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Course Name: Physics 2 Part II: Waves

Content : Chapter 1: Oscillation Motion

- Motion of a spring
- Energy of the Simple Harmonic Oscillator
- Comparing SHM with uniform motion

Ch. 1 : Oscillation Motion

***Objectives:**
The Student will be able to;

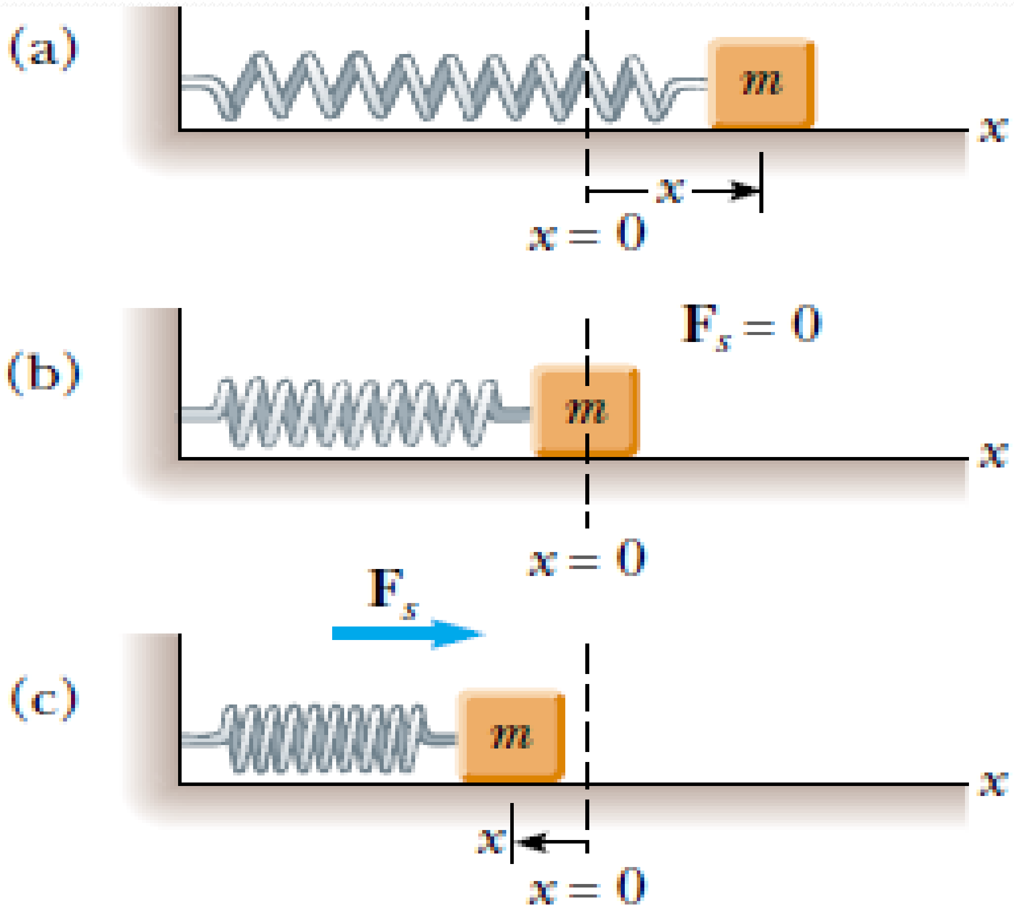
- Define the oscillatory motion.
- Represent the mathematical form of the SHM.
- Specify the energy of the oscillation.
- Verify the law of the conservation energy.
- Compare between SHM and the uniform motion.

Def: Periodic motion is a motion of an object that regularly repeats , the object returns to a given position after a fixed time interval.

For example the Earth returns to the same potion in its orbit around the Sun each year, resulting in the variation among the four seasons.

A special kind of **periodic motion occurs in mechanical systems:** when the force acting on an object is proportional to the position object relative to some equilibrium position.

If this **force** is always **directed toward the equilibrium position,** **the motion is called simple harmonic motion.**



a) A block is displaced to the right from the equilibrium position $x > 0$.

b) When the block is at its equilibrium $x = 0$

c) A block is displaced to the left of the equilibrium $x < 0$.

A block attached to a spring moving on the <u>frictionless surface</u>.

Motion of a spring:

Consider a block of mass m attached to the end of a spring, with the block free to move on a horizontal, **frictionless surface**.
When the block at the position called the "equilibrium position" of the system.

The block a force that is proportional to the position and given by **Hookye's law**

$$F_s = -kx$$

,

Applying **Newton's law** to the motion of the block, with last equation providing the net force in the direction

$$\sum F_x = ma_x \qquad -kx = ma_x \qquad a_x = -\frac{k}{m}x$$

•The acceleration is proportional to the position of the block, and its direction is opposite the direction of the displacement from the equilibrium position.

