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Turbulent Dynamo and Magnetic Helicity Transport in Strongly Magnetized , Collisionless Fusion Plasmas

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ABSTRACT

Based on a multiple time scales approach, the time evolution of the total magnetic helicity on the resistive diffusion time scale (RDMHD) has been derived. It was shown that for the case of strongly magnetized, collisionless fusion plasmas, dynamo α-effect is merely due to MHD mechanism in consistent with the experimental measurements which have been detected in reversed field pinch (RFP) plasmas. Furthermore, It was emphasized that the effect of turbulent dynamos on magnetic helicity transport depends critically on the nature of the turbulence. When the turbulence is electromagnetic, the dynamo α-effect converts helicity from turbulent, small-scale field to mean, large-scale field. When the turbulence is electrostatic the dynamo α-effect transports the mean field helicity across space without dissipation. In all cases, it was shown that on the resistive time scale, the α-effect conserves the total magnetic helicity against the resistive effect, the result which explains the long discharge time of the reversed field pinch (RFP).

1. INTRODUCTION

Turbulent dynamo plays an important role in magneto-hydrodynamic (MHD) flows or plasmas. Planetary magnetic fields are considered to be maintained by motions of electrically conducting fluids in plants [1,2]. Recently, the turbulent dynamo problem has attracted much attention in the study of plasma turbulence in nuclear fusion experiments such as spheromak plasmas [3] and reversed field pinch (RFP) plasmas [4-18]. Unlike many astrophysical systems which are driven by a combination of thermal, rotational, and gravitational energies, the laboratory pinch plasmas are driven magnetically [19]. When the system is overdriven, the resultant instabilities cause magnetic and flow fields to fluctuate, and their correlation induces turbulent electromotive forces along the mean magnetic field (or equivalently dynamo like α-effect). This α-effect drives mean parallel electric current, which in turn modifies the initial background mean magnetic structure towards the stable regime. This cycle or the so-called self-organization process happens in magnetized plasmas on a time scale much shorter than the classical resistive diffusion time scale [4]. This paper is devoted, in particular, to study the dynamo in reversed field pinch (RFP) plasmas, where many experimental [6-12] as well as theoretical studies have been performed [13-18]. The reversed field pinch (RFP) is an axisymmetric toroidal confinement device in which the plasma is confined by a poloidal magnetic field induced by strong toroidal plasma current and a toroidal magnetic field generated by a poloidal plasma current and external windings. Independently from an initial state, the RFP plasma relaxes to a minimum energy state, but never reaches the well-known fully relaxed state satisfying force-free field configuration $\{\nabla\times\boldsymbol{B} = \mu \boldsymbol{B}\}$, with μ uniform across the plasma as predicted by Taylor [20]. The departure from the fully relaxed state is indicated by the measured μ-profile, which is not uniform, but decreases toward the plasma edge where the toroidal magnetic field reverses its direction. Many RFP experiments have confirmed that the sustainment of the reversed toroidal field is essential for the duration of plasma confinement. Actually, in the case of cylindrical RFP it can be shown that toroidal field reversal implies the loss of axisymmetry [4]. Thus, a symmetric field reversed state cannot be maintained; and the reversed field profile would decay on a resistive diffusion time scale. The existence of a long plasma discharge on RFP experiments, however, suggests that the profiles are continually regenerated by a sort of dynamo mechanism [5], which depends critically upon the plasma collisionality. It was shown that, for a collisional plasmas diamagnetic dynamo, resulting from the fluctuating electron drift becomes dominant [6], while for a collsionless plasmas the MHD dynamo, resulting from fluctuating electromagnetic and electrostatic fields becomes dominant [7-10]. Both dynamo mechanisms were shown to be responsible for magnetic helicity transport across the plasma space through a fluctuation-induced helicity flux, keeping the overall magnetic helicity conserved [11-13]. In order to explain these experimental observations, theoretical [14-15] as well as numerical approaches [16-18], have been developed. In this paper, based upon a multiple time scales (MTS) approach [21] ,which has been developed mainly for the case of strongly magnetized, high-temperature, collisionless fusion plasma, the time evolution of the overall magnetic helicity over the resistive diffusion time scale is derived. It is proved that, for the case of collisionless fusion plasmas, the MHD dynamo mechanism is indeed the dominant. Moreover, it is shown that, during the evolution of the magnetic helicity on the resistive diffusion time scale, there exists a non dissipative transport of the magnetic helicity which counteract the resistive decay and lead to the conservation of the overall magnetic helicity and thus the sustainment of the magnetic field profile.

