REWITABLE GROUPS AND VARIOUS GENERALIZATIONS

By
Mustafa Ibrahim Abdelrahman Mohammed Elashiry

A Thesis Submitted in Partial Fulfillment of The Requirements for the Degree of
Doctor of Philosophy of Science

In
(Mathematics - Algebra)

Department of Mathematics
Faculty of Science - Fayoum University

FAYOUM UNIVERSITY

2012
Abstract.

Let $n > 1$ be an integer. A group $G$ is called $n$-rewritable if for every $n$-tuple $(x_1, x_2, \ldots, x_n)$ of the elements of $G$ there exist distinct permutations $\tau \cdot \sigma \in \text{Sym}_n$, such that $x_{\sigma(1)} x_{\sigma(2)} \cdots x_{\sigma(n)} = x_{\tau(1)} x_{\tau(2)} \cdots x_{\tau(n)}$. We also use $Q_n$ to denote the class of groups having this property. If one can always choose $\tau = 1$, one obtains the class of $P_n$-groups ($n$-permutable groups).

The complete classification of these classes were given in [6] and [19] respectively. It is shown there that the class of such groups coincides with the class of finite-by-abelian-by-finite groups.

Our goal is to continue the study of structure of these classes and its various generalizations. In fact, we have obtained significant new results on these groups depending on bounding the orders of certain subgroups and bounding the exponents of certain homomorphic images. Moreover, we proved the long standing conjecture of Blyth that $n$-rewritable groups are $m$-permutable for a suitable explicit function $m$ of $n$. These results improve the past results in the literature.