effect avoidance requires the integration of improved forecasting with smoothing ordering policy.

The results have confirmed that longer lead-time increases the variability amplification in the supply chain. A possible remedy is to reduce the lead-time but this is not always convenient in real applications. It has been found that increasing the smoothing level reduces the impact of the lead-time on the supply chain stability. The SPC with its two smoothing dimensions and improved forecasting mechanism has shown a lower sensitivity to the lead-time compared to \((R, S)\) with moving average under different smoothing levels.

The results have also shown that information sharing leads to improve the ordering and inventory stability in the supply chain regardless of the demand process. However, in order to eliminate the bullwhip effect, smoothing replenishment policies should be applied in such collaborative environments. The results have shown the supply chain performances are improved to a great extent under SPC in IESCM compared to the smoothing \((R, S)\).

Finally, the work mainly contributes to the development of ordering policies for improving supply chain performances. Specifically, a novel ordering policy has been developed that can allow order smoothing showing a significant value on the ordering and inventory stability in supply chains. The work further contributes to understand the impact of different operational factors such as demand process, lead-time, collaboration level and smoothing level on the bullwhip effect and inventory stability under different ordering policies.

### 8. Conclusions

This paper has formulated and evaluated a real-time inventory replenishment system with smoothing capability, denoted as SPC, that relies on a control chart approach to be used in dynamic and complex environments like multi-echelon supply chains. The SPC is an alternative smoothing inventory replenishment system in which two control charts are integrated for estimating expected demand and controlling the inventory position, respectively. The demand control chart is employed to work as a forecasting mechanism to estimate the expected demand and the inventory position control chart is employed to adjust the inventory position with providing the capability of order smoothing. The SPC has a flexible structure since it has two dimensions for order smoothing and can also be turned into a generalized \((R, S)\) policy.

The extensive evaluation of SPC has confirmed that SPC can improve the operational performance and customer service level in the supply chain, showing a superior performance to the generalized \((R, S)\), across a wide range of both the autoregressive demand parameter and the seasonal demand parameters. The forecasting mechanism integrated with SPC has shown a superior performance to the moving average integrated with \((R, S)\) and therefore it can be employed with other ordering policies to regu-
late the reaction to demand changes. Although both replenishment policies have shown a considerable sensitivity to the lead-time, SPC has shown a lower sensitivity to the lead-time than (R, S), and proving that increasing the smoothing level (with either (R, S) or SPC) reduces their sensitivity to the lead-time. It has also been found that the performance of both policies is improved in information enriched supply chains and that the performance of SPC outperforms (R, S) in this collaborative configuration as well. The results show that the bullwhip effect could be reduced through information sharing but applying smoothing replenishment rules is needed to eliminate it. Therefore, information sharing and appropriate smoothing inventory replenishment systems such as SPC helps improving supply chain performances in terms of bullwhip effect and inventory stability.

Future research can consider the application of other advanced control charts such as ARMA and EWMA for the estimation of expected lead-time demand. Furthermore, other combinations of traditional and SPC inventory systems can be developed and evaluated as mitigation solutions for the variability amplification in supply chains. Since the forecasting mechanism integrated with SPC has shown a superior performance to moving average method, it is worth evaluating and comparing it with the common forecasting systems in practice under the (R, S) policy, in terms of bullwhip effect and inventory stability. Finally, the impact of smoothing replenishment policies in seasonal supply chains should be further investigated.

References


