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# Physics 2 (Waves)

## Chapter: 2 Wave Motion

- Propagation of a Disturbance  Sinusoidal Waves

- The Speed of Waves on String

- Reflection and transmission.

- Rate of Energy Transfer by Sinusoidal Waves on Strings

- The Linear Wave Equation

# Wave Motion

**Objectives of part 1:**
**The student will be able to:**

- Define the propagation wave and the sinusoidal  wave.
- Define the wavelength, frequency and the phase difference.
- Demonstrate the speed of wave on the string.
- Define the reflection & transmission waves.
- Define the kinetic and the potential energies.
- Define the total energy on the string
- Determine the rat of the energy transfer.
- The liner equation of the wave.

Introduction:

Two main types being " ***mechanical* waves"** and ***electromagnetic* waves.**

- **Mechanical waves**, require some physical medium is being disturbed in our pebble and beach ball example, elements of water are disturbed.

- **Electromagnetic waves** do not require a medium to propagate:

some examples of electromagnetic waves are **visible light, radio waves, television signals, and x-rays**.

Here, in this part of the course , we study only **mechanical waves.**

All waves carry energy, but the amount of energy transmitted through a medium and the mechanism responsible for that transport of energy differ from case to case.

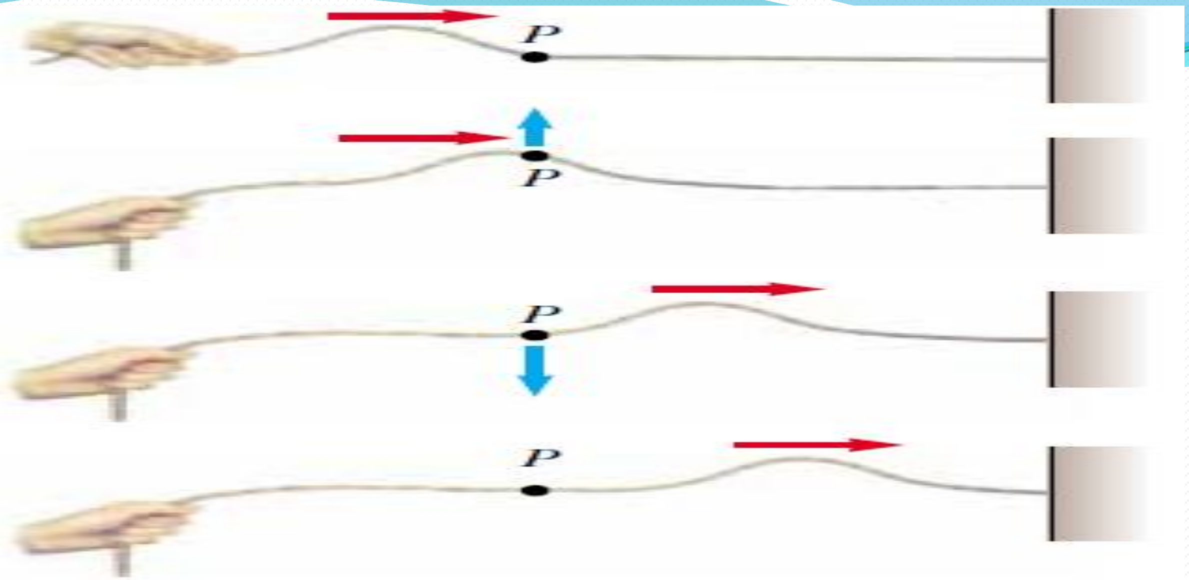
## 1 - Propagation of a disturbance:

**All mechanical waves require**:

(1) Some source of disturbance,

(2) A medium that can be disturbed, and

(3) Some physical mechanism through which elements of the medium can influence each other.




- Figure:1 illustrates this point for one particular element, labeled *P*. Note that no part of the rope ever moves in the direction of the propagation. **A traveling wave or pulse that causes the elements of the disturbed medium to move perpendicular to the direction of propagation** is called a **transverse wave.**

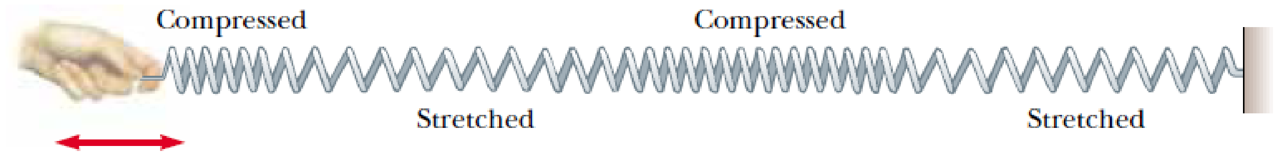


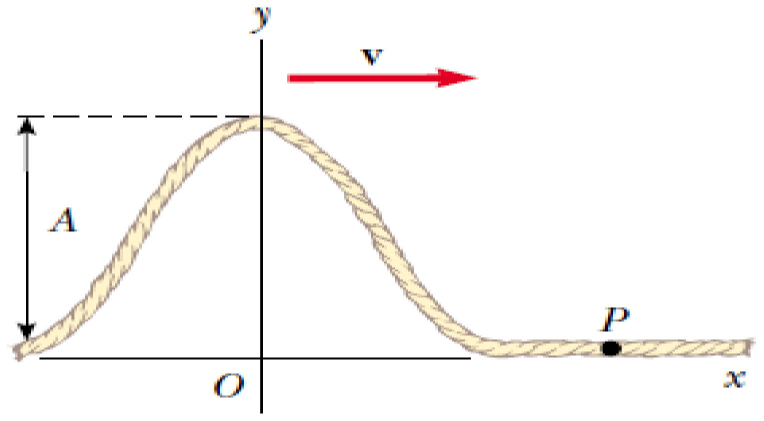

**Figure -2** The left end of the spring is pushed briefly to the right and then pulled briefly to the left.

This movement creates a sudden compression of a region of the coils. The compressed region travels along the spring. The **compressed region** is followed by a region where the coils are **extended**.