2. TURBULENT DYNAMO AND MAGNETIC HELICITY TRANSPORT

Magnetic helicity is a quantity which describes the amount of twist or writhe in the magnetic field of a given volume.[22]. The injection or ejection of magnetic helicity from a plasma often figures prominently in its dynamics and overall stability. In the context of plasma relaxation, Taylor conjectured [20] that in a slightly resistive plasma, the total helicity is well conserved during plasma relaxation in which the magnetic energy decays towards a minimum-energy state. This well-known hypothesis has been successful in explaining the magnetic structures in fusion plasmas, such as the reversed field pinch (RFP), spheromak, and multi-pinch. Based upon the multiple expansion approach [21], Taylor's conjecture was explicitly proven [23]. In the MST-RFP experiment [12], it was shown that the observed helicity change is larger than the simple MHD predictions due to enhanced fluctuation-induced helicity transport during the relaxation. This fluctuation-induced helicity transport is shown to be due to dynamo α-effect or equivalently the fluctuations-induced aligned electric field. Indeed, the dynamo α-effect drives parallel current which twists up the field lines, thus increasing the magnetic helicity on large-scale. Later on, it was shown that the dynamo α-effect converts magnetic helicity from the turbulent field to the mean field when the turbulence is electromagnetic while the magnetic helicity of the mean-field is transported across space when the turbulence is electrostatic or due to the electron diamagnetic effect. In all cases, however, the dynamo effect strictly conserves the total helicity except for resistive effects [15]. Therefore, it is crucial to know the type of turbulence which generates the dynamo effects in turbulent plasma in order to asses the role of dynamo effects on the magnetic helicity, even though the total helicity is always conserved. In the case of weakly collisional fusion plasmas (the MST-RFP), direct measurements [7, 12], indicated that the turbulence is predominantly electrostatic, thus causing helicity transport in the mean field with no effects on the turbulent field. In this section, using the multiple time scales derivative expansion scheme [21], the validity of the above-mentioned results will be investigated for the case of strongly-magnetized, collisionless fusion plasmas. In the frame of the multiple time scales expansion scheme, the overall *gauge invariant* magnetic helicity is defined as

$$K := \int \boldsymbol{A}\cdot\boldsymbol{B}\; dV - \oint_{\phi=0} \boldsymbol{A}\cdot\boldsymbol{e}_\theta d\theta \oint_{\theta=0} \boldsymbol{A}\cdot\boldsymbol{e}_\phi d\phi = \int \boldsymbol{A}\cdot\boldsymbol{B}\; dV - \Psi_t\Psi_p^h = \sum_{n=0}^{3} K_n\,, \tag{1}$$

where, Ψ_t and $\Psi_{\mathrm{p}}^{\mathrm{h}}$ are the toroidal flux and the poloidal flux threading through the hole of the torus respectively [23-25]. The integration is performed over the whole volume of the plasma. Applying the multiple time scales derivative expansion scheme [20], one obtains for the time evolution of the total magnetic helicity;

$$\left.\frac{\partial K}{\partial t}\right|_{RDMHD} = \delta^3 \left\langle \sum_{m=0}^{3}\int \boldsymbol{A_m}\cdot\left(\sum_{s=0}^{3-m}\frac{\partial \boldsymbol{B}_s}{\partial t_{3-m-ss}}\right) dV + \sum_{m=0}^{3}\int \boldsymbol{B_m}\cdot\left(\sum_{s=0}^{3-m}\frac{\partial \boldsymbol{A}_s}{\partial t_{3-m-s}}\right) dV - \sum_{m=0}^{3}\Psi_{tm}\sum_{s=0}^{3-m}\frac{\partial\left(\Psi_p^h\right)_s}{\partial t_{3-m-s}}\right\rangle_{\boldsymbol{x},\boldsymbol{t}} \tag{2}$$