•Systems that behave in this way is called **Simple Harmonic Motion.**

•**Object moves with Simple Harmonic Motion its acceleration is proportional to its position and is oppositely directed to the displacement from equilibrium.**

Mathematical Representation of Simple Harmonic Motion:

To develop the mathematical representation of motion, choose **x** as the axis along which the oscillation occurs, by definition

$$a = dv/dt = d^2x/dt^2$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$\omega^2 = \frac{k}{m}$$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

We seek a function *x(t) whose second derivative is the same as the original function with a negative sign* and multiplied by ‘ ω^2. **The trigonometric functions sine and cosine exhibit this behavior**, so we can build a solution around one or both of these. We chose the solution as,

$$x(t) = A\cos(\omega t + \phi)$$

where *A,ω and ϕ (are constants).*

$$x(t) = A\cos(\omega t + \varphi)$$

$$\frac{dx}{dt} = A\frac{d}{dt}\cos(\omega t + \varphi) = -\omega A\sin(\omega t + \varphi)$$

$$\frac{d^2x}{dt^2} = -\omega A\frac{d}{dt}\sin(\omega t + \varphi) = -A\omega^2\cos(\omega t + \varphi)$$

The constant angle ***Φ*** is called the phase constant, if the particle at max position *x = A* at *t = 0* , the phase constant is ***Φ*= *0*** , as the graphical representation of the motion

The period ***T*** of the motion is the time interval required for the particle to go through one full cycle of its motion.
The inverse of the period is called the frequency ***f***

$$T = 2\pi / \omega \qquad\qquad f = \frac{1}{T} = \frac{\omega}{2\pi}$$

We may represent the period and the frequency of the motion for the particle spring system in terms of the characteristic m and k of the system as,

$$T = {}^{2\pi}\!/\!_{\omega} = 2\pi / \sqrt{\frac{k}{m}} \qquad f = {}^{1}\!/\!_{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

The period and the frequency depend only on **the mass of the block and the force constant of the spring**, not on the **parameters of the motion** (*A* and φ)

How to evaluate the constants, A and **φ** from the initial conditions.

Suppose the motion initiate from the equilibrium by distance A and releasing it from rest at
t =0 , x(0) = A and v(0) =0:

$$x(0) = A\cos\varphi = A$$

$$v(0) = -\omega A\sin\varphi = 0$$

If we choose φ =0 , $x = A\cos\omega t$

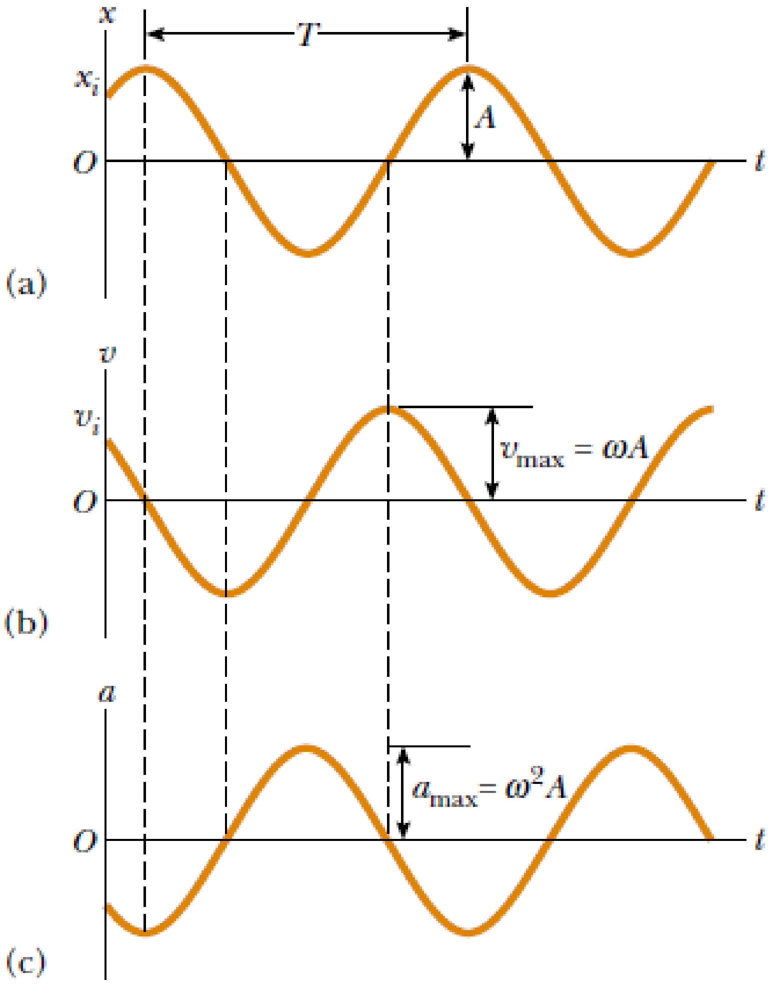
At *t=0 , x(0) =A,* because cos φ =1.

•The **velocity** and the **acceleration** of the particle undergoing simple harmonic motion from the equations

$$v = \frac{dx}{dt} = -\omega A \sin(\omega t + \varphi) \qquad a = \frac{d^2x}{dt^2} = -\omega^2 A \cos(\omega t + \varphi)$$

The **max values** of the **velocity** as, the sin function oscillate between $\pm$ 1 then

$$v_{\max} = \omega A = \sqrt{\frac{k}{m}} A \qquad a_{\max} = A\omega^2 = \frac{k}{m} A$$

