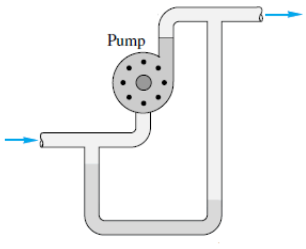
بسم الله الرحمن الرحيم

Mechanical Engineering (2)



Fayoum University

Faculty of Engineering
Mechanical Engineering Dept.

Lecture (10)

on

Thermal Applications

By

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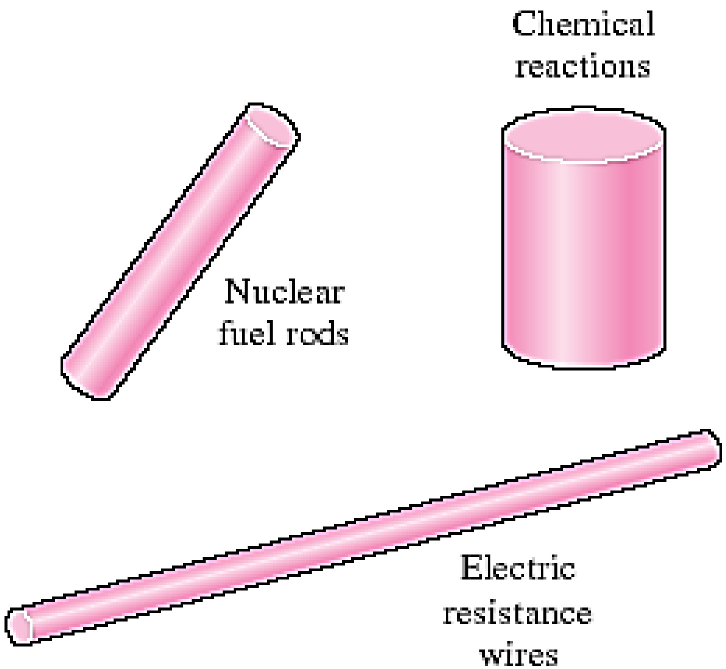
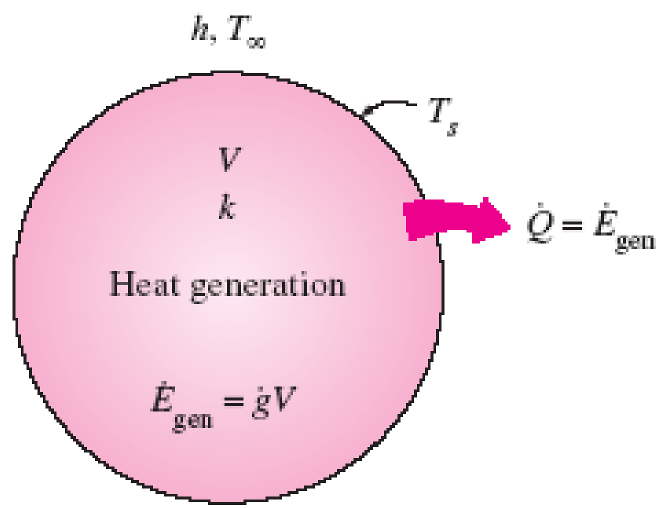
Cooling of Insulators, Machines and Cables

Heat generation in a solid



Heat generation in solids is commonly encountered in practice.



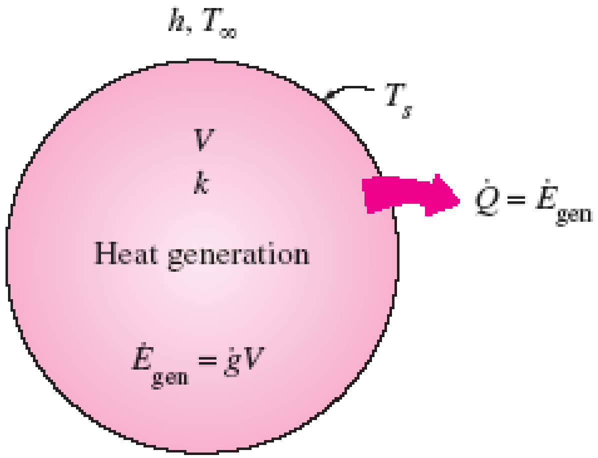
At steady conditions, the entire heat generated in a solid must leave the solid through its outer surface.

Cooling of Insulators, Machines and Cables

Heat generation in a solid

Heat generation is usually expressed per unit volume of the medium, and is denoted by $\dot{g}$, whose unit is W/m^3. For example, heat generation in an electrical wire of outer radius r_0 and length l can be expressed as

$$\dot{g} = \frac{\dot{E}_{g.electric}}{V_{wire}} = \frac{I^2 R_e}{\pi r_o^2 l}$$



where I is the electric current, V_{wire} is the wire volume and R_e is the electrical resistance of the wire.

Cooling of Insulators, Machines and Cables

Heat generation in a solid

Under *steady* conditions, the energy balance for this solid can be expressed as

$$\begin{pmatrix} \text{Rate of energy generation} \\ \text{within the solid} \end{pmatrix} = \begin{pmatrix} \text{Rate of heat transfer} \\ \text{from the solid} \end{pmatrix}$$