Where all physical variables are in dimensionless form , and the operator $\langle\cdots\rangle_{\mathbf{x},\mathbf{t}}$ refers to the average over a mean flux surface as well as over the preceding time scales , a second average operator $\{\ \}_{\mathrm{x}}$, which denotes averages over mean flux surface but not over time, is introduced to extract the non-spatial fluctuating components. When expressed in these average operators, a dimensionless variable can be separated into a mean and two fluctuating parts such as [26]

$$Q = Q_0 + \tilde{Q} = \langle Q\rangle_{x,t} + \{\tilde{Q}\}_x + \tilde{Q}^L = Q_0(r,t_2,t_3) + \sum_{m=1}^{3}\delta^m\tilde{Q}_m(\mathbf{x},t_1,t_2,t_3) \tag{3a}$$

where δ is an expansion parameter [21]. In terms of Fourier modes with spatial mode *(m, l)* the two fluctuating parts $\{\tilde{Q}\}_{\mathbf{x}}$ and $\tilde{Q}'$ correspond to *(m,l)= (0,0)* mode and the higher spatial harmonics, respectively. If we consider, for instance, the case of periodic cylinder geometry [26], equation (3a) thus reads

$$Q = Q_0(r,t_2,t_3) + \sum_{m=1}^{3}\delta^m\sum_{n=0}^{3}Q_{mn}(r,t_2,t_3)\left[1+\exp\{i(m\theta+lz)\}\right]\cdot\exp(i2\pi nt_1). \tag{3b}$$

Where t_1 , t_2 , and t_3 are the Alfven time scale, the MHD-collision time scale (CMHD), and the resistive diffusion time scale (RDMHD), respectively. The overall magnetic helicity is considered the sum of the magnetic helicity in the mean field K_{mf} and the magnetic helicity in the turbulent field K_{tf} [14]. In this sense, the dimensionless time evolution of the magnetic helicity on the resistive diffusion time scale (RDMHD), reads;

$$\left.\frac{\partial K}{\partial t}\right|_{RDMHD} = \left.\frac{\partial K_{mf}}{\partial t}\right|_{RDMHD} + \left.\frac{\partial K_{tf}}{\partial t}\right|_{RDMHD}, \tag{4}$$

Where ,

$$\begin{aligned}\left.\frac{\partial K_{mf}}{\partial t}\right|_{RDMHD} &= \delta^3\left\langle \int \boldsymbol{A_0}\cdot\sum_{m=0}^{3}\frac{\partial \boldsymbol{B_m}}{\partial t_{3-m}}\ dV + \int \boldsymbol{B_0}\cdot\sum_{m=0}^{3}\frac{\partial \boldsymbol{A}_m}{\partial t_{3-m}}\ dV - \Psi_{t0}\sum_{m=0}^{3}\frac{\partial\left(\Psi_p^h\right)_m}{\partial t_{3-m}}\right\rangle_{\boldsymbol{x},\boldsymbol{t}} \\ &+ \delta^3\left\langle \int \boldsymbol{A}_2\cdot\sum_{m=1}^{1}\frac{\partial \boldsymbol{B_m}}{\partial t_{1-m}} dV + \int \boldsymbol{B}_2\cdot\sum_{m=0}^{1}\frac{\partial \boldsymbol{A}_m}{\partial t_{1-m}} dV - \Psi_{t2}\sum_{m=0}^{1}\frac{\partial\left(\Psi_p^h\right)_m}{\partial t_{1-m}}\right\rangle_{\boldsymbol{x},\boldsymbol{t}}\end{aligned}, \tag{5a}$$

$$\left.\frac{\partial K_{tf}}{\partial t}\right|_{RDMHD} = \delta^3\left\langle \int \boldsymbol{A}_1\cdot\sum_{m=0}^{2}\frac{\partial \boldsymbol{B_m}}{\partial t_{2-m}}\ dV + \int \boldsymbol{B}_1\cdot\sum_{m=0}^{2}\frac{\partial \boldsymbol{A}_m}{\partial t_{2-m}}\ dV - \Psi_{t1}\sum_{m=0}^{2}\frac{\partial\left(\Psi_p^h\right)_m}{\partial t_{2-m}}\right\rangle_{\boldsymbol{x},\boldsymbol{t}}. \tag{5b}$$