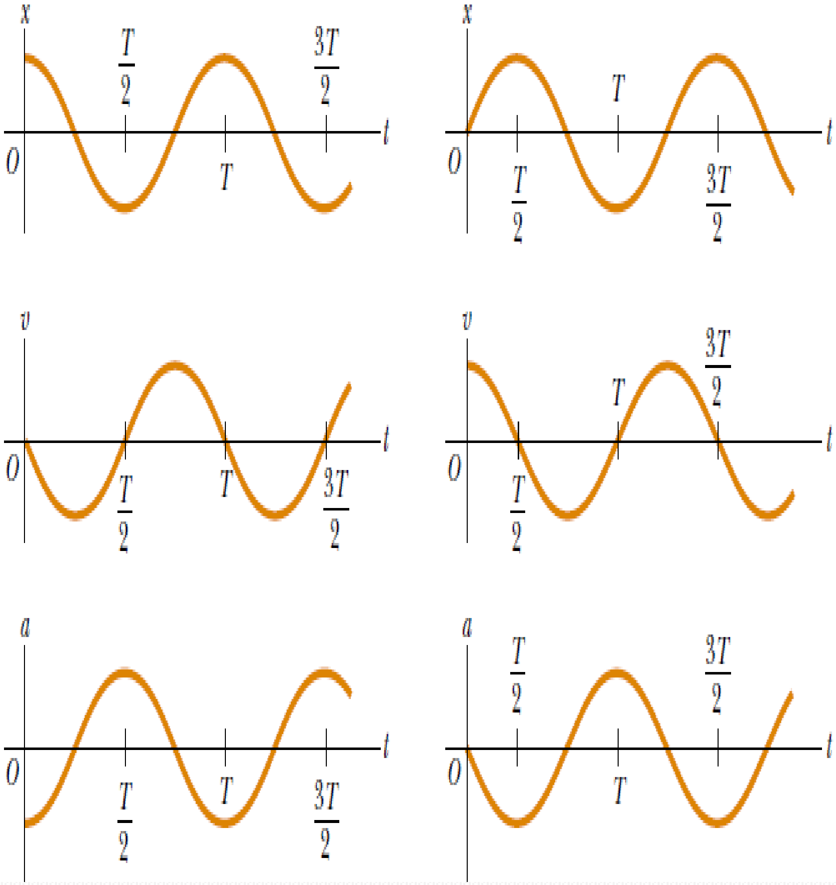


Graphical representation of SHM.

a) Shows the relation between the position versus time.

b) Velocity versus time and the velocity is 90° out of phase with the position

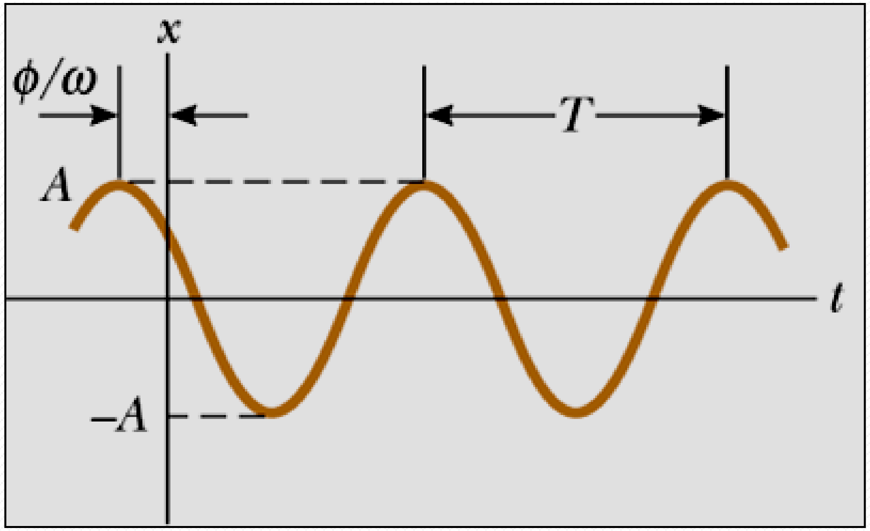
c) Acceleration versus time and the acceleration is 180° out of phase with the position.



(a) the position, velocity and acceleration Vrs. Time for a block undergoing SHM under the initial conditions at t=0, x(0) =A and V(0) = 0.

B) THE Position, velocity and acceleration Vrs. Time for a block undergoing SHM under the initial conditions at t=0, x(0) =0 and V(0) = v_f.

Properties of simple harmonic motion



Position of particle at time t:

$$x(t) = A\cos(\omega t + \phi)$$

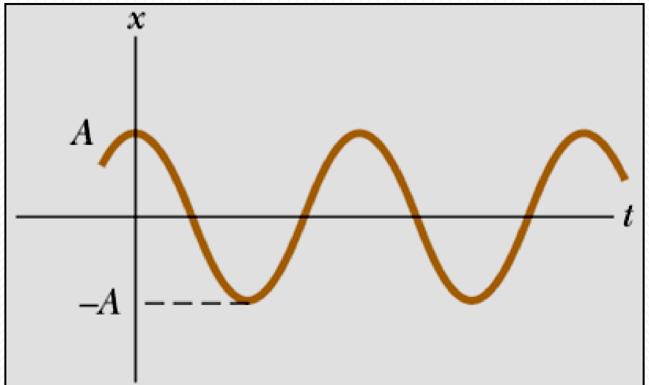
A…amplitude

ω…Angular frequency

ϕ…phase constant, phase angle

T…period, time to complete one full cycle

(ωt + Φ)…phase



$$x(t) = A\cos(\omega t)$$

Properties of simple harmonic motion

Displacement: $x(t) = A\cos(\omega t + \phi)$

Period T: $T = \dfrac{2\pi}{\omega}$

Frequency: $f = \dfrac{1}{T} = \dfrac{\omega}{2\pi}$

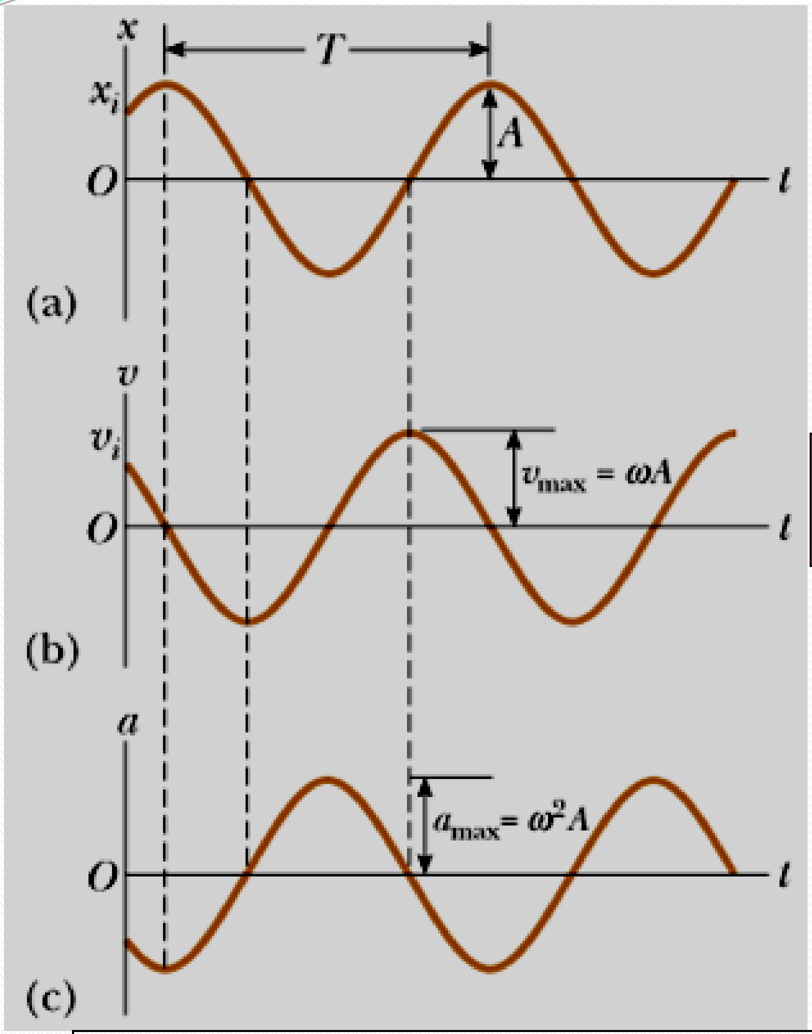
Units: 1/s = 1 Hz

Angular frequency: $\omega = 2\pi f = \dfrac{2\pi}{T}$

Velocity: $v(t) = -\omega A\sin(\omega t + \phi)$

Acceleration: $a(t) = -\omega^2 A\cos(\omega t + \phi)$

Properties of simple harmonic motion



$$x_{max} = A$$

$$v_{max} = -\omega A$$

$$a_{max} = -\omega^2 A$$

- Displacement, velocity and acceleration vary sinusoidally.
- Acceleration of particle is proportional to the displacement, but is in the opposite direction ($a = - \omega^2 \cdot x$).
- The frequency and period of the motion are independent of the amplitude.

Phase of velocity differs by $\pi/2$ or 90° from phase of displacement.

Phase of acceleration differs by π or 180° from phase of displacement.

Energy of the Simple Harmonic Oscillator:

To express the mechanical energy of the spring system.
Assume we have frictionless surface and a massless spring. **The kinetic energy of the block is**,

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \varphi)$$

-**The potential energy stored in the spring**,

$$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \cos^2(\omega t + \varphi)$$

- the total energy of the simple harmonic oscillator as,

$$E = K + U = \frac{1}{2}kA^2\left[\sin^2(\omega t + \varphi) + \cos^2(\omega t + \varphi)\right]$$

then the **equation reduced** to

$$E = \frac{1}{2}kA^2$$

The last equation **is the total mechanical energy** of a simple harmonic oscillation is a **constant** of the motion and is **proportional to the square of the amplitude.**

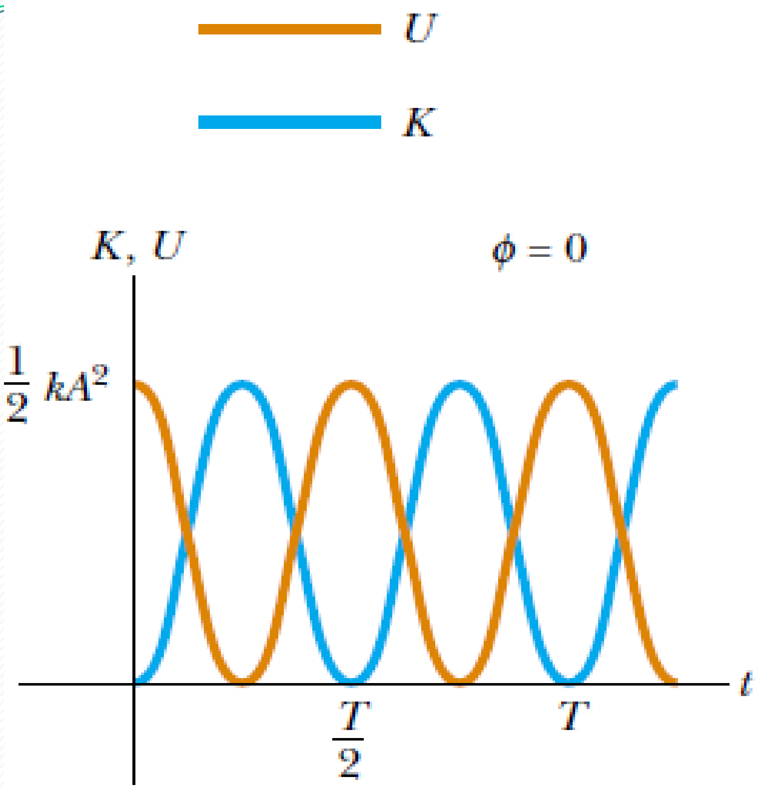
Note that:

U is small when K is a large, and vice versa, that the sum must be constant.