$$\dot{Q} = V\,\dot{g}$$

$$\dot{Q} = h\,A_s\,(T_s - T_\infty)$$

$$T_s = T_\infty + \frac{\dot{g}V}{h\,A_s}$$

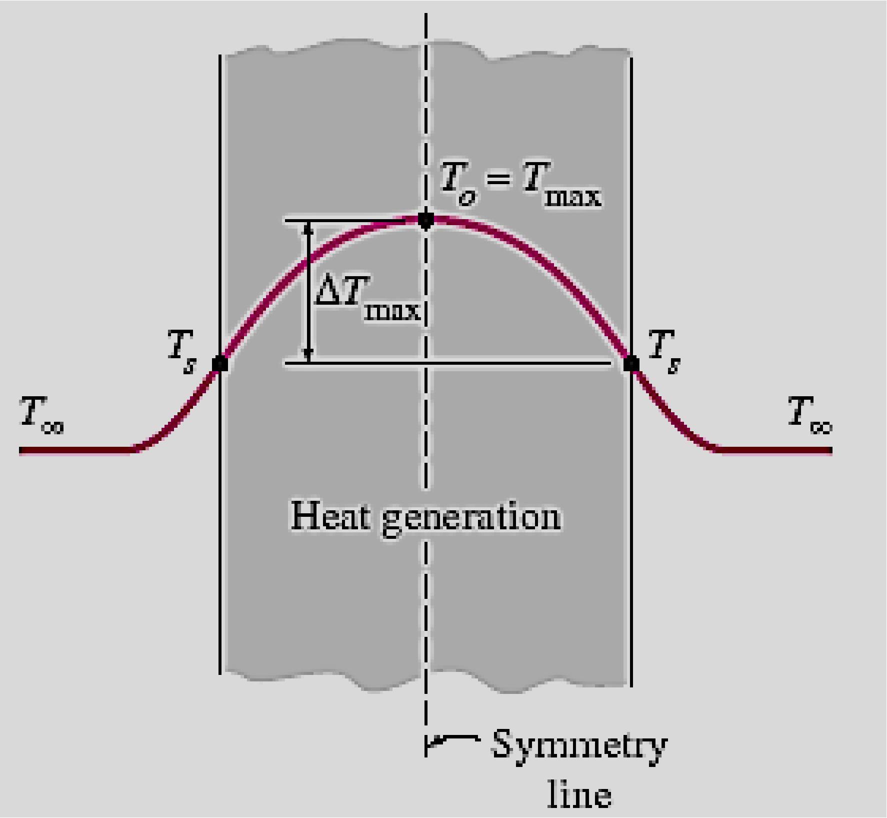
$$T_{s,\,plane\ wall} = T_\infty + \frac{\dot{g}\,l}{h}$$ For Plane wall

$$T_{s,cylinder} = T_\infty + \frac{\dot{g}\,r_o}{2h}$$ For Cylinder

$$T_{s,\,sphere} = T_\infty + \frac{\dot{g}\,r_o}{3h}$$ For Sphere

Cooling of Insulators, Machines and Cables

Maximum temperature in a symmetrical solid with uniform heat generation



Cooling of Insulators, Machines and Cables

Maximum temperature in a symmetrical solid with uniform heat generation

$$-kA_r \frac{dT}{dr} = \dot{g} V_r$$

where $A_r = 2\pi r l$ and $V_r = \pi r^2 l$ at any location r. Substituting these expressions into Eq. (5.33) and separating the variables, we get

$$-k(2\pi r l)\frac{dT}{dr} = \dot{g}\left(\pi r^2 l\right) \rightarrow dT = -\frac{\dot{g}}{2k} r dr$$

Integrating from $r = 0$ where $T(0) = T_0$ to $r = r_o$ where $T(r_o) = T_s$ yields

$$\Delta T_{\max, cylinder} = T_o - T_s = \frac{\dot{g} r_o^2}{4k}$$

Cooling of Insulators, Machines and Cables

Maximum temperature in a symmetrical solid with uniform heat generation

The approach outlined above can also be used to determine the maximum temperature rise in a plane wall of thickness $2L$ and a solid sphere of radius r_o, with these results:

$$\Delta T_{max,\ planewall} = \frac{\dot{g}L^2}{2k}$$

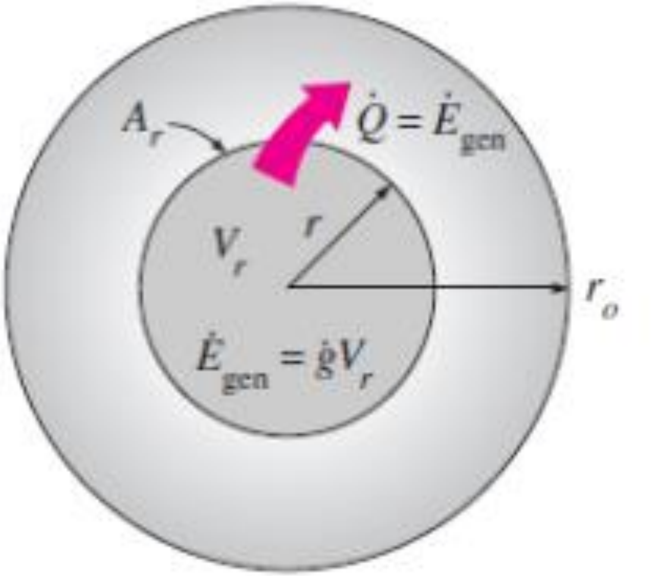
For Plane wall

$$\Delta T_{max\ ,cylinder} = T_o - T_s = \frac{\dot{g}\,r_o^2}{4k}$$

For Cylinder

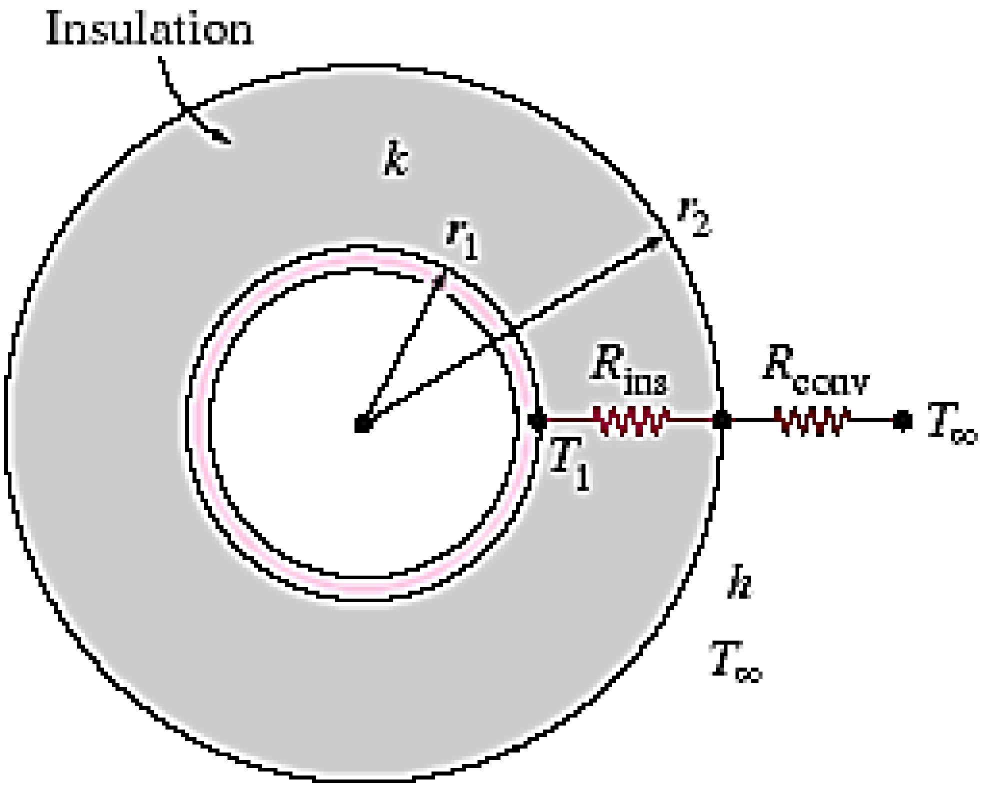
$$\Delta T_{max,\ sphere} = \frac{\dot{g}\,r_o^2}{6k}$$

