1.6. Solved Examples

Example 1.1

Dimensions and Units

A body weighs 1000 Ibf when exposed to a standard earth gravity $g = 32.174$ ft/s^2. (*a*) What is its mass in kg? (*b*) What will the weight of this body be in N if it is exposed to the moon's standard acceleration $g_{moon} = 1.62$ m/s^2? (*c*) How fast will the body accelerate if a net force of 400 lbf is applied to it on the moon or on the earth?

Solution:

F = weight and $a = g_{earth}$:

$F=W=mg=1000$ lbf $= $ (m slugs) (32.174 ft/s^2)

or

$m = 1000/32.174 = $ (31.08 slugs)(14.5939 kg/slug) $= 453.6$ kg   ***Ans.* (a)**

The change from 31.08 slugs to 453.6 kg illustrates the proper use of the conversion factor 14.5939 kg/slug.

The mass of the body remains 453.6 kg regardless of its location.

$F = W_{moon} = m.g_{moon} = $ (453.6 kg)(1.62 m/s^2) $= 735$ N   ***Ans. (b)***

This problem does not involve weight or gravity or position and is simply a direct application of Newton's law with an unbalanced force:

$F = 400$ lbf $= m.a = $ (31.08 slugs)(a ft/s^2)

or

a $=400/31.08 = 12.43$ ft/s$^2 = 3.79$ m/s^2 ***Ans. (c)***

This acceleration would be the same on the moon or earth or anywhere.

Example 1.2

Dimensions and Units

An early viscosity unit in the cgs system is the poise (abbreviated **P**), or g/(cm.s), named after J. L. M. Poiseuille, a French physician. The viscosity of water (fresh or salt) at 293.16 K $= 20°$C is approximately $\mu = 0.01$ P. Express this value in (a) SI and (b) BG units.

Solution:

μ = [0.01 g/(cm . s)] (1 kg/1000 g) (100cm/m) = 0.001 kg/(m.s) **Ans. (a)**

μ = [0.01 kg/(m . s)] (1 slug/14.59 kg) (0.3048 m/ft)

   = 2.09×10^{-5} slug/(ft.s)   **Ans. (b)**

Note: Result (b) could have been found directly from (a) by dividing (a) by the viscosity conversion factor 47.88 listed in Table (1.2).

Example 1.3

Properties of a Fluid

Suppose that the fluid being sheared in Figure (1.5) is SAE 30 oil at 20°C. Compute the shear stress in the oil if u = 3 m/s and h = 2 cm.

Solution:

The shear stress is found from Eq. (1.13) by differentiating Eq. (1.14):

$$\tau = \mu \frac{du}{dy} = \frac{\mu u}{h} \qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(E1.1)}$$

From Table (1.5) for SAE 30 oil, μ = 0.29 kg/(m . s). Then, for the given values of u and h, Eq. (E1.1) predicts

$$\tau = \frac{[0.29\,kg/(m.s)](3\,m/s)}{0.02\,m} = 43\,kg/(m.s^2) = 43N/m^2 = 43\,Pa \qquad Ans.$$

Although oil is very viscous, this is a modest shear stress, about 2400 times less than atmospheric pressure. Viscous stresses in gases and thin liquids are even smaller.

Example 1.4 (2014 final Exam)

Properties of a Fluid

The velocity profile is a laminar flow through a round pipe is expressed as, $u = 2U\left[1-\left(r^2/r_0^2\right)\right]$ where U = average velocity, r_0 = radius of pipe. (a) Draw dimensionless shear stress profile (τ/τ_0) against (r/r_0) where τ_0

is wall shear stress. (b) Find τ_0, when oil flows with absolute viscosity 4×10^{-2} N.s/m^2 and velocity of 4 m/s in a pipe of diameter 150 mm.

Solution:

Given $u = 2U\left[1 - \left(r^2/r_0^2\right)\right]$

Then $\dfrac{du}{dr} = -\dfrac{4Ur}{r_0^2}$ and $\tau = \mu\dfrac{du}{dr} = -\dfrac{4\mu Ur}{r_0^2}$

And $\tau_0 = \mu\dfrac{du}{dr}\bigg|_{r=0} = -\dfrac{4\mu U}{r_0}$ .... (E1.2)

So, $\dfrac{\tau}{\tau_0} = \dfrac{r}{r_0}$

and the plot is shown in Figure (E4.1)   ***Ans.*** (*a*)