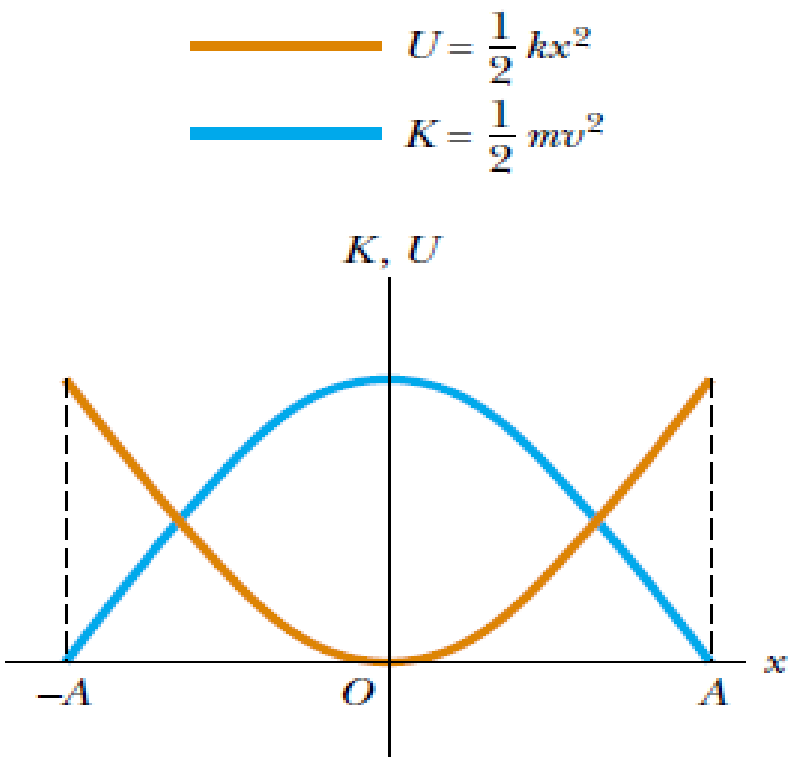
1- **When x= ±A** the total energy is equal to the max potential energy stored in the spring because *v* =0, there is no kinetic energy, the total energy is equal

$$E = \frac{1}{2}kA^2$$

2- At the **equilibrium position** x = 0, the total energy is kinetic energy which is equal $E = \frac{1}{2}kA^2$

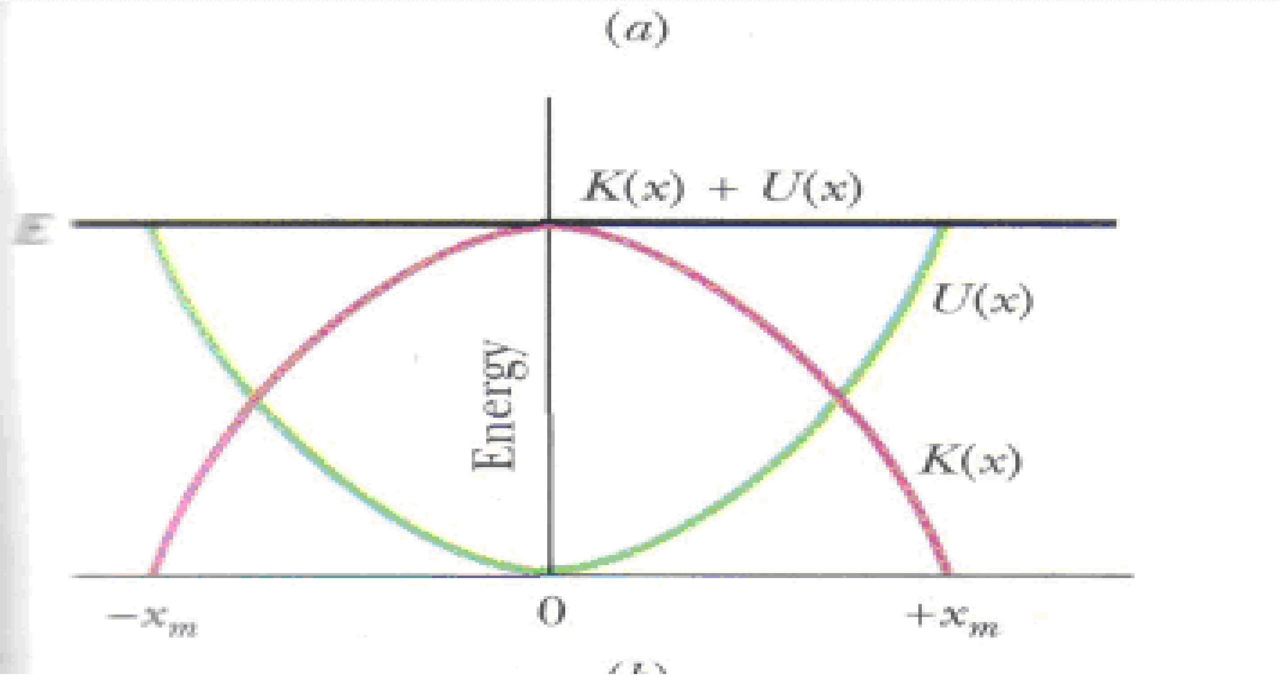


Kinetic energy and potential energy vs time for simple harmonic oscillator with Φ=0



