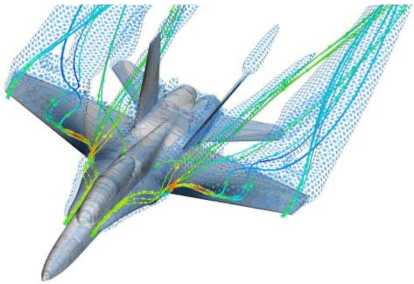
بسم الله الرحمن الرحيم

Fluid Mechanics I



Fayoum University

Faculty of Engineering
Mechanical Engineering Dept.

Lecture (6)
on

Dimensional Analysis and Similarity

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2015 - 2016

Dimensional Homogeneity

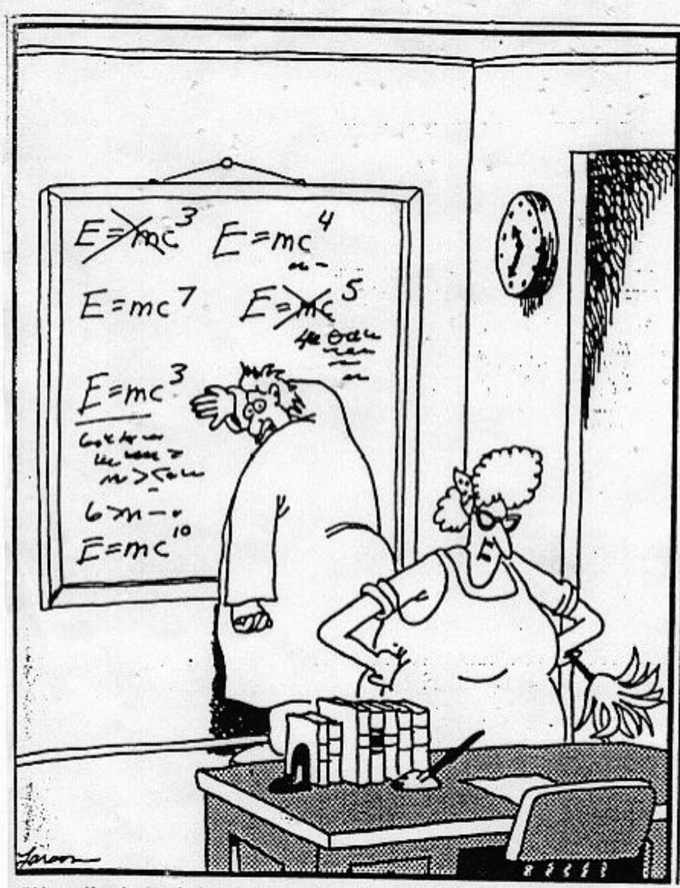
Homogeneity: the quality or state of being homogeneous (of the same or similar nature). So i other words, the structure of the dimensions must be equal to each other.

For example X(F,L,T) = y(F,L,T)

If we have Energy = Force * Distance.

Here we can rewrite Force as mass*acceleration. Acceleration is L/T^2. So we can rewrite E=ML/T^2 *L = M*L^2*T^-2.

We can even use Dimensional homogeneity to find Dimensionless numbers e.g. F/(P*A) where F=force, P=pressure and A=area. We can rewrite this as F/(F/L^2)*L^2) = F/F = 1. Therefore dimensionless number.



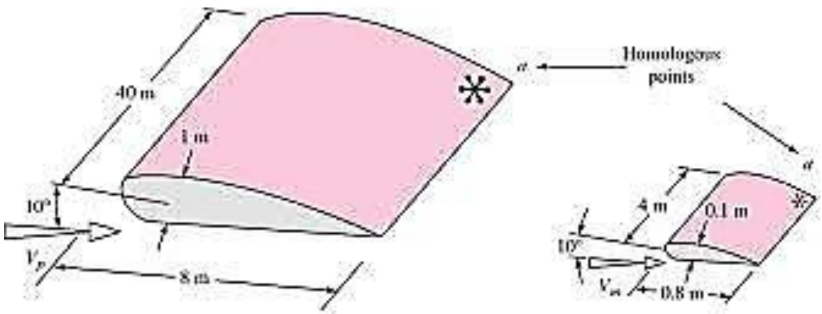
"Now that desk looks better. Everything's squared away, yessir, squaaaaaared away."