Notice that the **direction of the displacement of the coils is *parallel* to the direction of propagation of the compressed region**. A traveling wave or pulse that causes the elements of the medium to move parallel to the direction of propagation is called **longitudinal wave.**

The disturbance in a sound wave are an example of longitudinal waves of series of high-pressure and low-pressure regions that travel through air.

The **motion of water** elements on the surface of deep water in which a wave is propagating is a **combination** of **transverse** and **longitudinal** displacements, each element is displaced both horizontally and vertically from its equilibrium position.

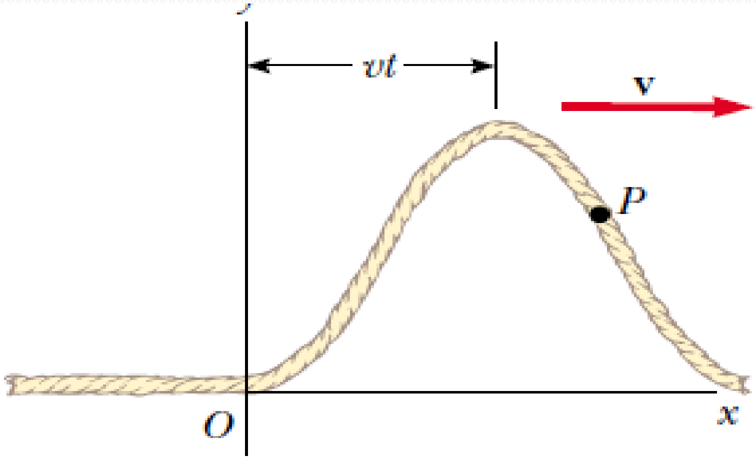


(a) Pulse at $t = 0$

Figure -3a represents the shape and position of the pulse at time *t* = 0. The shape of the pulse can be represented by some mathematical function which we will write as

$$y(x,0) = f(x).$$

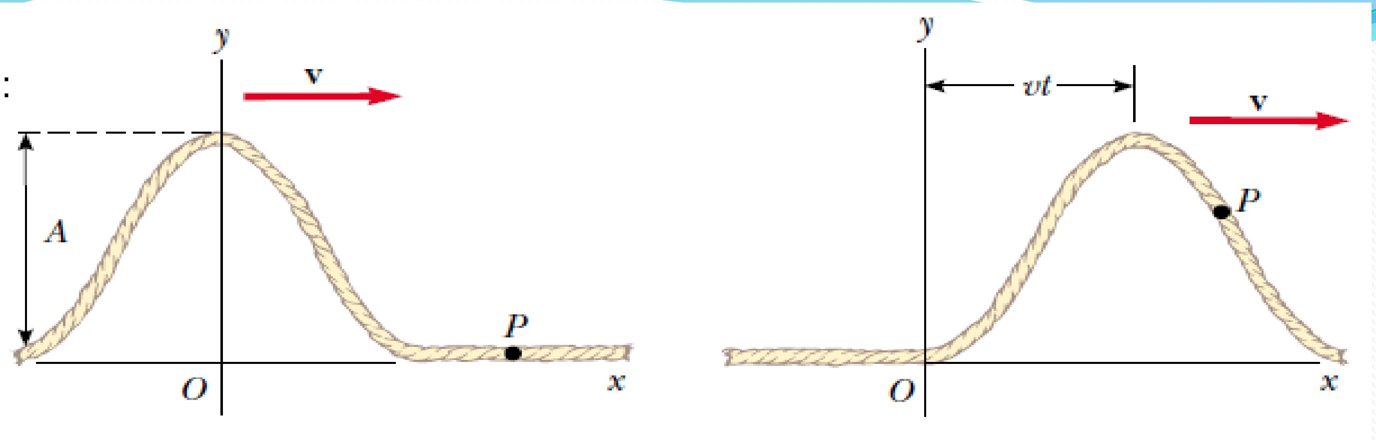
The **speed** of the pulse is ***v***, the pulse has traveled to the right **a distance *vt*** at the time ***t***



(b) Pulse at time $t$

(Fig. -3b). We assume that the shape of the pulse does not change with time.

* Thus, at time $t$, the shape of the pulse is the same as it was at time $t = 0$




(a) Pulse at $t = 0$ (b) Pulse at time $t$

Fig 4 an element of the string at *x* at this time has the same *y* position as an element located at ($x - vt$ had at time $t$ = 0

$$y(x,t) = y(x - vt)$$

If the pulse travels to the left, the transverse positions of elements of the string are described by

$$y(x,t) = f(x + vt)$$

If the pulse travels to the left, the transverse positions of elements of the string are described by

$$y(x,t) = f(x+vt)$$

The function *y, called the* **wave function**, *depends on the two variables x and t.* **For this reason, it is often written y(x, t), which is read "y as a function of x and t."**

Ex: Consider the sinusoidal wave, which shows the position of the wave at $t = 0$. Because the wave is sinusoidal, we expect the wave function at this instant to be expressed as **$y(x, 0) = A \sin ax$**,

Where ***A*** is the **amplitude** and ***a*** is a **constant** must be determined from initial conditions:.

At $x = 0$, we see that $y(0, 0) = A \sin a(0) = 0$, consistent with Fig. -4a. The next value of $x$ for which $y$ is zero is $x = \lambda/2$. Thus,

$$y(\frac{\lambda}{2},0) = A \sin a(\frac{\lambda}{2}) = 0$$

$a(\lambda/2) = \pi$, or $a = 2\pi/\lambda$. Thus, the function describing the positions of the elements of the medium through which the sinusoidal wave is traveling can be written as

$$y(x,0) = A \sin(\frac{2\pi}{\lambda} x)$$

If the wave moves to the right with a speed *v*, then the wave function at some later time *t* is

$$y(x,t) = A\sin\left[\frac{2\pi}{\lambda}(x - vt)\right]$$

The relation between wave speed, wavelength, and period are related by the expression

