THREE-DIMENSIONAL ANALYSIS OF SINGLE PILE IN SAND USING DRUCKER-PRAGER MODEL

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Abstract
This paper discusses the load-displacement behavior of axially loaded pile in sandy soil using three dimensional finite element numerical analysis. A three-dimensional finite beam element was used to model the single pile. Three dimensional solid element was used to model the soil. Drucker-Prager model was chosen to simulate the sandy soil. A comparison was made between the results of the finite element analysis and laboratory model test results which are quite close. In addition the effect of sand dilation angle on the load displacement curve using numerical Drucker-Prager model was studied. It was concluded that the dilated angle has great effect on load-displacement curve at ultimate load. Deflections and stresses contours during different stages of pile loading were drawn and discussed.

Introduction
Several attempts have been made during the last decades to explain the behavior of piles in soil. Some of these attempts are based on numerical approaches and others on the analysis of experimental or field tests on piles. The behavior of piled foundation can be summarized in some aspects such as, load-displacement relationship of a single pile and ultimate pile capacity. One of the main numerical methods is the finite element approach. A three dimensional model was chosen to model the pile behavior in sandy soil. Smith (1980), and Chow (1992) summarized the numerical methods used in the analysis of vertically loaded single pile. Dalerci and Del Grosso, (1981) proposed a finite element procedure to simulate the load transfer mechanism between bored pile and soil for cohesion-less soil. The numerical application, carried out on a test case represented by a bored pile in sand, has shown the validity of the procedure within the limits of the hypothesis of linearly elastic soil. To investigate the behavior of the pile near the ultimate load a nonlinear soil model should be introduced. Armaleh and Desai (1987) introduced a simple analytical method based on a one-dimensional finite element idealization, together with a new procedure for finding the nonlinear response of the point resistance in axially loaded single pile in cohesion-less soil. A generalized Ramberg-Osgood model was utilized to simulate nonlinear soil response around the pile shaft and the pile tip resistance response. They concluded that the new method for tip resistance response, provided satisfactory predictions of the load-displacement behavior in cohesion-less soils. Trochanis et al. (1991) studied the role of pile-soil slippage and separation, and the overall nonlinear soil behavior on the response of single pile and pairs of piles. A three-dimensional finite element elasto-plastic model (generalized Drucker-Prager material)

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that includes interface elements to represent slippage and pile-soil separation was used. Numerical results indicate that the interaction can be expected to be much smaller when slippage is taken into consideration than when the pile is assumed to be bonded to the soil. Experimental methods can be useful to formulate some empirical relationships for pile behavior prediction such as those of Meyerhof (1959) and Baligh (1978) for piles in sand. In this study, analysis of one free-standing cap pile embedded in homogeneous soil medium is studied. The pile element was assumed to be three-dimensional beam element. The soil element was assumed to be three dimensional solid element. Drucker-Prager model was used to simulate the nonlinear soil behavior. A comparison was made between finite element results and measured results in the laboratory model made by Baligh (1978) to verify the validation of the numerical model.

Numerical Models

The finite element program ANSYS (1997) is used in this study. Drucker-Prager model was used to simulate the nonlinear elasto-perfectly plastic soil behavior. The main elements of the problem are the single pile and the sandy soil. In establishing the analytical model, following assumptions were considered for each element:

1. Non-Cohesive Soil Model

The 3D Solid-45 element is used for modeling the homogeneous soil. The element is defined by eight nodes having three degrees of freedom at each node; translation in the nodal x, y, and z direction. The element has plasticity, creep, swelling, stress stiffening, large deflection, and large strain capabilities. The geometry, node locations, and the coordinate system for this element are shown in Figure (1). The soil is assumed to be homogeneous isotropic, and elastic–perfectly plastic.

![Figure (1) Soil modeling by ANSYS 5.4, (3-D Structural Solid-45 Element).](image)

The following are the soil parameters used by the model:
Friction angle of sandy soil, \( \phi' = 42^\circ \),
Dilated angle, $\psi = 7^\circ$

Poisson’s ratio for the non-cohesive soil, $\nu_s = 0.35$

Soil modulus of elasticity, $E_s = 70 \text{ N/mm}^2$

Young’s modulus in sandy soil was chosen according to the Egyptian code of soil mechanics and foundation engineering, third part shallow foundation, (1995).

2. Pile Modeling

Pile was modeled as a three dimensional elastic beam which is uniaxial element with tension, compression, torsion, and bending capabilities. The pile is modeled as solid element with six degrees of freedom at each node; translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z axes. Young’s modulus of steel pile material, $E_p = 2.1 \times 10^5 \text{ N/mm}^2$

Pile length, $L = 370 \text{ mm}$, pile diameter, $d = 10 \text{ mm}$, and pile Poisson’s ratio for steel material, $\nu_p = 0.33$, interaction friction angle, $\delta = 20^\circ$. Only loaded pile with a vertical concentrated load was investigated.

3. Pile Soil Interface Element

The interface element contact 52 is used to model pile-soil surfaces. The element has three degrees of freedom at each node translation in the nodal x, y, and z directions. The element may be initially preloaded in the normal direction or it may be given a gap specification. The interface coefficient of friction angle $\delta$ was chosen $20^\circ$. No slip is assumed to occur between the pile and adjacent soil at the beginning of loading. With increasing of loading level, sliding between pile and soil is allowed.