For Sphere



Cooling of Insulators, Machines and Cables

Critical radius of insulation



Cooling of Insulators, Machines and Cables

Critical radius of insulation

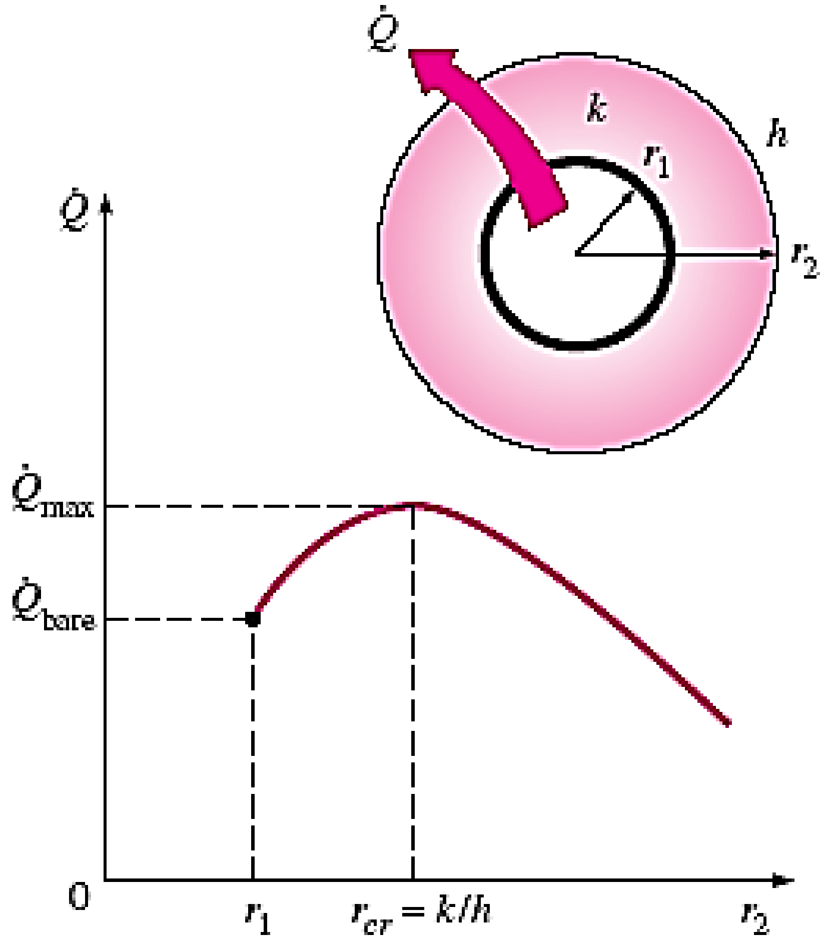
$$\dot{Q} = \frac{T_1 - T_\infty}{R_{ins} + R_{conv}} = \frac{T_1 - T_\infty}{\frac{\ln(r_2/r_1)}{2\pi Lk} + \frac{1}{h(2\pi r_2 L)}}$$

$$d\dot{Q}/dr_2 = 0 \text{ (zero slope)}$$

$$r_{cr,\,cylinder} = \frac{k}{h} \qquad \text{(m)}$$

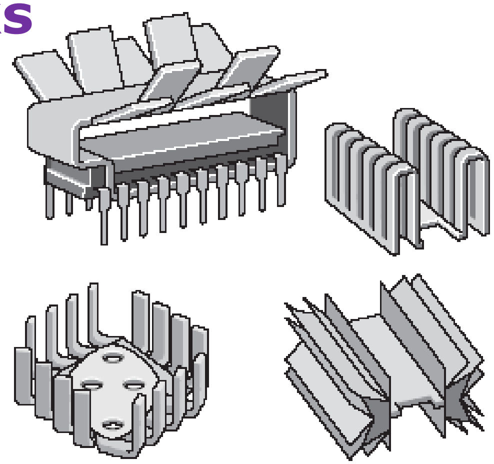
$$\boxed{r_{cr,\,max} = \frac{k_{max,\,insulation}}{h_{min}}}$$