From Eq. (E1.2), $\tau_0 == -\dfrac{4 \times 4 \times 10^{-2} \times 4}{0.075} = 8.534\ N/m^2$ ***Ans.*** (*b*)
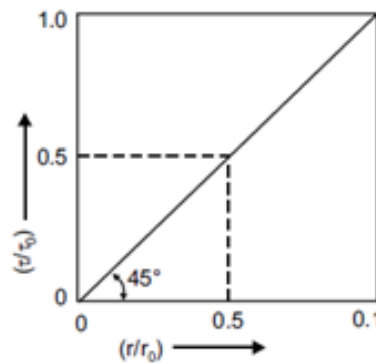

Fig. E1.4: dimensionless shear stress profile $\left(\tau/\tau_0\right)$ against $\left(r/r_0\right)$.

Example 1.5

Properties of a Fluid

Derive an expression for the change in height **h** in a circular tube of a liquid with surface tension $\boldsymbol{\sigma}$ and contact angle $\boldsymbol{\theta}$, as in Figure (E1.5).

Solution:

The vertical component of the ring surface-tension force at the interface in the tube must balance the weight of the column of fluid of height **h**

$\pi R\sigma\cos\theta = \pi R^2\rho gh$

Solving for **h**, we have the desired result

$$h = \frac{2\sigma \cos \theta}{\rho gR}$$ ***Ans.***

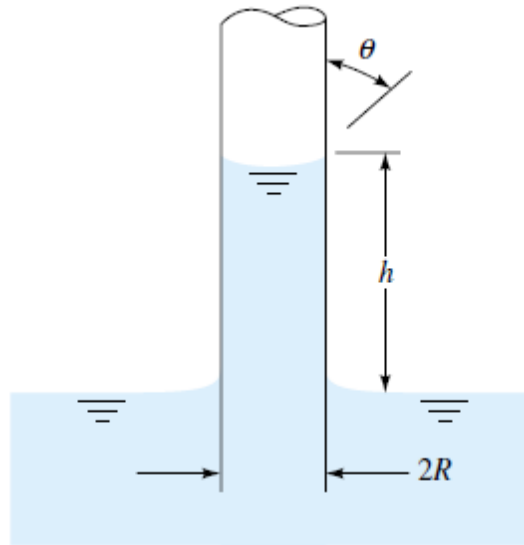Fig. E1.5

Thus the capillary height increases inversely with tube radius **R** and is positive if $\theta < 90°$ (wetting liquid) and negative (capillary depression) if $\theta > 90°$.

  Suppose that R = 1 mm. Then the capillary rise for a water-air-glass interface, $\theta \approx 0°$, σ = 0.073 N/m, and ρ = 1000 kg/m³ is

$$h = \frac{2\,(0.073\,N\,/\,m)(\cos 0°)}{(1000\,kg\,/\,m^3)(9.81\,m\,/\,s^2)(0.001m)} = 0.015\,(N.s^2)\,/\,kg = 0.015\,m = 1.5\,cm$$

For a mercury-air-glass interface, with θ = 130°,, σ = 0.48 N/m, and ρ = 113600, the capillary rise is

h = -0.46 cm

When a small-diameter tube is used to make pressure measurements, these capillary effects must be corrected for.

Example 1.6

Properties of a Fluid

A cylinder 7.5 cm radius and 60 cm in length rotate coaxially inside a fixed cylinder of the same length and 9 cm inner radius as shown in Figure (E1.6). Glycerin $\mu = 8$ Poise fills the space between to cylinders. A Torque 0.4 N.m is applied to the inner cylinder. After a constant velocity is attended, calculate the following: (a) velocity gradient at the cylinder walls, (b) the velocity rustling and (c) the power dissipated by the fluid resistance.
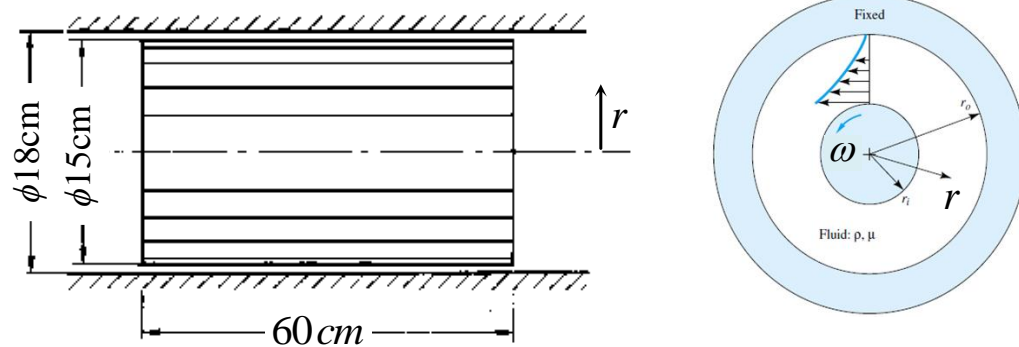