$$v = \frac{\lambda}{T}$$

In general we have

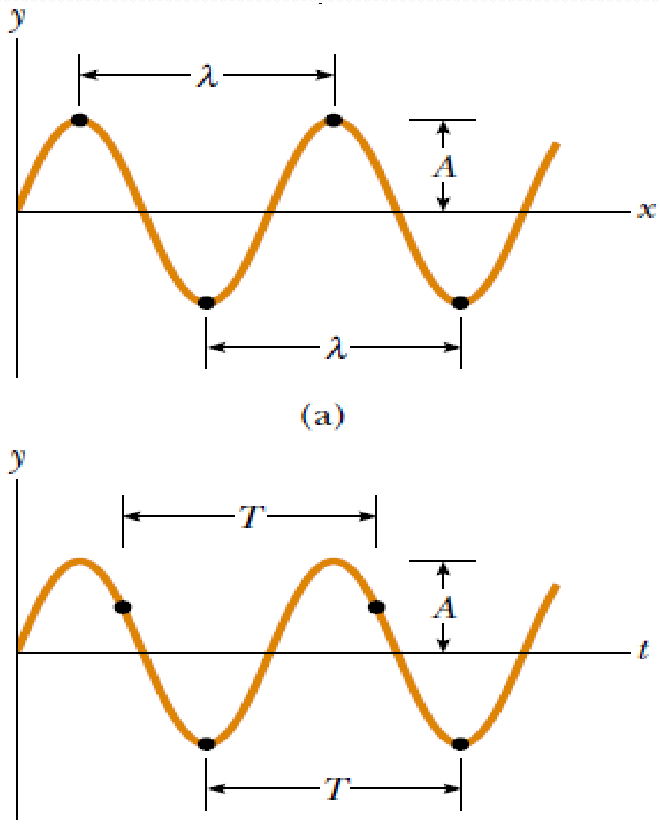
$$y(x,t) = A\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$$

We defined two other quantities, the angular wave number ***k*** (usually called simply the **wave number**) and the **angular frequency**

$$k = \frac{2\pi}{\lambda} \qquad \omega = \frac{2\pi}{T}$$

## 2 - Sinusoidal Waves:

The wave represented by this curve is called a **sinusoidal wave** because the curve is the same as that of the function *sin* plotted against *t*. On a rope, a **sinusoidal wave** could be established by shaking the end of the rope up and down in **simple harmonic motion.( Fig )**



Definitions:

**Wavelength**: The distance from **one crest** to the **next is** called the wavelength ***"λ"***. More generally, the wavelength is the **minimum distance between any two identical points** (such as the crests) on adjacent waves.

**Frequency**: of a periodic wave is **the number of crests** (or troughs, or any other point on the wave) **that pass a given point in a unit time interval**. **The frequency of a sinusoidal wave is related to the period by the expression** $f = 1/T$

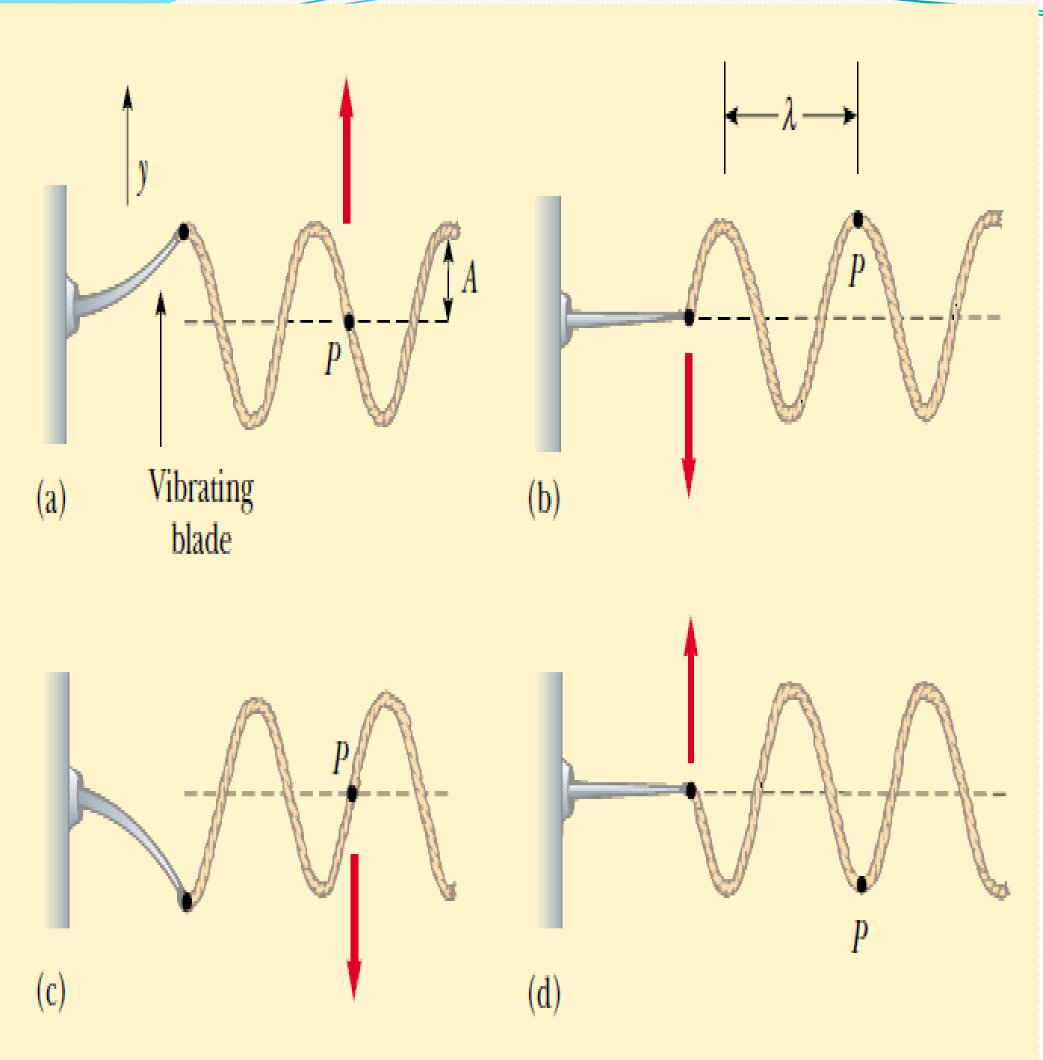
The unit for frequency is second$^{-1}$, or hertz (Hz).