$$\boxed{r_{cr,\,sphere} = \frac{2k}{h}}$$



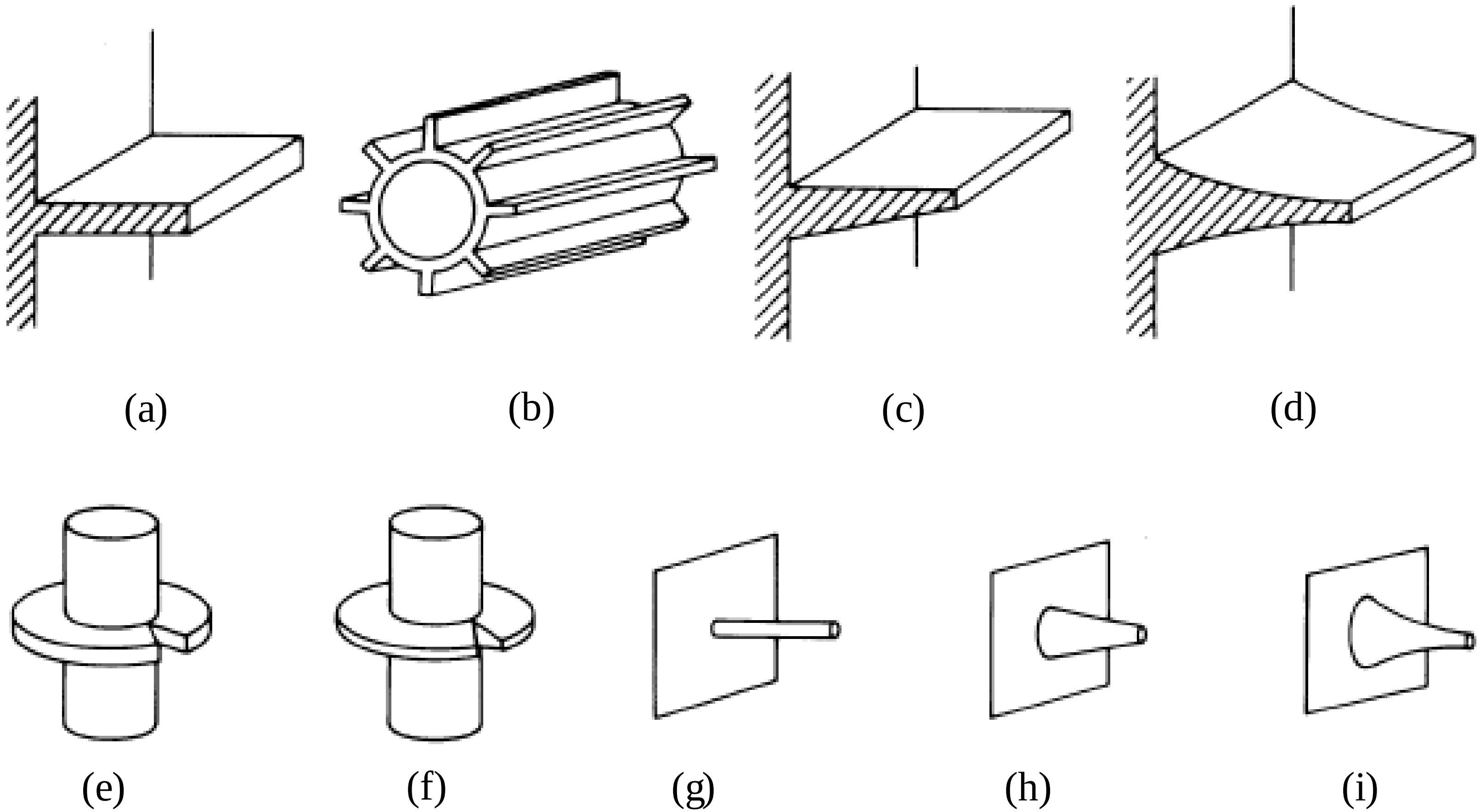
Cooling of Insulators, Machines and Cables

Finned or extended surfaces and heat sinks



Cooling of Insulators, Machines and Cables

Finned or extended surfaces and heat sinks



Cooling of Insulators, Machines and Cables

Finned or extended surfaces and heat sinks

Fin Equation

$$\begin{pmatrix}\text{Rate of } \textit{heat} \\ \textit{conduction} \text{ into} \\ \text{the element at } x\end{pmatrix} = \begin{pmatrix}\text{Rate of } \textit{heat} \\ \textit{conduction} \text{ from the} \\ \text{element at } x + \Delta x\end{pmatrix} + \begin{pmatrix}\text{Rate of } \textit{heat} \\ \textit{convection} \text{ from} \\ \text{the element}\end{pmatrix}$$

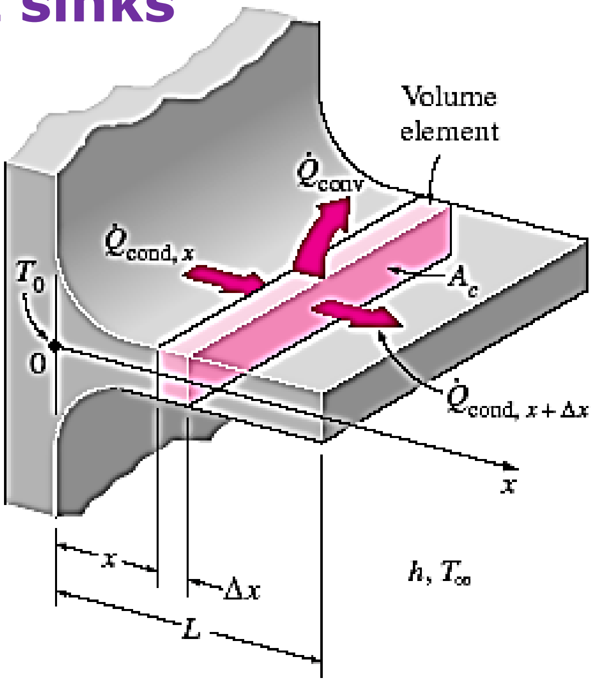
$$\dot{Q}_{\text{cond},\,x} = \dot{Q}_{\text{cond},\,x+\Delta x} + \dot{Q}_{\text{conv}}$$

$$\dot{Q}_{\text{conv}} = h(p\,\Delta x)(T - T_\infty)$$

$$\frac{\dot{Q}_{\text{cond},\,x+\Delta x} - \dot{Q}_{\text{cond},\,x}}{\Delta x} + hp(T - T_\infty) = 0$$

Taking the limit as $\Delta x \to 0$ gives

$$\frac{d\dot{Q}_{\text{cond}}}{dx} + hp(T - T_\infty) = 0$$



Volume element of a fin at location x having a length of Δx, cross-sectional area of A_c, and perimeter of p.

Finned or extended surfaces and heat sinks

Fin Equation

$$\dot{Q}_{\text{cond}} = -kA_c \frac{dT}{dx}$$

$$\frac{d}{dx}\left(kA_c \frac{dT}{dx}\right) - hp(T - T_\infty) = 0$$

$$\frac{d^2\theta}{dx^2} - a^2\theta = 0 \quad \text{where} \quad a^2 = \frac{hp}{kA_c}$$

and $\theta = T - T_\infty$ is the *temperature excess*. At the fin base we have $\theta_b = T_b - T_\infty$.