Kinetic energy and potential energy vs position for simple harmonic oscillator,
the total energy is constant

Conservation of Energy For A Spring in Horizontal Motion



E = Kinetic + Elastic Potential

E = ½ mv^2 + ½ kx^2 = Constant

At maximum displacement, velocity is zero and all energy is elastic potential, so total energy is equal to ½ kA^2

Self study: To find the velocity @ any displacement… do conservation of Energy… @ some point at max displacement, the energy equal

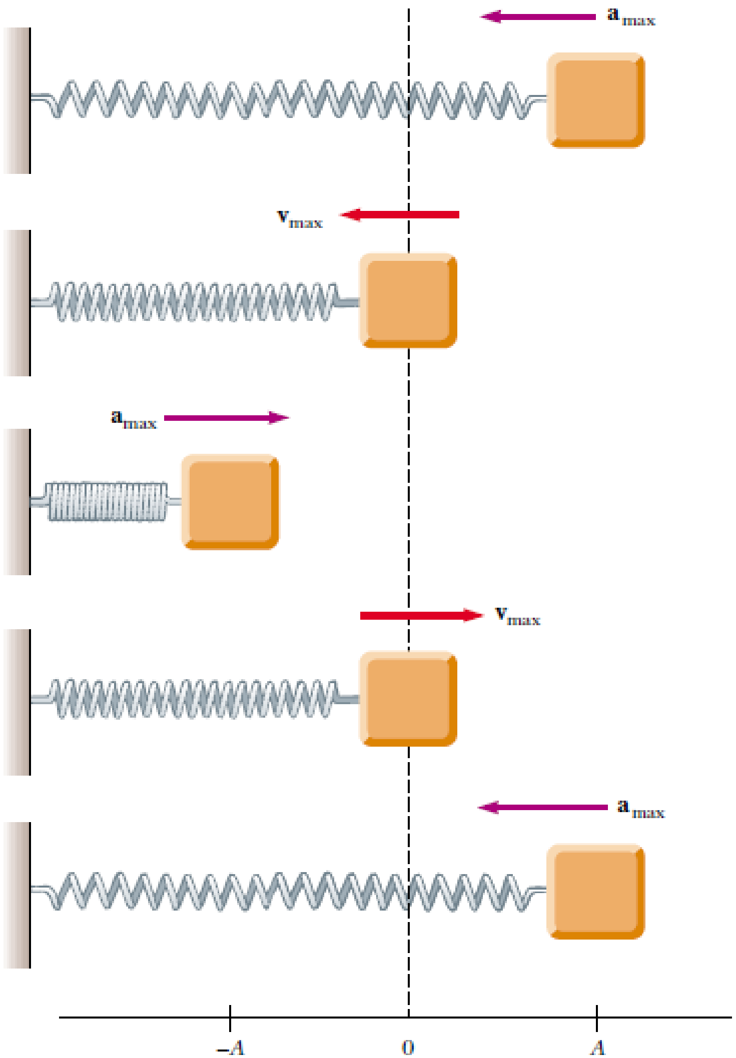
$\frac{1}{2} mv^2 + \frac{1}{2} kx^2 = \frac{1}{2} kA^2$

Solving for v

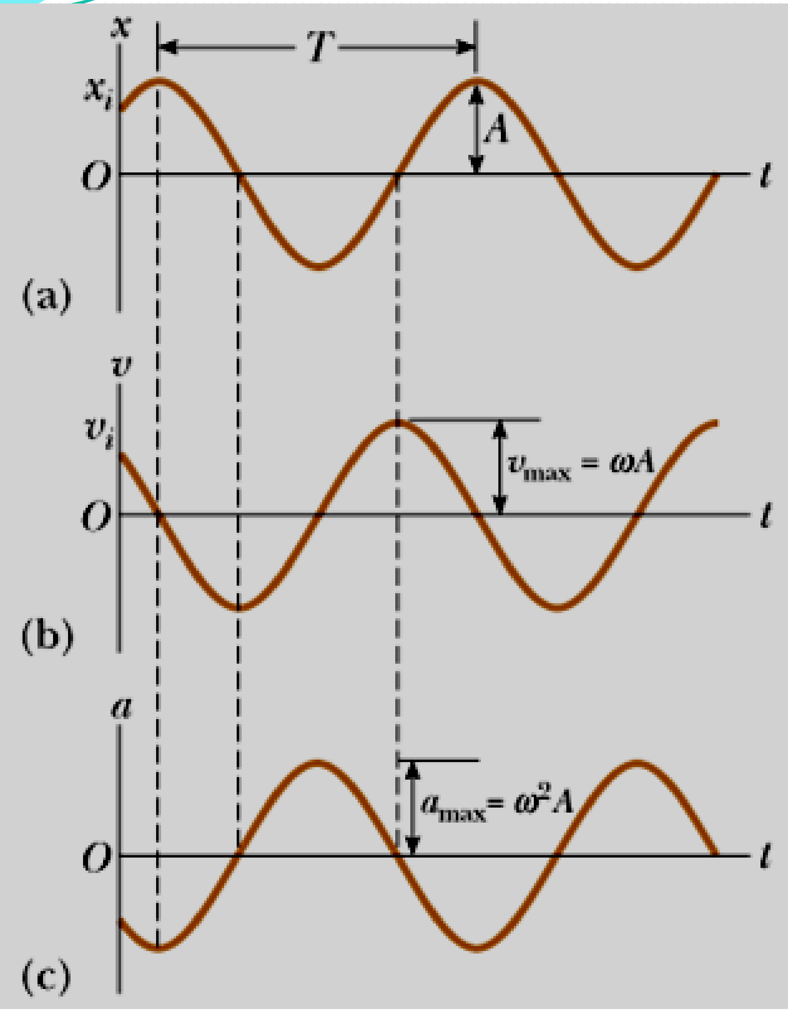
$$v = \pm\sqrt{\frac{k}{m}\left(A^2 - x^2\right)}$$