## 3- Sinusoidal Waves on Strings

Note that although each **element oscillates in the *y* direction**, the wave **travels in the *x* direction with a speed *v***. Of course, this is the definition of a transverse wave.

If the wave at $t = 0$ is as described in Figure, and then the wave function can be written as

$$y = A\sin(kx - \omega t)$$




http://phet.colorado.edu/en/simulation/wave-on-a-string

One method for producing a sinusoidal wave on a string. The left end of the string is connected to a blade that is set into oscillation. Every element of the string, such as that at point *P*, oscillates with simple harmonic motion in the vertical direction.

$$y = A\sin(kx - \omega t)$$

$$v_y = \frac{dy}{dt}\Big|_{x=constant} = \frac{\partial y}{\partial t} = -\omega A\cos(kx - \omega t)$$

$$a_y = \frac{dv_y}{dt}\Big|_{x=constant} = \frac{\partial v_y}{\partial t} = -\omega^2 A\sin(kx - \omega t)$$

The maximum values of the transverse speed and transverse acceleration are

simply the absolute values of the coefficients of the cosine and sine functions:

$v_{y,\,max} = \omega A$ , $a_{y,\,max} = \omega^2 A$

# Sinusoidal Waves

Equation of motion of a simple harmonic oscillation is a sine function.

The function describing the position of particles, located at x, of the medium through which the sinusoidal wave is traveling can be written at t=0

$$y = A\sin\left(\frac{2\pi}{\lambda}x\right)$$

Amplitude ($A$)

Wave Length ($\lambda$)

The wave form of the wave traveling at the speed v in +x at any given time t becomes

$$y = A\sin\left(\frac{2\pi}{\lambda}(x - vt)\right)$$

By definition, the speed of wave in terms of wave length and period T is

$$v = \frac{\lambda}{T}$$

Thus the wave form can be rewritten

$$y = A\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$$

Defining, angular wave number $k$ and angular frequency w,

$$k \equiv \frac{2\pi}{\lambda};\ \omega = \frac{2\pi}{T}$$

The wave form becomes

$$y = A\sin(kx - \omega t)$$

Frequency, $f$,

$$f = \frac{1}{T}$$

Wave speed, $v$

$$v = \frac{\lambda}{T} = \frac{\omega}{k}$$

General wave form

$$y = A\sin(kx - \omega t + \phi)$$

**Objectives of part 2:**

**The student will be able to**

- **Demonstrate the speed of wave on the string.**
- **Define the reflection & transmission waves.**
- **The liner equation of the wave.**

- **Define the kinetic and the potential energies.**
- **Define the total energy on the string**
- **Determine the rat of the energy transfer.**

# Speed of Waves on Strings

Which law does this hypothesis based on?

Newton's second law of motion

Based on the hypothesis we have laid out above, we can construct a hypothetical formula for the speed of wave

$$v = \sqrt{\frac{T}{\mu}}$$

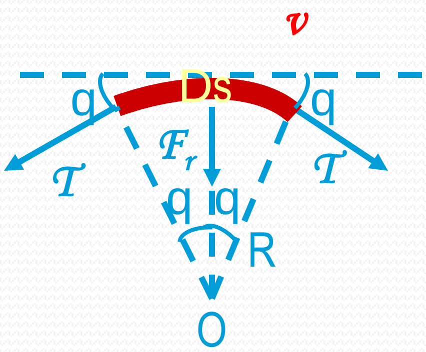
T: Tension on the string
m: Unit mass per length

Is the above expression dimensionally sound?

T=[$MLT^{-2}$], m=[$ML^{-1}$]

$(T/m)^{1/2}=[L^2T^{-2}]^{1/2}=[LT^{-1}]$

# Speed of Waves on Strings




Let's consider a pulse moving to right and look at it in the frame that moves along with the the pulse.

Since in the reference frame moves with the pulse, the segment is moving to the left with the speed $v$, and the centripetal acceleration of the segment is

$$a_r = \frac{v^2}{R}$$

Now what do the force components look in this motion when q is small?

$$\sum F_t = T\cos\theta - T\cos\theta = 0$$

$$\sum F_r = 2T\sin\theta \approx 2\mathrm{T}\theta$$

What is the mass of the segment when the line density of the string is m?