Finned or extended surfaces and heat sinks

Fin Equation

the general solution of the differential equation

$$\theta(x) = C_1 e^{ax} + C_2 e^{-ax}$$

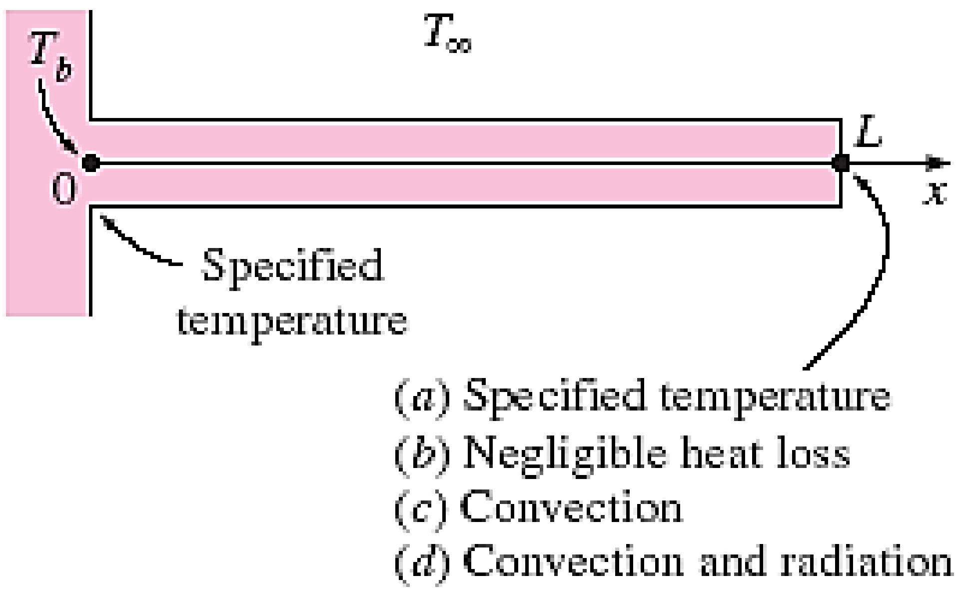
where C_1 and C_2 are arbitrary constants whose values are to be determined from the boundary conditions at the base and at the tip of the fin. Note that we need only two conditions to determine C_1 and C_2 uniquely.

Boundary condition at fin base: $\theta(0) = \theta_b = T_b - T_\infty$

Finned or extended surfaces and heat sinks

Fin Equation

Boundary conditions at the fin base and the fin tip.



Cooling of Insulators, Machines and Cables

Finned or extended surfaces and heat sinks

Fin Equation

Negligible Heat Loss from the Fin Tip
(Insulated fin tip, $\dot{Q}_{\text{fin tip}} = 0$)

Boundary condition at fin tip:

$$\left.\frac{d\theta}{dx}\right|_{x=L} = 0$$

Adiabatic fin tip:

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh a(L - x)}{\cosh aL}$$

Adiabatic fin tip:

$$\dot{Q}_{\text{insulated tip}} = -kA_c \left.\frac{dT}{dx}\right|_{x=0}$$

$$= \sqrt{hpkA_c}\,(T_b - T_\infty)\tanh aL$$

Cooling of Insulators, Machines and Cables

Finned or extended surfaces and heat sinks

Fin Equation

Infinitely Long Fin ($T_{\text{fin tip}} = T_\infty$)

Boundary condition at fin tip: $\theta(L) = T(L) - T_\infty = 0 \quad \text{as} \quad L \to \infty$

Very long fin:

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-ax} = e^{-x\sqrt{hp/kA_c}}$$

Very long fin:

$$\dot{Q}_{\text{long fin}} = -kA_c \left.\frac{dT}{dx}\right|_{x=0} = \sqrt{hpkA_c}\,(T_b - T_\infty)$$

Cooling of Insulators, Machines and Cables

Finned or extended surfaces and heat sinks

Fin Efficiency

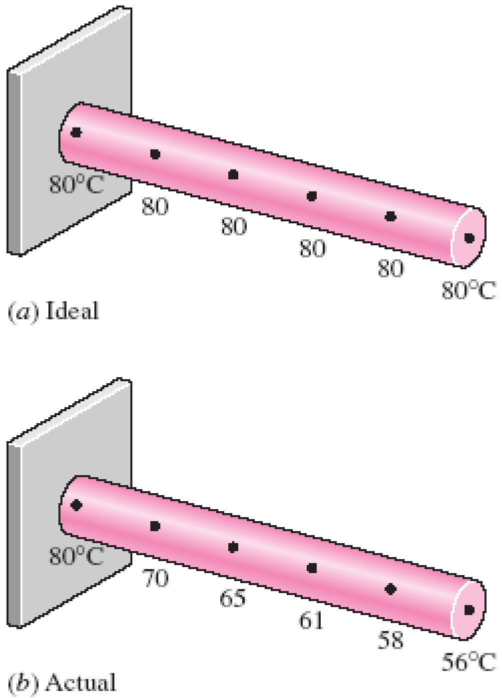
$$\dot{Q}_{\text{fin, max}} = hA_{\text{fin}}\,(T_b - T_\infty)$$

$$\eta_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\text{Actual heat transfer rate from the fin}}{\text{Ideal heat transfer rate from the fin if the entire fin were at base temperature}}$$

$$\dot{Q}_{\text{fin}} = \eta_{\text{fin}}\,\dot{Q}_{\text{fin, max}} = \eta_{\text{fin}}\,hA_{\text{fin}}\,(T_b - T_\infty)$$

$$\eta_{\text{insulated tip}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)\tanh aL}{hA_{\text{fin}}\,(T_b - T_\infty)} = \frac{\tanh aL}{aL}$$

$$\eta_{\text{long fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)}{hA_{\text{fin}}\,(T_b - T_\infty)} = \frac{1}{L}\sqrt{\frac{kA_c}{hp}} = \frac{1}{aL}$$