![Figure (2) Contact-52, 3-D point to point pile-soil contact element.](image)

Effect of Dilatancy Angle $\psi$

The elastic perfectly-plastic Drucker-Prager model is a simplification of the Mohr-Coulomb model. The Drucker-Prager model presents the soil behavior in three dimensional-non linear analysis. One of the parameters influencing the behavior of pile-soil-systems is the dilatancy angle, $\psi$. Apart from heavily over-consolidated layers, clayey and silty soils tend to show little dilatancy ($\psi = 0$). On the other hand the dilatancy of sand depends on both, the density and the friction angle, $\phi'$. Bolton (1986) supposed that $\phi_{\text{crit.}}$, is the angle of shearing observed
in a simple shear test on soil loose enough to be in a critical state. For quartz sands, critical state friction angle, $\phi_{\text{crit}}$, equal to $33^\circ$, dilation angle, $\psi$, can be assumed as $\psi = \phi - \phi_{\text{crit}}$. Rowe's (1969) stress-dilatancy relationship for plane shear is $0.8\psi = \phi - \phi_{\text{crit}}$.

In this study and according to Bolton, dilated angle is suggested to be $9^\circ$. Figure (3) shows that the dilation angle $\psi$ has a great effect on load displacement curve in Drucker-Prager model. Load settlement curve was calculated using dilated angle, $\psi$, equal to 3, 5, 6, 7, 8, 9, and 11. The ultimate pile load changes to two and half the value when $\psi$ increased from 7 to 8 for the same settlement. Drucker-Prager model is very sensitive to the dilatant angle at plastic region, while at elastic range the effect of dilatance angle $\psi$ has not the same effect as shown in Figure (3).

A comparison between numerical results calculated using finite element and measured results in the laboratory model made by Baligh (1978) is presented. As shown in Figure (3), dilatance angle $\psi$ equal to 6 or 7 gave values close to those measured at same model dimension for sand-container, soil, and pile. Bolton (1986) suggestion gave high values of ultimate load at failure.

![Figure (3) The effect of dilated angle value on the load-deflection curve.](image)

**Load-Settlement Pile Analysis**

Figure (4) shows the load-settlement relationship and the ultimate bearing capacity of pile in sandy soil using the three dimensional finite element program. In experimental studies made by Baligh (1978) on a small-scale laboratory model, a square container with of 580mm side and 580mm depth, was used to detect the maximum load carried by a single pile. The sand used was yellow siliceous of medium size passing sieve No.10 (2mm). The results of tests to define the soil properties was as follows: $C_u=2.56$, $G_s=2.63$, $\gamma_d=1.72$ t/m$^3$, $\gamma_w=1.8$ t/m$^3$, $R_d=0.668$, $\phi=42^\circ$). Piles were modeled by cylindrical 10mm structural steel rods. The pile embedded length was 294mm. It can be noticed as shown in Figure (4) that three-dimensional non linear analysis presented the load-settlement relationship of single pile and it could predict the ultimate pile bearing capacity.
Deformation Distribution Contours Around the Pile

ANSYS has the ability to calculate pile and soil deformation around the pile and at bottom at different level of loading. Maximum and minimum deflections were computed at each loading levels Contour lines for the soil deformation along the pile are also obtained. Three loading levels 60, 120, and 180 N were chosen to figure out the distribution of deflection. The deflection is concentrated around the pile head as shown in Figures (6, 7, and 8), and decreases around the pile in lateral and vertical direction away of the pile head. Deflection increases as the level of loading increase. The ultimate load is defined as the load which causes a settlement of one tenth of the pile diameter $D_p$, Terzaghi, (1942) and Tomlinson (1980). It is difficult to define displacement corresponding to load near failure, because at that stage small increment of load causes substantial displacement which is difficult to measure. The maximum deflection near the ultimate load was equal to 0.06% as shown in Figure (5). Dilation at pile tip might be due to the dense sand at pile tip as bearing resistance existed. Dilation contour lines G, H, and I at pile tip decreases in lateral direction away of the pile. Maximum dilation is less than 0.01% of pile diameter $D_p$ as shown in Figure (5) which is negligible.

Figure (4) Comparison between calculated and measured load-settlement
(Dilated angle = 6 and 7)

Figure (5) Maximum deformation around the pile.
Deformation Contours

Figure (6) Vertical soil movement contours at pile load 60 N

Figure (7) Vertical soil movement contours at pile load 120 N
Load transfer mechanism through the soil is studied as stress contours around the pile. The shape, size and values of stress contours are degraded with the increasing of loading level. The stress intensity around the pile (friction and/or bearing) increases as the loading level increases as shown in Figure (9). The concentration of friction stress can be noticed around the pile as shown in figure (10), for a normalized load equal to 0.3. For higher loading degree the bearing stress below pile tip joined the friction around the pile as shown in Figure (11). At loading level near the ultimate pile load, at which normalized pile loading, $P/P_u$ equal to 0.85, the soil friction resistance around the pile is exhaustive and the end bearing soil resistance existed and bulb of stress formed lower than pile tip as shown in Figure (12). The bulb of stress concentrated at the end of the soil model, which is assumed double the pile length, for this reason, the bulb of stress extended more than twice the pile length.
Stress Contours

Figure (10) Vertical soil stresses contours at pile load 60 N

Figure (11) Vertical soil stresses contours at pile load 120 N
Conclusions
Based on the results of this paper, the following conclusions are obtained:

- Numerical three-dimensional non linear analysis may simulate the load-settlement relationship of single pile in sand and it could estimate the ultimate pile capacity.
- Drucker-Prager numerical model is sensitive to the dilatance angle at the beginning of plastic behavior, while at the beginning of loading the elastic behavior dilatance angle does not have the same effect.
- Dilatance angle, $\psi$, between 6 to 7 gave good estimation of vertical load-settlement relationship of single pile in sand when using Drucker-Prager numerical model.
- Dilatence angle suggested by Bolton (1986) gave very high values for ultimate pile load values.

REFERENCES