$$m = \mu\Delta s = \mu R 2\theta = 2\mu R\theta$$

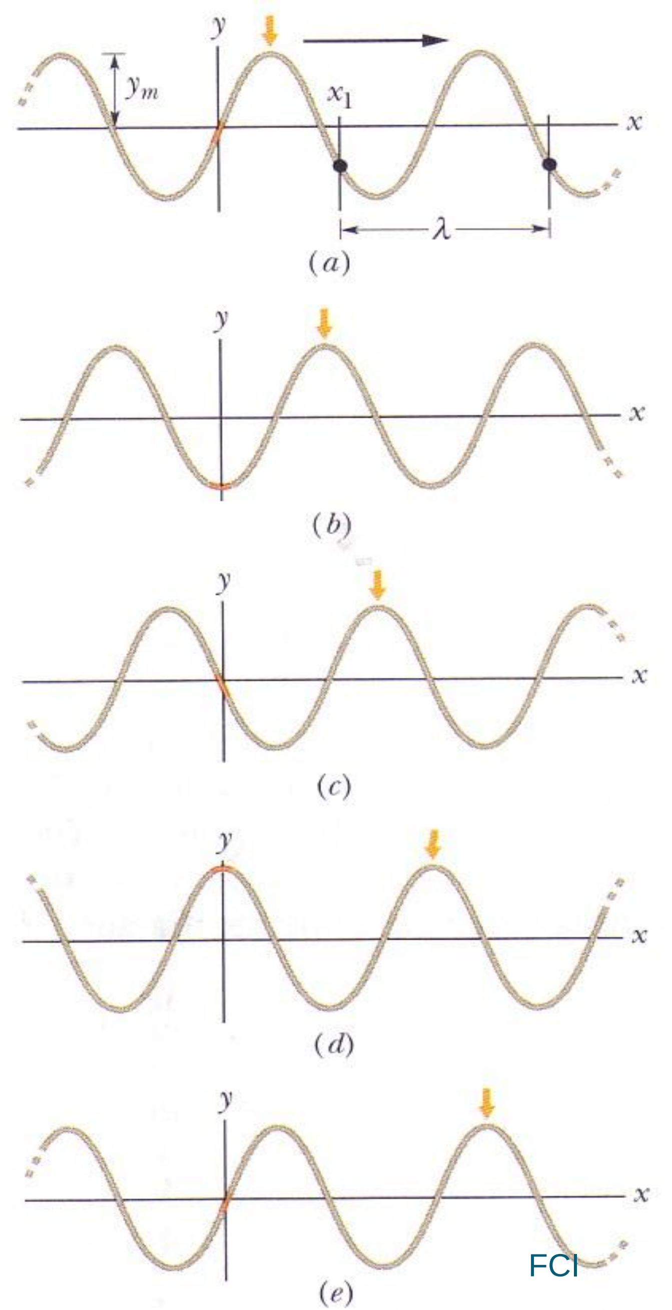
Using the radial force component

$$\sum F_r = ma = m\frac{v^2}{R} = 2\mu R\theta\frac{v^2}{R} = 2T\theta$$

Therefore the speed of the pulse is

$$v = \sqrt{\frac{T}{\mu}}$$

# The speed of a traveling wave




A fixed point on a wave has a constant • value of the phase, *i.e.*

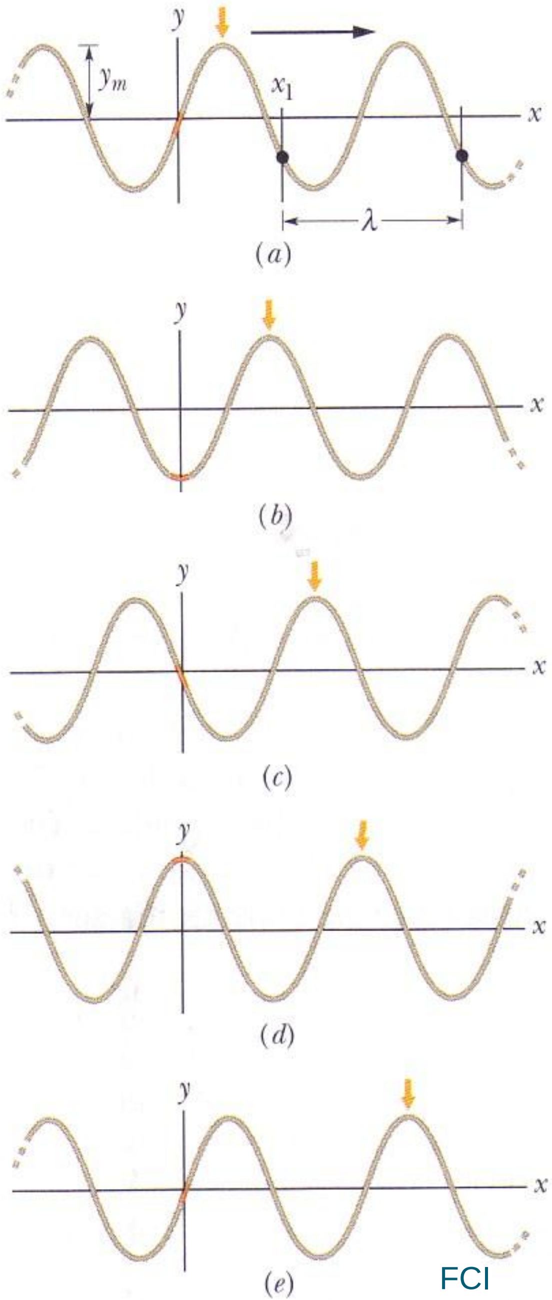
$$kx - \omega t = \text{constant}$$

$$\Rightarrow k\frac{dx}{dt} - \omega = 0 \quad \text{or} \quad \frac{dx}{dt} = v = \frac{\omega}{k}$$

Or

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = f\lambda$$

# The speed of a traveling wave




- For a wave traveling in the opposite direction, we simply set time to run backwards, *i.e.* replace *t* with -*t*.

$$kx + \omega t = \text{constant}$$

$$\Rightarrow k\frac{dx}{dt} + \omega = 0 \qquad \text{or} \qquad \frac{dx}{dt} = v = -\frac{\omega}{k}$$

$$y(x,t) = y_m \sin\left(kx + \omega t\right)$$

- So, general sinusoidal solution is:

$$y(x,t) = y_m \sin\left(kx \pm \omega t\right)$$

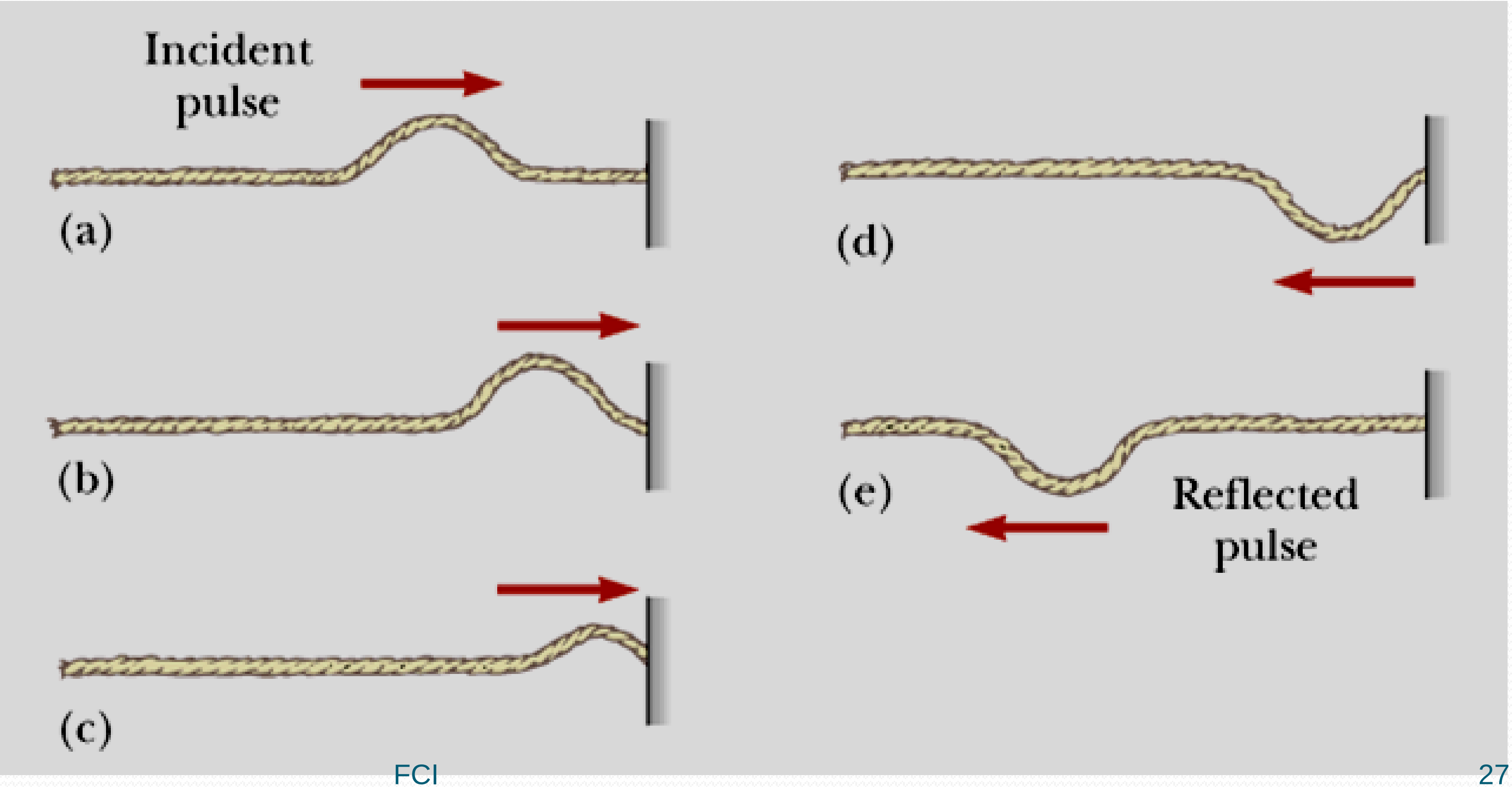
- In fact, any function of the form

is a solution.

# Reflection of a traveling wave on rigid wall

- If a wave encounters a "denser", new medium, or a rigid wall, it gets reflected.

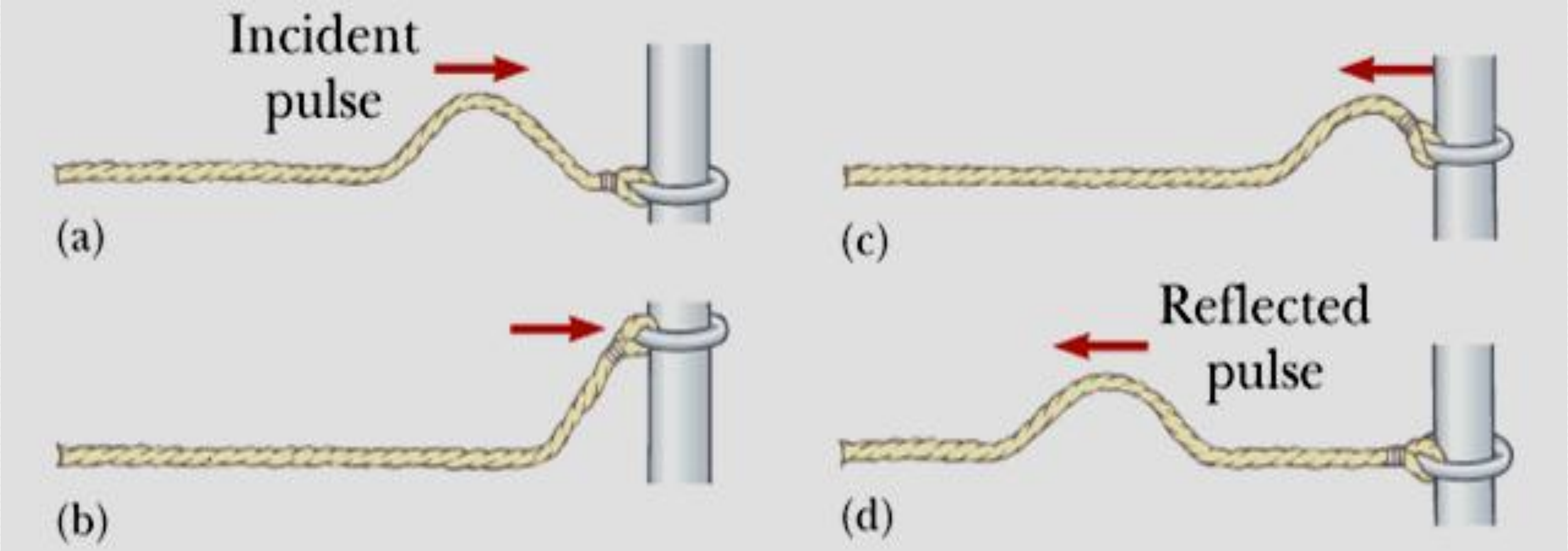
- In this case the reflected pulse is inverted upon reflection



# Reflection of a traveling wave on a loose end

- If a wave encounters a "less dense" medium or an end it also gets reflected.

