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## Viscosity











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### Newton's law of viscosity.

$$\tau \propto \frac{\delta \theta}{\delta t} \qquad \qquad \tan \delta \theta = \frac{\delta u \, \delta t}{\delta y}$$

In the limit of infinitesimal changes, this becomes a relation between shear strain rate and velocity gradient







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### Newton's law of viscosity.

$$\tau = \mu \frac{\mathrm{d}u}{\mathrm{d}y} \longrightarrow \tau = \frac{F}{A} = \mu \frac{U}{h}$$

This relation was found by Newton through experiment, and is called Newton's law of viscosity.

The proportional constant  $\mu$  is called the viscosity, the coefficient of viscosity or the dynamic viscosity.

In this case, the force per unit area necessary for moving the plate, i.e. the shearing stress (Pa), is proportional to U and inversely proportional to h. Using a proportional constant  $\mu$ , it can be expressed as follows:







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#### Change in viscosity of air and of water under 1 atm.





## **Example 1: Fluid Properties**

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The velocity distribution for the flow of a Newtonian fluid between two wide, parallel plates (see Fig. E1.5) is given by the equation

$$u = u(y)$$
  $\longrightarrow$   $u = \frac{3V}{2} \left[ 1 - \left(\frac{y}{h}\right)^2 \right]$ 

where V is the mean velocity. The fluid has a viscosity of  $0.04 \text{ lb} \cdot \text{s/ft}^2$ . When V = 2 ft/s and h = 0.2 in. determine: (a) the shearing stress acting on the bottom wall, and (b) the shearing stress acting on a plane parallel to the walls and passing through the centerline (midplane).





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### **Example 1: Fluid Properties**

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Along the bottom wall y = -h so that (from Eq. 2)

$$\frac{du}{dy} = \frac{3V}{h}$$

$$\tau_{\text{bottom}} = \mu \left(\frac{3V}{h}\right) = \frac{(0.04 \text{ lb} \cdot \text{s/ft}^2)(3)(2 \text{ ft/s})}{(0.2 \text{ in.})(1 \text{ ft/12 in.})}$$

$$= 14.4 \text{ lb/ft}^2 \text{ (in direction of flow)}$$

(b) Along the midplane where y = 0 it follows from Eq. 2 that

$$\frac{du}{dy} = 0$$

and thus the shearing stress is

$$\tau_{\mathrm{midplane}} = 0$$





### **Example 2: Fluid Properties**

A cylinder 7.5 cm radius and 60 cm in length rotate coaxially inside a fixed cylinder of the same length and 9 cm inner radius as shown in Figure (E1.6). Glycerin  $\mu = 8$  Poise fills the space between to cylinders. A Torque 0.4 N.m is applied to the inner cylinder. After a constant velocity is attended, calculate the following: (a) velocity gradient at the cylinder walls, (b) the velocity rustling and (c) the power dissipated by the fluid resistance.





### **Example 2: Fluid Properties**





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### **Example 2: Fluid Properties**

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The shear stress is found from Eq. (1.13)

$$\tau = \frac{F}{A} = \mu \frac{U}{h} \quad \dots \tag{E1.3}$$

$$Torque = F r = \tau A r = \tau r (2\pi r L) = 2\tau r^2 \pi L .... (E1.4)$$

where L is the cylinder length

then

from Eq. (E1.4)  $0.4(N.m) = 2\tau r^2 \pi \times 60 \times 10^{-2} (m)$  $\tau = \frac{0.1062}{r^2} = \mu \frac{du}{dy}$ 





### **Example 2: Fluid Properties**

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$$\frac{du}{dy}\Big|_{inner wall} = \frac{0.13275}{r^2} = \frac{0.13275}{(7.5 \times 10^{-2})^2} = 23.6$$

$$Ans.(a)$$

$$\frac{du}{dy}\Big|_{outer wall} = \frac{0.13275}{r^2} = \frac{0.13275}{(9 \times 10^{-2})^2} = 16.38$$

$$Ans.(a)$$

From Eq. (E1.4) and where dv = -dr

$$\frac{du}{dy} = -\frac{du}{dr} = -\frac{0.13275}{r^2}$$
  
$$du = -\frac{0.13275}{r^2} dr \qquad (E1.6)$$
  
by integrating Eq. (E1.6):

$$\int_{0}^{u} du = -0.13275 \int_{0.09}^{0.075} \frac{1}{r^{2}} dr$$



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## **Example 2: Fluid Properties**