$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

$$v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)} = \pm\omega\sqrt{(A^2 - x^2)}$$



t	x	v	a	K	U
0	A	0	$-\omega^2 A$	0	$\frac{1}{2}kA^2$
$T/4$	0	$-\omega A$	0	$\frac{1}{2}kA^2$	0
$T/2$	$-A$	0	$\omega^2 A$	0	$\frac{1}{2}kA^2$
$3T/4$	0	ωA	0	$\frac{1}{2}kA^2$	0
T	A	0	$-\omega^2 A$	0	$\frac{1}{2}kA^2$



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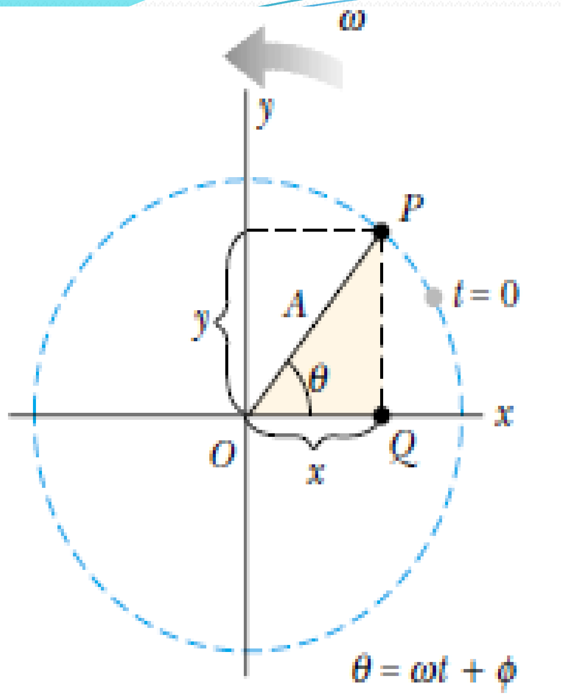
$T/2$	$-A$	0	$\omega^2 A$	0	$\frac{1}{2}kA^2$
$3T/4$	0	ωA	0	$\frac{1}{2}kA^2$	0
T	A	0	$-\omega^2 A$	0	$\frac{1}{2}kA^2$

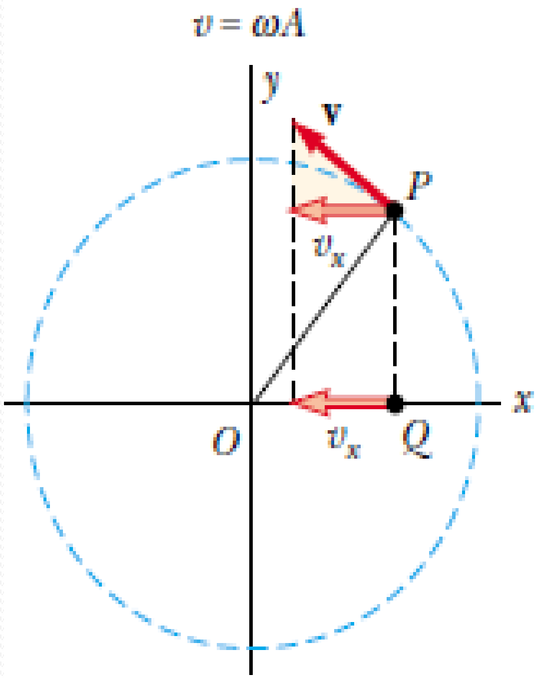
ω

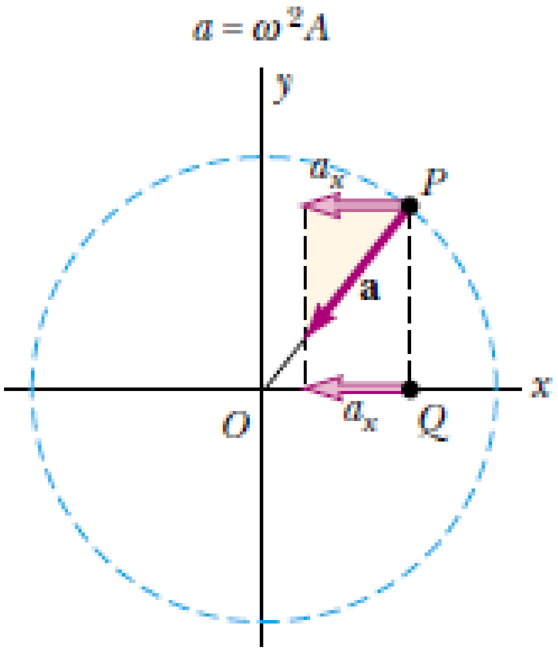
Comparing SHM with uniform motion:

Consider a particle at point *P* moving in a circle of radius *A* with constant angular velocity .ω

At time t the angle between *OP* and the x axis is , $(\omega t+\varphi)$ where φ is the angle the *Op* makes with x axis at *t=0* . The projection of P on the x axis point Q, moves back and forth along a line parallel to the diameter of the circle, between the limits .We see that the x coordinate of p and Q is given by $x = A\cos(\omega t+\varphi)$







The x coordinate of point **P** and **Q** are equal and vary in time according to the expression

$$x = A\cos(\omega t + \varphi)$$

The x component of the velocity of **P** equals the velocity of **Q**

The x component of the acceleration of **P** equals the acceleration of **Q**

Simple harmonic motion along straight line can be represented by the projection of uniform circular motion along a diameter"

The relation between linear and angular velocity for circular motion is $v = \omega r$

The acceleration of the point P on the circle is directed readily inward toward O and has a magnitude given by

$$\frac{v^2}{A} = \omega^2 A$$

Self Study:

Object hanging from a vertical spring with mass m, the object will stretch to a new position y =0. Verify the net force acting on the mass m.