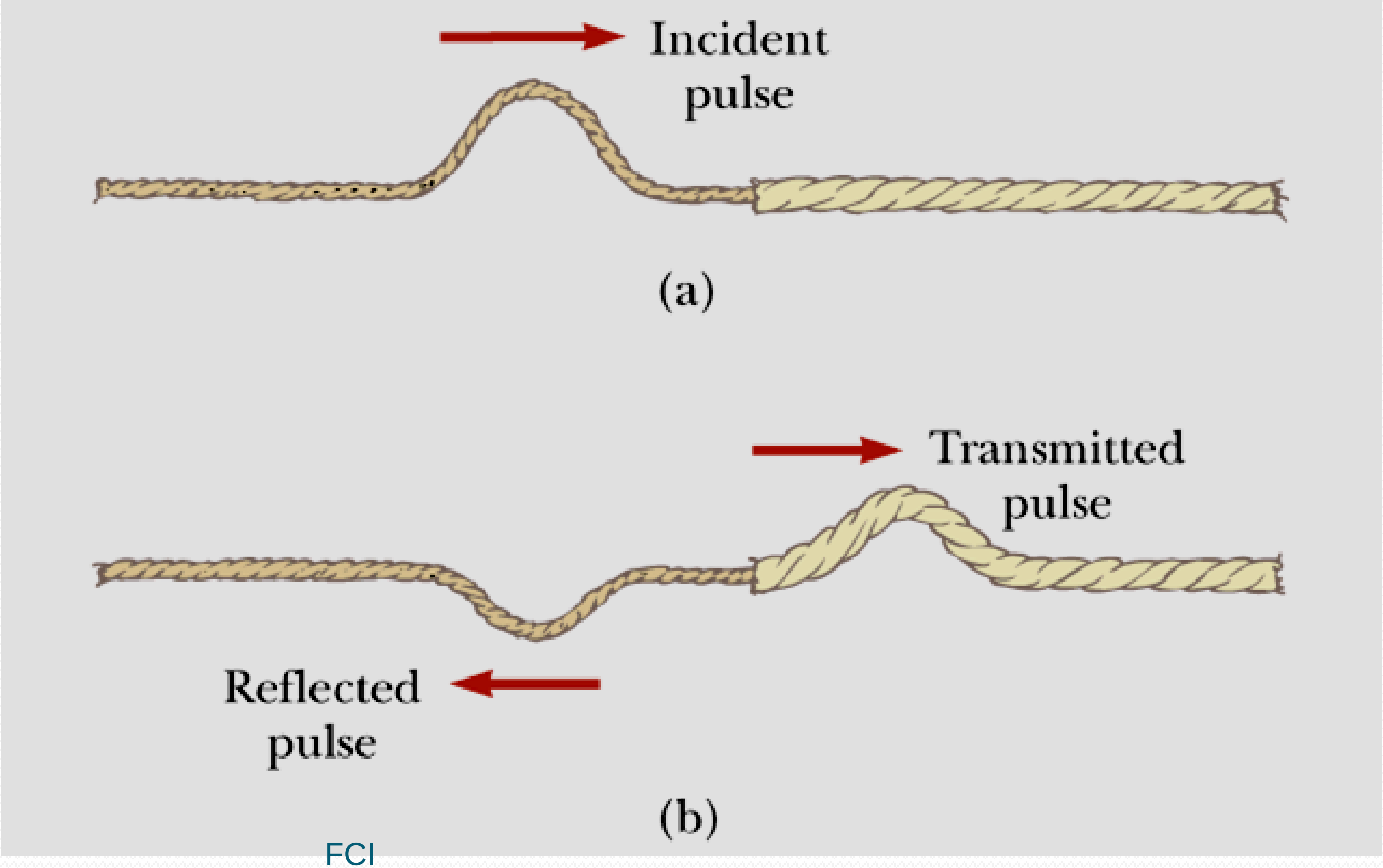
- In this case the reflected pulse is not inverted upon reflection.



# Transmission: Light string → heavier string

The transmitted pulse is not inverted.

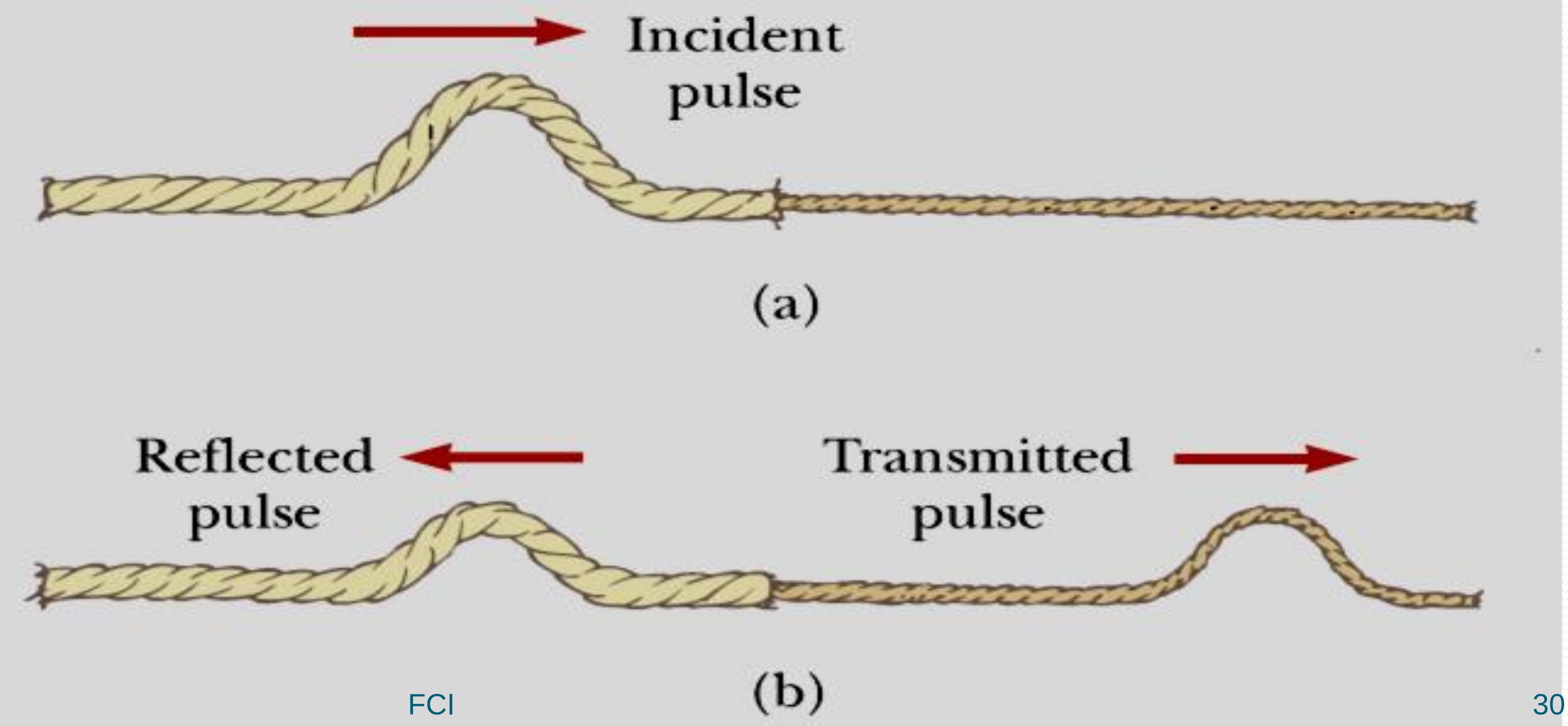
The reflected pulse is inverted.