Purposes of Dimensional Analysis

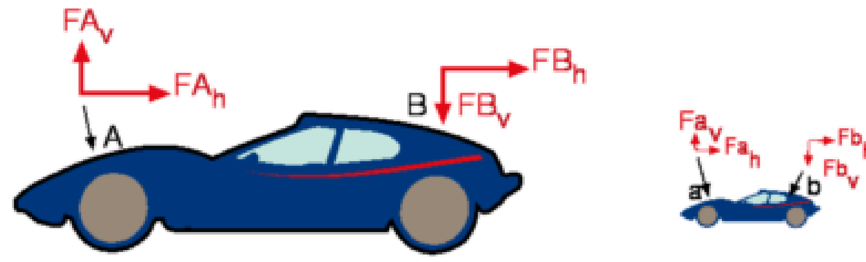
Nondimensionalization of an equation by inspection is useful only when we know the equation to begin with. However, in many cases in real-life engineering, the equations are either not known or too difficult to solve; oftentimes experimentation is the only method of obtaining reliable information.

In most experiments, to save time and money, tests are performed on a geometrically scaled **model**, rather than on the full-scale **prototype**. In such cases, care must be taken to properly scale the results. We introduce here a powerful technique called **dimensional analysis**. While typically taught in fluid mechanics, dimensional analysis is useful in all disciplines, especially when it is necessary to design and conduct experiments. You are encouraged to use this powerful tool in other subjects as well, not just in fluid mechanics.

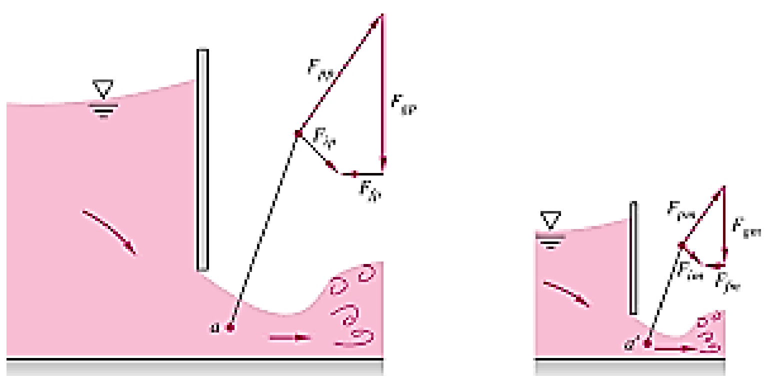
Purposes of Dimensional Analysis



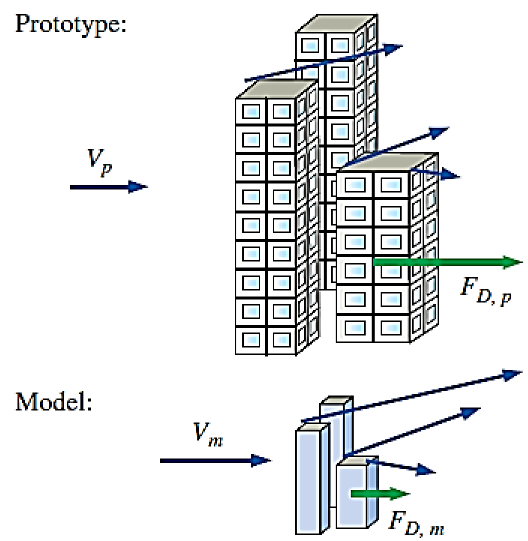
Prototype Model



Prototype Model



Prototype Model



Purposes of Dimensional Analysis

The three primary purposes of dimensional analysis are

1. To generate nondimensional parameters that help in the design of experiments (physical and/or numerical) and in the reporting of experimental results
2. To obtain scaling laws so that prototype performance can be predicted from model performance
3. To (sometimes) predict trends in the relationship between parameters

Dimensional Analysis Concept

Before discussing the technique of dimensional analysis, we first explain the underlying concept of dimensional analysis; the principle of similarity.

There are three necessary conditions for complete similarity between a model and a prototype:

Geometric similarity: the model must be the same shape as the prototype, but may be scaled by some constant scale factor.

$$\frac{L_p}{L_m} = \frac{B_p}{B_m} = \frac{D_p}{D_m} = L_r$$

- Where: L_p, B_p and D_p are Length, Breadth, and diameter of prototype and L_m, B_m, D_m are Length, Breadth, and diameter of model.
- Lr= Scale ratio

Dimensional Analysis Concept

Kinematic similarity: which means that the velocity at any point in the model flow must be proportional (by a constant scale factor) to the velocity at the corresponding point in the prototype flow. Specifically, for kinematic similarity the velocity at corresponding points must scale in magnitude and must point in the same relative direction. You may think of geometric similarity as length-scale equivalence and kinematic similarity as time-scale equivalence. Geometric similarity is a prerequisite for kinematic similarity.