Fig. E1.6

Solution:

The shear stress is found from Eq. (1.13)

$$\tau = \mu \frac{du}{dy} \quad \text{...} \quad (E1.3)$$

$$Torque = F\,r = \tau\,A\,r = \tau\,r(2\pi\,r\,L) = 2\tau\,r^2\pi\,L \quad \text{....................} \quad (E1.4)$$

where L is the cylinder length

then

from Eq. (E1.4)

$$0.4\,(N.m) = 2\tau\,r^2\pi \times 60 \times 10^{-2}\,(m)$$

$$\tau = \frac{0.1062}{r^2} = \mu \frac{du}{dy}$$

$$\frac{du}{dy} = \frac{0.1062}{\mu r^2} = \frac{0.1062}{8(Poise) \times 10 \times r^2} = \frac{0.13275}{r^2} \quad N/m \quad \text{........…………..……} \quad \text{(E1.5)}$$

$$\left.\frac{du}{dy}\right|_{innerwall} = \frac{0.13275}{r^2} = \frac{0.13275}{(7.5 \times 10^{-2})^2} = 23.6 \qquad\qquad \textbf{\textit{Ans.(a)}}$$

$$\left.\frac{du}{dy}\right|_{outerwall} = \frac{0.13275}{r^2} = \frac{0.13275}{(9 \times 10^{-2})^2} = 16.38 \qquad\qquad \textbf{\textit{Ans.(a)}}$$

From Eq. (E1.4) and where $dy = -dr$

$$\frac{du}{dy} = -\frac{du}{dr} = -\frac{0.13275}{r^2}$$

$$du = -\frac{0.13275}{r^2} dr \quad \text{……………………………………………} \quad \text{(E1.6)}$$

by integrating Eq. (E1.6):

$$\int_0^u du = -0.13275 \int_{0.09}^{0.075} \frac{1}{r^2} dr$$

Where $u(r) = 0$ at $r=0.09$ m
Then

$$u = \left[\frac{0.13275}{r^2}\right]_{0.09}^{0.075} = 29.48 \; m/s \qquad\qquad \textbf{\textit{Ans.(b)}}$$

Where $u = \omega r = \frac{2\pi N}{60} r$ (**N**: revolution per minute)

$$29.48 = \frac{2\pi N}{60} r = \frac{2\pi N}{60} 7.5 \times 10^{-2} \qquad \rightarrow \quad N = 37.5 rpm$$

$$Power = Torque \times \omega = 0.0021 HP \qquad\qquad \textbf{\textit{Ans.(c)}}$$

1.7. Problems

1. Derive the SI unit of force from base units.

2. Explain dynamic viscosity and kinematic viscosity. Give their dimensions.

3. Explain the phenomenon of capillarity. Obtain an expression for capillary rise of a fluid.

4. Express the viscosity and the kinematics' viscosity in SI units.

5. For low-speed (laminar) steady flow through a circular pipe, the velocity u varies with radius and takes the form

$$u = B \frac{\Delta p}{\mu} \left[r_0^2 - r^2 \right]$$

   where μ is the fluid viscosity and Δp is the pressure drop from entrance to exit. What are the dimensions of the constant B?

6. The density of water at 4°C and 1 *atm* is 1000 *kg/m³*. Obtain the specific volume.

7. The specific weight of a certain liquid is 10 *KN/m³*. Determine its density and specific gravity.

8. A liquid when poured into a graduated cylinder is found to weigh 8 *N* when occupying a volume of 500 *ml* (milliliters). Determine its specific weight, density, and specific gravity.

9. Obtain the pressure in SI (*Pa*) necessary for shrinking the volume of water by 1% at normal temperature and pressure. Assume the compressibility of water $\beta = 4.85 \times 10^{-9}$ *Pa⁻¹*.

10. A block of weight W slides down an inclined plane while lubricated by a thin film of oil, as in Figure (1.P10). The film contact area is A and its thickness is h. Assuming a linear velocity distribution in the film, derive an expression for the "terminal" (zero-acceleration) velocity V of the block.