## Transmission: Heavy string → light sting

The transmitted pulse is not inverted.

The reflected pulse is not inverted.



# Reflection and Transmission

A pulse or a wave undergoes various changes when the medium it travels changes.

Depending on how rigid the support is, two radically different reflection patterns can be observed.

1. The support is **rigidly fixed**: The reflected pulse will be **inverted to the original** due to the force exerted on to the string by the support in reaction to the force on the support due to the pulse on the string.
2. The support is **freely moving**: The reflected pulse will **maintain the original shape** but moving in the reverse direction.

If the boundary is intermediate between the above two extremes, part of the pulse reflects, and the other undergoes transmission, passing through the boundary and propagating in the new medium.

When a wave pulse travels from medium A to B:

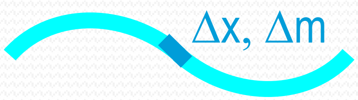
- $\mathbf{v_A} > \mathbf{v_B}$ (or $m_A < m_B$), the pulse is inverted upon reflection.
- $\mathbf{v_A} < \mathbf{v_B}$ (or $m_A > m_B$), the pulse is not inverted upon reflection.

# Rate of Energy Transfer by Sinusoidal Waves on Strings

Waves traveling through medium carries energy.

When an external source performs work on the string, the energy enters into the string and propagates through the medium as wave.

What is the potential energy of one wave length of a traveling wave?




Elastic potential energy of a particle in a simple harmonic motion

$$U = \frac{1}{2} k y^2$$

Since $\omega^2 = k/m$

$$U = \frac{1}{2} m \omega^2 y^2$$

The energy $\Delta U$ of the segment $\Delta m$ is

$$\Delta U = \frac{1}{2} \Delta m \omega^2 y^2 = \frac{1}{2} \mu \Delta x \omega^2 y^2$$

# Rate of Energy Transfer by Sinusoidal Waves on Strings

As $\Delta x \rightarrow 0$, the energy $\Delta U$ becomes

$$dU = \frac{1}{2}\mu\omega^2 y^2 dx$$

Using the wave function, the energy is

$$dU = \frac{1}{2}\mu\omega^2 A^2 \sin^2(kx - \omega t)dx$$

For the wave at t=0, the potential energy in one wave length, $\lambda$, is

$$U_\lambda = \frac{1}{2}\mu\omega^2 A^2 \int_{x=0}^{x=\lambda} \sin^2 kx dx = \frac{1}{2}\mu\omega^2 A^2 \int_{x=0}^{x=\lambda} \frac{1-\cos 2kx}{2} dx$$

$$= \frac{1}{2}\mu\omega^2 A^2 \left[\frac{1}{2}x - \frac{1}{4k}\sin\frac{4\pi x}{\lambda}\right]_{x=0}^{x=\lambda} = \frac{1}{4}\mu\omega^2 A^2 \lambda$$

Recall $k = 2\pi/\lambda$

# Rate of Energy Transfer by Sinusoidal Waves

How does the kinetic energy of each segment of the string in the wave look?

Since the vertical speed of the particle is $$v_y = -\omega A\cos(kx - \omega t)$$

The kinetic energy, ΔK, of the segment Δm is

$$\Delta K = \frac{1}{2}\Delta m v_y^2 = \frac{1}{2}\mu\Delta x\omega^2 A^2 \cos^2(kx - \omega t)$$

As Δx→0, the energy ΔK becomes $$dK = \frac{1}{2}\mu\omega^2 A^2 \cos^2(kx - \omega t)dx$$

For the wave at t=0, the kinetic energy in one wave length, λ, is

$$K_\lambda = \frac{1}{2}\mu\omega^2 A^2 \int_{x=0}^{x=\lambda} \cos^2 kx dx = \frac{1}{2}\mu\omega^2 A^2 \int_{x=0}^{x=\lambda} \frac{1+\cos 2kx}{2} dx$$

$$= \frac{1}{2}\mu\omega^2 A^2 \left[\frac{1}{2}x + \frac{1}{4k}\sin\frac{4\pi x}{\lambda}\right]_{x=0}^{x=\lambda} = \frac{1}{4}\mu\omega^2 A^2 \lambda$$

Recall $k = 2\pi\lambda$

Just like harmonic oscillation, the total mechanical energy in one wave length, λ, is

$$E_\lambda = U_\lambda + K_\lambda = \frac{1}{2}\mu\omega^2 A^2 \lambda$$

# Rate of Energy Transfer by Sinusoidal Waves

As the wave moves along the string, the amount of energy passes by a given point changes during one period. So the power, the rate of energy transfer becomes

$$P = \frac{E_\lambda}{\Delta t} = \frac{1}{2}\mu\omega^2 A^2 \frac{\lambda}{T}$$

$$= \frac{1}{2}\mu\omega^2 A^2 v$$

P of any sinusoidal wave is proportion to the square of angular frequency, the square of amplitude, density of medium, and wave speed.

# The Linear Wave Equation

If the wave function has the form

- $y = A\sin(kx - \omega t + \varphi)$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

$$v^2 = \frac{\omega^2}{k^2} = \frac{T}{\mu}$$

$$v = \sqrt{\frac{T}{\mu}}$$

This is the linear wave equation as it applies to waves on a string.