Cooling of Insulators, Machines and Cables

Finned or extended surfaces and heat sinks

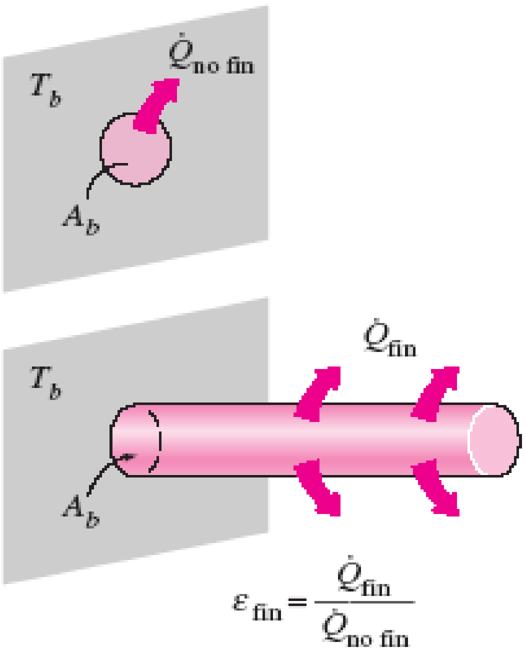
Fin Effectiveness

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_b(T_b - T_\infty)} = \frac{\text{Heat transfer rate from the fin of } base\ area\ A_b}{\text{Heat transfer rate from the surface of } area\ A_b}$$

1) An effectiveness of $\varepsilon_{fin} = 1$ indicates that the **addition of fins to the surface does not affect heat transfer at all**. That is, heat conducted to the fin through the base area A_b is equal to the heat transferred from the same area A_b to the surrounding medium.

2) An effectiveness of $\varepsilon_{fin} < 1$ indicates that **the fin actually acts as *insulation***, slowing down the heat transfer from the surface. This situation can occur when fins made of low thermal conductivity materials are used.

3) An effectiveness of $\varepsilon_{fin} > 1$ indicates that **fins are *enhancing* heat transfer from the surface**, as they should. However, the use of fins cannot be justified unless ε_{fin} is sufficiently larger than 1.



Cooling of Insulators, Machines and Cables

Finned or extended surfaces and heat sinks

Fin Effectiveness

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_b(T_b - T_\infty)} = \frac{\eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty)}{hA_b(T_b - T_\infty)} = \frac{A_{\text{fin}}}{A_b}\eta_{\text{fin}}$$

$$\varepsilon_{\text{long fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)}{hA_b(T_b - T_\infty)} = \sqrt{\frac{kp}{hA_c}}$$

$$\varepsilon_{\text{insulated tip}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)\tanh aL}{hA_b(T_b - T_\infty)} = \sqrt{\frac{kp}{hA_c}}\tanh aL$$

Finned or extended surfaces and heat sinks

Fin Effectiveness

To increase fin effectiveness, one can conclude:

1. The thermal conductivity of the fin material must be as high as possible
2. The ratio of perimeter to the cross-sectional area p/Ac should be as high as possible
3. The use of fin is most effective in applications that involve low convection heat transfer coefficient, i.e. natural convection.

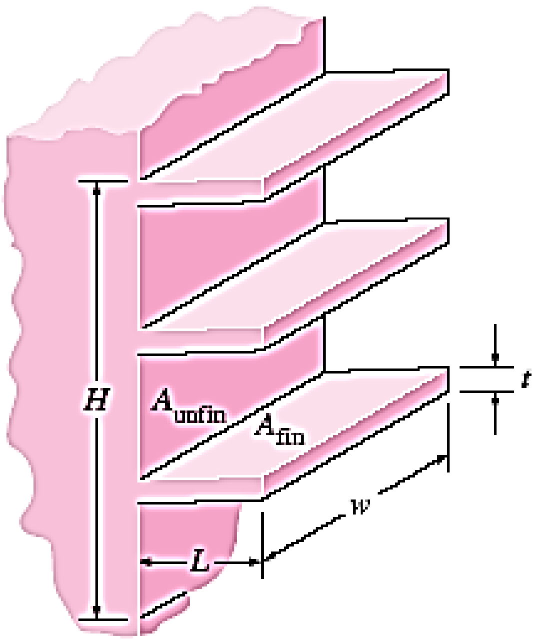
Cooling of Insulators, Machines and Cables

Finned or extended surfaces and heat sinks

Fin Effectiveness

$$\dot{Q}_{\text{total, fin}} = \dot{Q}_{\text{unfin}} + \dot{Q}_{\text{fin}}$$
$$= hA_{\text{unfin}}(T_b - T_\infty) + \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty)$$
$$= h(A_{\text{unfin}} + \eta_{\text{fin}} A_{\text{fin}})(T_b - T_\infty)$$

$$\varepsilon_{\text{fin, overall}} = \frac{\dot{Q}_{\text{total, fin}}}{\dot{Q}_{\text{total, no fin}}} = \frac{h(A_{\text{unfin}} + \eta_{\text{fin}} A_{\text{fin}})(T_b - T_\infty)}{hA_{\text{no fin}}(T_b - T_\infty)}$$



$$A_{\text{no fin}} = w \times H$$
$$A_{\text{unfin}} = w \times H - 3 \times (t \times w)$$
$$A_{\text{fin}} = 2 \times L \times w + t \times w \text{ (one fin)}$$
$$= 2 \times L \times w$$

Cooling of Insulators, Machines and Cables

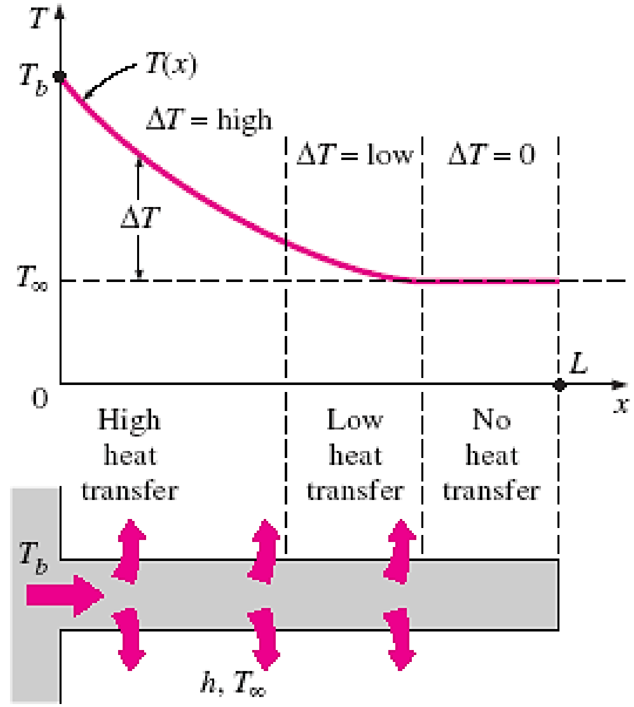
Finned or extended surfaces and heat sinks