11. Derive an expression for the capillary height change **h** for a fluid of surface tension **σ** and contact angle **θ** between two vertical parallel plates a distance **W** apart, as in Figure (P1.11). What will **h** be for water at 20°C if **W** = 0.5 mm?
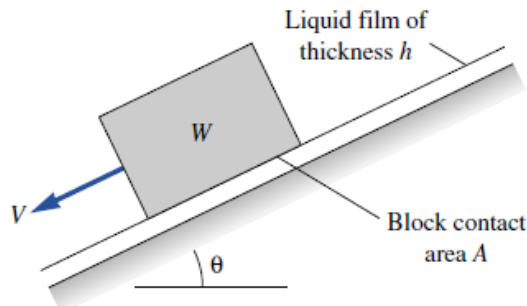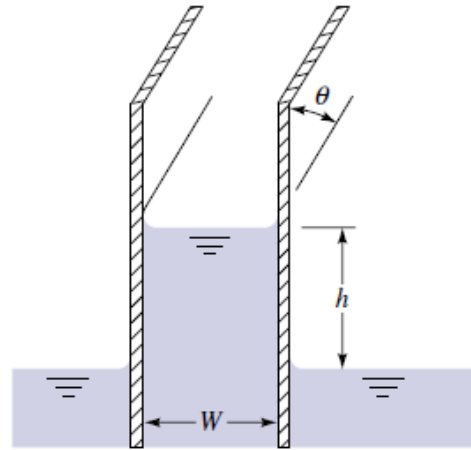

Fig. 1.P10.   Fig. 1.P11.

12. Find surface tension of a soap bubble of 48 mm diameter while pressure inside is 3.12 Pa higher than atmospheric one.

13. A Newtonian fluid having a specific gravity of 0.92 and a kinematics viscosity of 4×10^{-4} m²/s flows past a fixed surface. Due to the no-slip condition, the velocity at the fixed surface is zero (as shown), and the velocity profile near the surface is shown in Figure(1.P13). Determine the magnitude and direction of the shearing stress developed on the plate. Express your answer in terms of **U** and **δ**, with **U** and **δ** expressed in units of meters per second and meters, respectively.

$$\frac{u}{U} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$
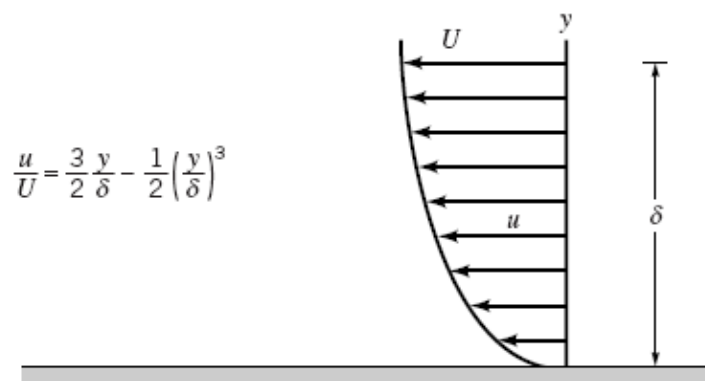

Fig. 1.P13.

14. As shown in Figure (1.P14), a cylinder of diameter 122*mm* and length 200*mm* is placed inside a concentric long pipe of diameter 125 *mm*. An oil film is introduced in the gap between the pipe and the cylinder. What force is necessary to move the cylinder at a velocity of   1 m/s? Assume that the kinematic viscosity of oil is 30 cSt and the specific gravity is 0.9.
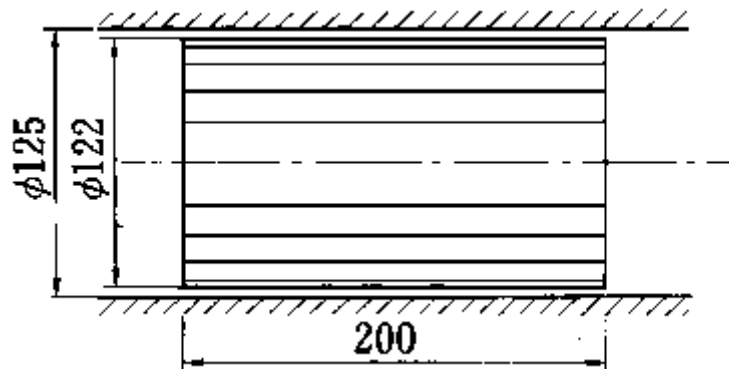

Fig. 1.P14