Determination of spatially dependent diffusion parameters in bovine bone using Kalman filter

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Abstract

Although many studies have been made for homogenous constant diffusion, bone is an inhomogeneous material. It has been suggested that bone porosity decreases from the inner boundaries to the outer boundaries of the long bones. The diffusivity of substances in the bone matrix is believed to increase as the bone porosity increases. In this study, an experimental set up is used where bovine bone samples, saturated with potassium chloride (KCl), were put into distilled water and the conductivity of the water was followed. Chloride ions in the bone samples escaped out in the water through diffusion and the increase of the conductivity was measured. A one-dimensional, spatially dependent mathematical model describing the diffusion process is used. The diffusion parameters in the model are determined using a Kalman filter technique. The parameters for spatially dependent at endosteal and periosteal surfaces are found to be \((12.8 \pm 4.7) \times 10^{-11}\) and \((5 \pm 3.5) \times 10^{-11}\) \(\text{m}^2/\text{s}\) respectively. The mathematical model function using the obtained diffusion parameters fits very well with the experimental data with mean square error varies from \(0.06 \times 10^{-6}\) to \(0.18 \times 10^{-6}\) (\(\mu\text{S}/\text{m}\)).

1. Introduction

Diffusion can be defined as the process by which matter is transported from one part of a system to another as a result of random molecular motion (Crank, 1975). The commonly used mathematical representation of the diffusion process that can be used to find the diffusion parameter is the so-called Fick’s law. Based on this mathematical representation, many researchers were able to derive and use models based on diffusion in many different fields (Cayan et al., 2009; Margetis, 2009; Naceri, 2009; Yildirim et al., 2011).

One of the fields where Fick’s laws can be applied is for representing diffusion in blood cell membranes. More specifically, such diffusion involves the diffusion of nutrients and oxygen from high concentrations in the blood vessels to less concentrations in the blood cells. For example, models including diffusion have been presented in studies of bone healing where bone is repaired after fracture (Adam, 2002; Ambard and Swider, 2006; Chou et al., 2013; Gomez-Benito et al., 2005; Sapotnick and Nackenhorst, 2012). Other models have been presented for bone remodeling, in which old bone tissue is replaced by a new one (Adachi et al., 2012; Stadelmann et al., 2009). Stress or strain driven diffusion have been suggested to play an important role in the transportation of substances from the inner surface, endosteal, that covers medullar canal in the long bone to the outer, periosteal, bone surface. A one-dimensional model for strain driven transport of bone nutrients has been presented by Banks-Sills et al. (2011). This study was followed up by another study (Lindberg et al., 2013) where a two-dimensional model describing the stress driven diffusion was presented.

Several studies have investigated the bone structure and the apparent diffusion coefficient using magnetic resonance imaging (Ababneh et al., 2009; Balliu et al., 2009; Capuani, 2013). The latter method is very suitable for obtaining information about the structural properties of the bone. A less complex method for determining the diffusion coefficient has been introduced by Lindberg et al. (2014). They used an experimental set up where the increased conductivity in distilled water due to the diffusing chloride ions escaping from saturated bone samples was measured over time. Further, the diffusion parameter was determined from fitting the parameters in an analytical model according to the experimental data using a Kalman filtering technique. Kalman filters could be used as a recursive method in order to extract unknown parameters from noisy experimental measurements. In previous studies, despite the very complex structure of bone, the diffusion parameter was assumed to be independent of the position in the bone wall. The diffusion of solutes that promote