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Where 
$$u(r) = 0$$
 at  $r=0.09$  m  
Then  
 $u = \left[\frac{0.13275}{r^2}\right]_{0.09}^{0.075} = 29.48 \ m/s$  Ans.(b)

Where 
$$u = \omega r = \frac{2\pi N}{60}r$$
 (N: revolution per minute)  
 $29.48 = \frac{2\pi N}{60}r = \frac{2\pi N}{60}7.5 \times 10^{-2} \rightarrow N = 37.5rpm$ 

*Power* = *Torque*  $\times \omega$  = 0.0021 *HP* 

Ans.(c)





### **Surface tension**

At the interface between a liquid and a gas, or between two immiscible liquids, forces develop in the liquid surface which cause the surface to behave as if it were a "skin" or "membrane" stretched over the fluid mass. Although such a skin is not actually present, this conceptual analogy allows us to explain several commonly observed phenomena. For example, a steel

any line in the surface. The intensity of the molecular attraction per unit length along any line in the surface is called the *surface tension* and is designated by the Greek symbol  $\sigma$  (sigma). For a given liquid the surface tension depends on temperature as well as the other fluid it is in contact with at the interface. The dimensions of surface tension are  $FL^{-1}$  with







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Surface tension







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### **Surface tension**

Forces acting on one-half of a liquid drop.









### Wettability











#### Hydrophobic vapor condensing



#### **Packing in cooling towers**





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Whenever water or alcohol is in direct contact with a glass tube in air under normal temperature,  $\theta \simeq 0$ . In the case of mercury,  $\theta = 130^{\circ}-150^{\circ}$ . In the case where a glass tube is placed in liquid,

for water	h = 30/d
for alcohol	h = 11.6/d
for mercury	h=-10/d





## **Example 3: Fluid Properties**

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Derive an expression for the change in height h in a circular tube of a liquid with surface tension  $\sigma$  and contact angle  $\theta$ , as in the following figure







### **Example 3: Fluid Properties**

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The vertical component of the ring surface-tension force at the interface in the tube must balance the weight of the column of fluid of height h $\pi R\sigma \cos \theta = \pi R^2 \rho g h$ 

Solving for h, we have the desired result



Ans.





## **Example 3: Fluid Properties**

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Thus the capillary height increases inversely with tube radius *R* and is positive if  $\theta < 90^{\circ}$  (wetting liquid) and negative (capillary depression) if  $\theta > 90^{\circ}$ .

Suppose that R = 1 mm. Then the capillary rise for a water-air-glass interface,  $\theta \approx 0^{\circ}$ ,  $\sigma = 0.073$  N/m, and  $\rho = 1000$  kg/m<sup>3</sup> is

$$h = \frac{2(0.073 N/m)(\cos 0^{\circ})}{(1000 kg/m^{3})(9.81m/s^{2})(0.001m)} = 0.015(N.s^{2})/kg = 0.015m = 1.5 cm$$

For a mercury-air-glass interface, with  $\theta = 130^{\circ}_{22} \sigma = 0.48$  N/m, and  $\rho = 113600$ , the capillary rise is

h = -0.46 cm

When a small-diameter tube is used to make pressure measurements, these capillary effects must be corrected for.





#### Compressibility

Stress =  $\frac{F}{A}$ Strain =  $\frac{\Delta L}{L_0}$ 

Bulk modulus of elasticity =  $\frac{\text{Stress}}{\text{Stresn}}$ 

Bulk modulus of elasticity,

$$K = \frac{\Delta p}{\Delta V/V} = -V \frac{\mathrm{d}p}{\mathrm{d}V} = -\frac{(\mathrm{P}_2 - \mathrm{P}_1)}{\frac{\mathrm{V}_2 - \mathrm{V}_1}{\mathrm{V}_1}}$$

**Compressibility**,  $\beta = 1/K$ 







#### Quiz



# Q (1)

- **49**
- 1. What is specific gravity? How is it related to density?
- 2. Define the Reynolds number and explain how it detect the flow field behavior?







#### Homework



### HW (1)

Derive an expression for the capillary height change h for a fluid of surface tension  $\sigma$  and contact angle  $\theta$  between two vertical parallel plates a distance W apart, as in the following figure. What will h be for water at 20°C if W = 0.5 mm?