$$\frac{V_{p1}}{V_{m1}} = \frac{V_{p2}}{V_{m2}} = V_r; \quad \frac{a_{p1}}{a_{m1}} = \frac{a_{p2}}{a_{m2}} = a_r$$

- Where: V_{p1}& V_{p2} and a_{p1} & a_{p2} are velocity and accelerations at point 1 & 2 in prototype and V_{m1}& V_{m2} and a_{m1} & a_{m2} are velocity and accelerations at point 1 & 2 in model.
- V_r and a_r are the velocity ratio and acceleration ratio

Dimensional Analysis Concept

Dynamic similarity: Dynamic similarity is achieved when all forces in the model flow scale by a constant factor to corresponding forces in the prototype flow (force-scale equivalence). As with geometric and kinematic similarity, the scale factor for forces can be less than, equal to, or greater than one.

$$\frac{(F_i)_p}{(F_i)_m} = \frac{(F_v)_p}{(F_v)_m} = \frac{(F_g)_p}{(F_g)_m} = F_r$$

- Where: $(F_i)_p$, $(F_v)_p$ and $(F_g)_p$ are inertia, viscous and gravitational forces in prototype and $(F_i)_m$, $(F_v)_m$ and $(F_g)_m$ are inertia, viscous and gravitational forces in model.
- F_r is the Force ratio

Dimensional Analysis Concept

- Kinematic similarity is a necessary but insufficient condition for dynamic similarity.
- It is thus possible for a model flow and a prototype flow to achieve both geometric and kinematic similarity, yet not dynamic similarity.
- All three similarity conditions must exist for complete similarity to be ensured.
- In a general flow field, complete similarity between a model and prototype is achieved only when there is geometric, kinematic, and dynamic similarity.

Dimensional Analysis Concept

Characteristics		Unit (SI)	Dimension (MLT)	Dimension (FLT)
Geometry	Length	m	L	
	Area	m^2	L^2	
	Volume	m^3	L^3	
Kinematic	Time	S	T	
	Velocity	m/S	L/T	
	Acceleration	m/S^2	L/T^2	
	Discharge	m^3/S	L^3/T	
Dynamic	Mass	Kg	M	$FL^{-1}T^2$
	Force	N ($Kg\text{-}m/S^2$)	MLT^{-2}	F
	Pressure	Pa (N/m^2)	$ML^{-1}T^{-2}$	FL^{-2}
	Energy	J (N-m)	ML^2T^{-2}	FL
	Power	Watt (N-m/S)	ML^2T^{-3}	FLT^{-1}

Methodology of Dimensional Analysis

The basic principle is Dimensional Homogeneity, which means the dimensions of each terms in an equation on both sides are equal.

So such an equation, in which dimensions of each term on both sides of equation are same, is known as Dimensionally Homogeneous Equation. Such equations are independent of system of units. For example;

Lets consider the equation $V=(2gH)^{1/2}$

- Dimensions of LHS$=V=L/T=LT^{-1}$
- Dimensions of RHS$=(2gH)^{1/2}=(L/T^2xL)^{1/2}=LT^{-1}$
- *Dimensions of LHS = Dimensions of RHS*

So the equation $V=(2gH)^{1/2}$ is dimensionally homogeneous equation.

Methods of Dimensional Analysis

If the number of variables involved in a physical phenomenon are known, then the relation among the variables can be determined by the following two methods:

1. Rayleigh's Method
2. Buckingham's - $\prod$ Theorem

Rayleigh's Methods of Dimensional Analysis

An elementary method for finding a functional relationship with respect to a parameter in interest is the Rayleigh Method, and will be illustrated with an example, using the **MLT** system.

Say that we are interested in the drag, D, which is a force on a ship. What exactly is the drag a function of? These variables need to be chosen correctly, though selection of such variables depends largely on one's experience in the topic. It is known that drag depends on

Quantity	Symbol	Dimension
Size	l	L
Viscosity	μ	M/LT
Density	ρ	M/L^3
Velocity	V	L/T
Gravity	g	L/T^2

Rayleigh's Methods of Dimensional Analysis

This means that $D = f\,(l, \rho, \mu, V, g)$ where f is some function.

With the Rayleigh Method, we assume that

$$D = C l^a \rho^b \mu^c V^d g^e$$

where C is a dimensionless constant, and a,b,c,d, and e are exponents, whose values are not yet known.

Note that the dimensions of the left side, force, must equal those on the right side. Here, we use only the three independent dimensions for the variables on the right side: M, L, and T.