Proper Length of a Fin

Heat transfer ratio:

$$\frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{long fin}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)\tanh aL}{\sqrt{hpkA_c}\,(T_b - T_\infty)} = \tanh aL$$

aL	$\dfrac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{long fin}}} = \tanh aL$
0.1	0.100
0.2	0.197
0.5	0.462
1.0	0.762
1.5	0.905
2.0	0.964
2.5	0.987
3.0	0.995
4.0	0.999
5.0	1.000

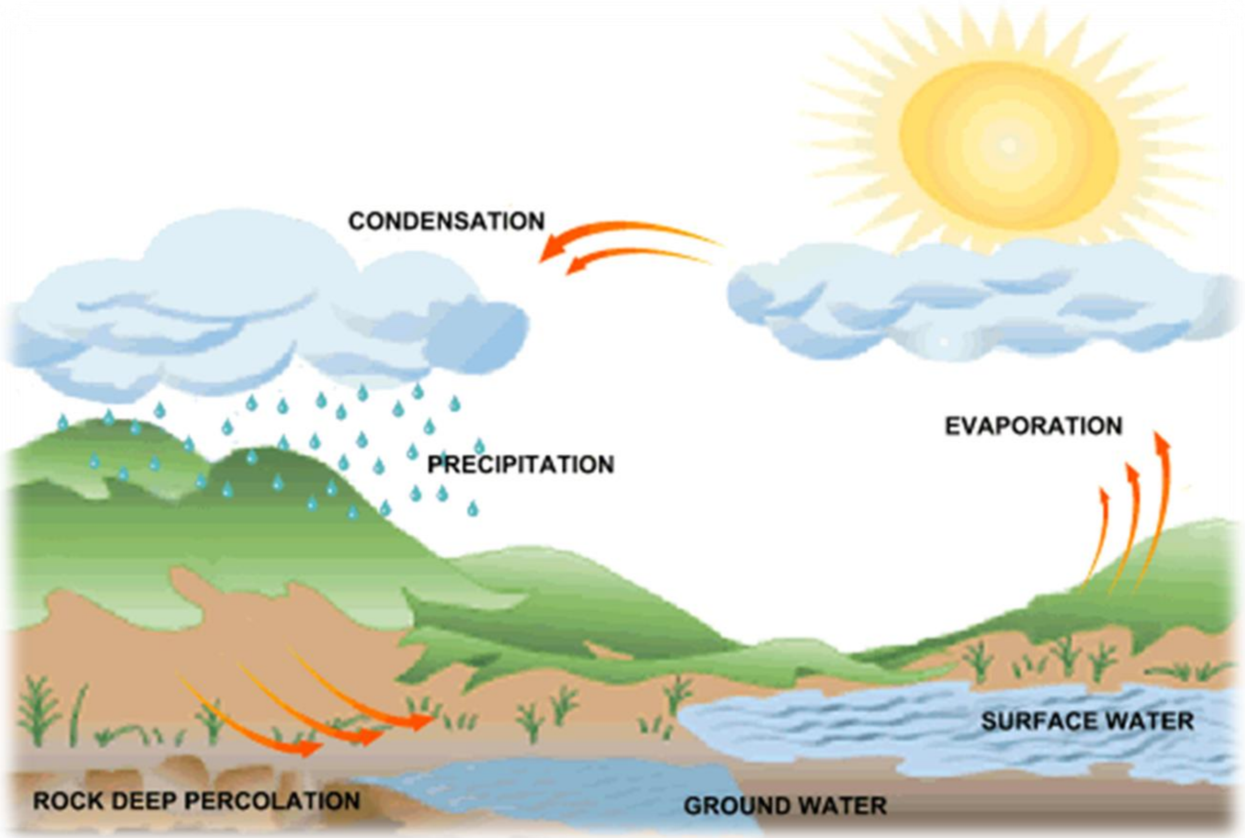


Finned or extended surfaces and heat sinks

The following must be noted for a proper fin selection:

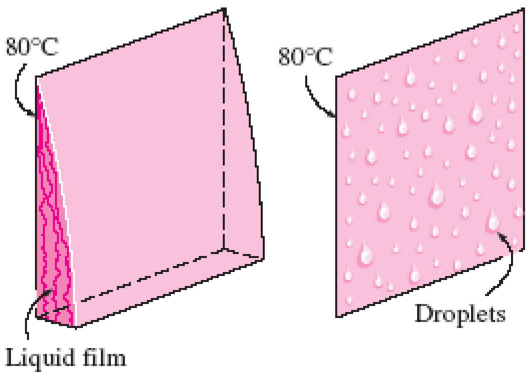
1. The longer the fin, the larger the heat transfer area and thus the higher the rate of heat transfer from the fin.
2. The larger the fin, the bigger the mass, the higher the price, and larger the fluid friction.
3. Also, the fin efficiency decreases with increasing fin length because of the decrease in fin temperature with length.

Evaporation, and Condensation Processes



CONDENSATION HEAT TRANSFER on solid surfaces

Condensation occurs when the temperature of a vapor is reduced *below* its saturation temperature T_{sat}. This is usually done by bringing the vapor into contact with a solid surface whose temperature T_s is *below* the saturation temperature T_{sat} of the vapor. But condensation can also occur on the free surface of a liquid or even in a gas when the temperature of the liquid or the gas to which the vapor is exposed is below T_{sat}.