Quiz 1:

A block on the end of a spring is pulled to position $x = A$ and released. In one full cycle of its motion, through what total distance does it travel?

(a) *A/2* (b) *A* (c) *2A* (d) *4A*

Ans:

(d). From its maximum positive position to the equilibrium position, the block travels a distance *A. It then goes* an equal distance past the equilibrium position to its maximum negative position. It then repeats these two motions in the reverse direction to return to its original position and complete one cycle.

Quiz 2:

An object of mass m is hung from a spring and set into oscillation. The period of the oscillation is measured and recorded as T. The object of mass m is removed and replaced with an object of mass $2m$. When this object is set into oscillation, the period of the motion is

- $2T$
- $\sqrt{2}T$
- T
- $T/\sqrt{2}$
- $T/2$

Ans:

According to the equation, the period is proportional to the square root of the mass.

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

Example 1:

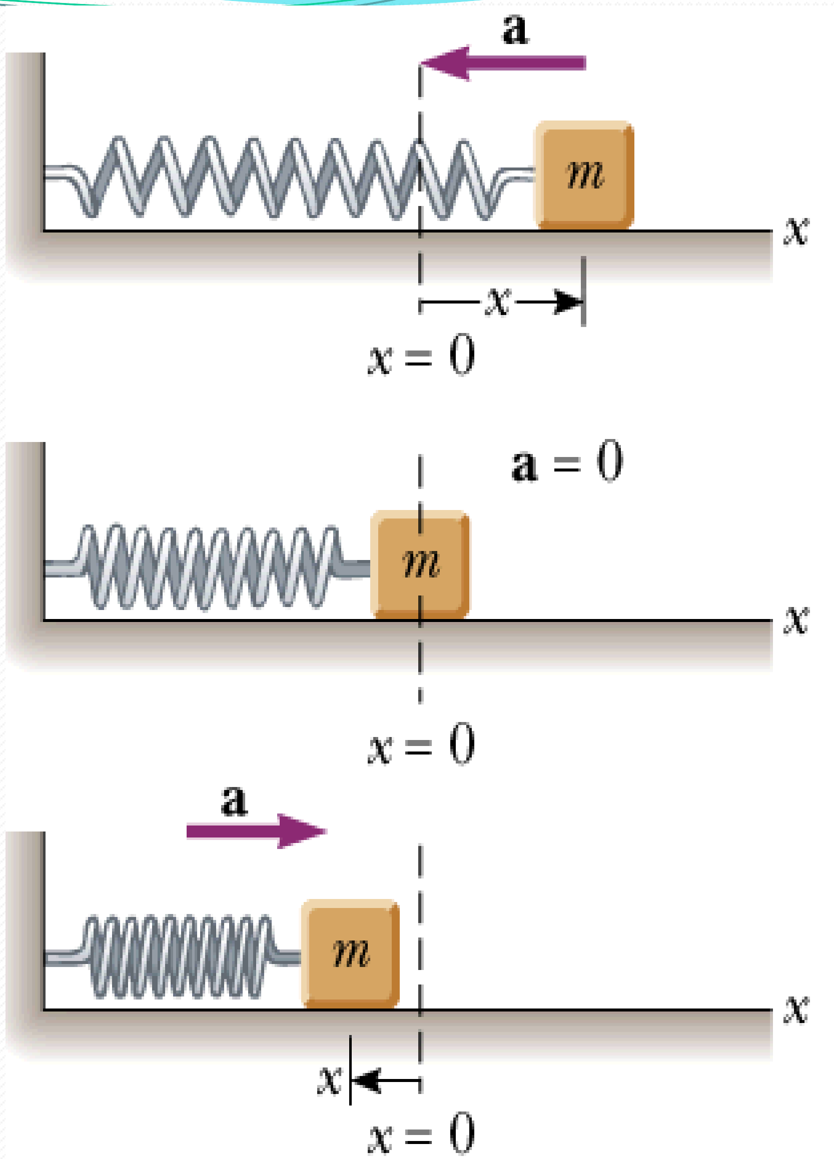
A mass m oscillates with an amplitude of 4.00 m, a frequency of 0.5 Hz and a phase angle of $\pi/4$.

(a) What is the period T?

(b) Write an equation for the displacement of the particle.

(c) Determine the position, velocity and acceleration of the object at time t = 1.00s.

(d) Calculate the maximum velocity and acceleration of the object.



Problem1: At what point during the oscillation of a spring is the force on the mass greatest?

Ans: Recall that $F = -kx$. Thus the force on the mass will be greatest when the displacement of the block is maximum, or

when $x = \pm x_m$.

Problem 2: What is the period of oscillation of a mass of 40 kg on a spring with constant $k = 10$ N/m?

Ans: We have derived that.

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

To find the period of oscillation we simply plug into this quation:

$T = 4\pi$ seconds

شكرا لحسن استماعكم