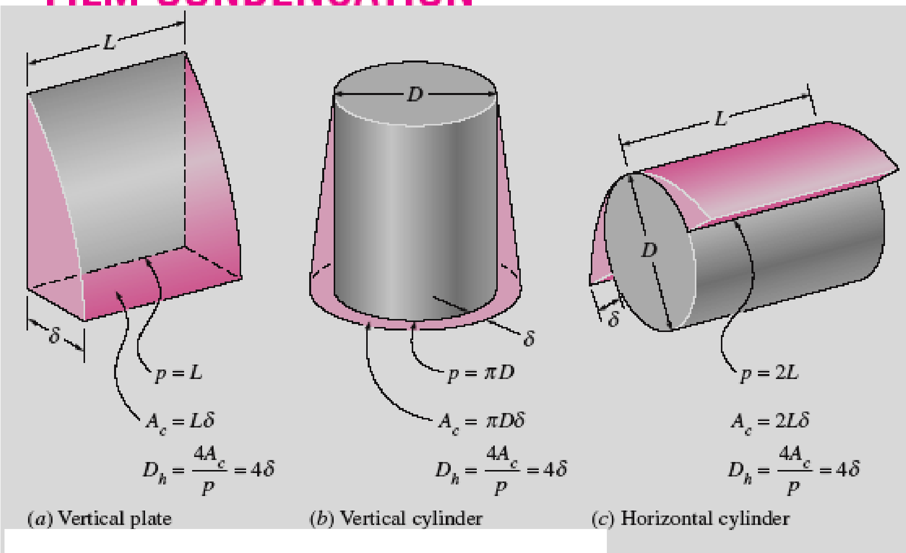


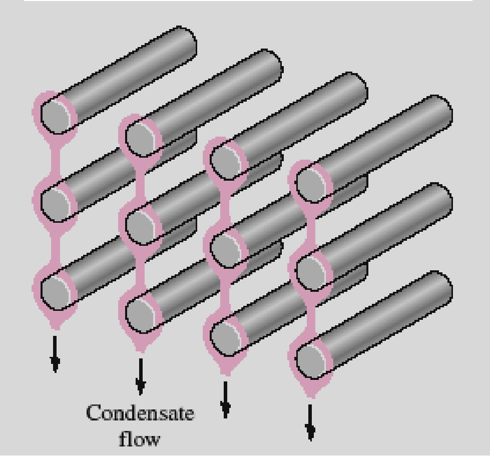
(*a*) Film condensation

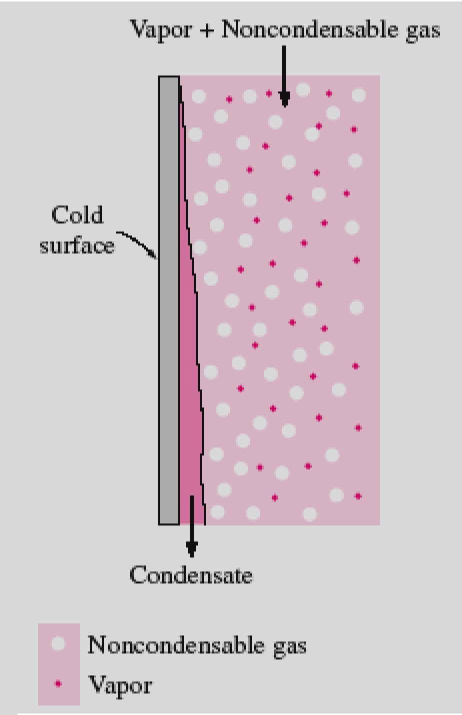
(*b*) Dropwise condensation

CONDENSATION HEAT TRANSFER on solid surfaces

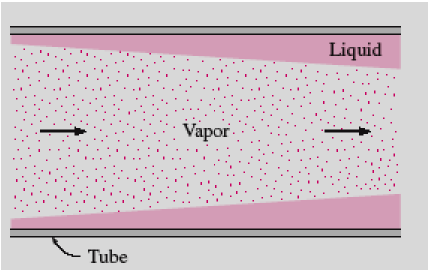
FILM CONDENSATION



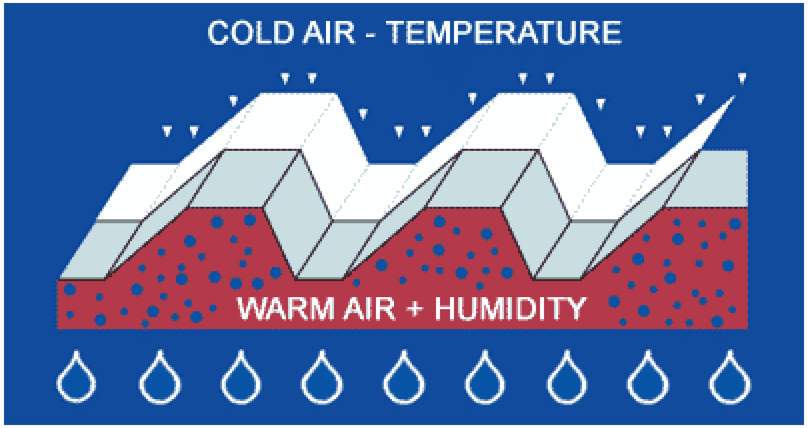




FILM CONDENSATION INSIDE HORIZONTAL TUBES



Desirable and undesirable condensation

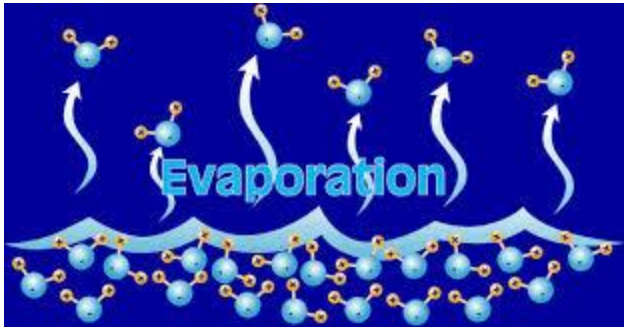


Evaporation Processes

The process of evaporation involves the vaporization of a liquid. However, it should be noted that a key distinction for evaporation is that it only happens at the surface of the liquid. For example, this is different from boiling because boiling affects the whole volume of liquid instead of just the top surface.

Additionally, evaporation is considered to be part of a phase transition. This phase transition refers to how molecules in a liquid or water state suddenly become gaseous or suddenly turn to water vapor. This phase transition is notable as a gradual reduction of a liquid from matter due to the exposure of a considerable amount of gas.

Generally, the molecules of water in a glass do not naturally possess a sufficient amount of energy in the form of heat to escape or remove themselves from the liquid.



Industrial applications include

1. Assist with the removal of contaminants and byproducts,
2. Water desalination,
3. Printing and coating processes,
4. Recovering salts from solutions,
5. Drying a variety of materials such as lumber, paper, cloth and chemicals,
6. Evaporative coolers.