The application of the dimensionless Maxwell's equations [21] for the third-order (i.e., n=3), one obtains;

$$\begin{aligned}\left.\frac{\partial K_{mf}}{\partial t}\right|_{RDMHD} &= -\frac{\delta^3}{M}\left\langle 2\int \boldsymbol{B_0}\cdot\boldsymbol{E_2}\ dV + 2M\int \boldsymbol{B_0}\cdot\nabla\chi_2\, dV + \int \boldsymbol{A_0}\times\frac{\partial \boldsymbol{A}_0}{\partial t_3}\cdot \boldsymbol{dS} + M\,\Psi_{t0}\frac{\partial\left(\Psi_p^h\right)_0}{\partial t_3}\right\rangle_{\boldsymbol{x},\boldsymbol{t}} \\ &- \frac{\delta^3}{M}\left\langle 2\int \boldsymbol{B}_2\cdot\boldsymbol{E}_0\ dV + 2M\int \boldsymbol{B_2}\cdot\nabla\chi_0\, dV + \int \boldsymbol{A}_2\times\frac{\partial \boldsymbol{A}_0}{\partial t_1}\cdot \boldsymbol{dS} + M\Psi_{t2}\frac{\partial\left(\Psi_p^h\right)_0}{\partial t_1}\right\rangle_{\boldsymbol{x},\boldsymbol{t}}\end{aligned},$$

(6a)

$$\left.\frac{\partial K_{tf}}{\partial t}\right|_{RDMHD} = -\frac{\delta^3}{M}\left\langle 2\int \boldsymbol{B_1}\cdot\boldsymbol{E_1}\ dV + 2M\int \boldsymbol{B_1}\cdot\nabla\chi_1\, dV\int \boldsymbol{A_1}\times\frac{\partial \boldsymbol{A}_1}{\partial t_1}\cdot \boldsymbol{dS} + M\,\Psi_{t1}\frac{\partial\left(\Psi_p^h\right)_1}{\partial t_1}\right\rangle_{\boldsymbol{x},\boldsymbol{t}}, \tag{6b}$$

where $\chi_0, \chi_1,$ and χ_2 refer to the zero-, the first-, and the second-order electrostatic potentials. The application of the dimensionless Ohm's laws [21] for the first- and the second-order respectively, one finally, after a straightforward calculations, ends up with

$$\left.\frac{\partial K_{mf}}{\partial t}\right|_{RDMHD} = -\frac{2\delta^3}{M}\left\langle \frac{\Omega_i^{-1}}{\tau_A}\int \eta_2\,\boldsymbol{B_0}\cdot\boldsymbol{J_0}\ dV - \left(\frac{\delta_e}{\delta_i}\right)^{\frac{1}{2}}\int \boldsymbol{B_0}\cdot\boldsymbol{u}_{e1}\times\boldsymbol{B_1}\ dV + M\Psi_{t0}\frac{\partial\left(\Psi_p^h\right)_0}{\partial t_3}\right\rangle_{\boldsymbol{x},\boldsymbol{t}}, \tag{7a}$$

$$\left.\frac{\partial K_{tf}}{\partial t}\right|_{RDMHD} = -\frac{\delta^3}{M}\left\langle 2\int \boldsymbol{B_1}\cdot\boldsymbol{E_1}\, dV + 2M\int \boldsymbol{B_1}\cdot\nabla\chi_1\, dV + \int \tilde{\boldsymbol{A}}_1'\times\frac{\partial\tilde{\mathbf{A}}_1'}{\partial t_1}\cdot \boldsymbol{dS} + 2M\,\Psi_{t1}\frac{\partial\left(\Psi_p^h\right)_1}{\partial t_1}\right\rangle_{x,t} .$$

(7b)

Where Ω_i is the ion Larmour gyration frequency, and M is a dimensionless constant of order unity. The second term in equation (7a) stands for the well-known turbulent dynamo α-effect, or equivalently, the fluctuations-induced field aligned electric field. Employing the dimensionless first-order Ohm's law [21], this term reduces to

$$\left(\frac{\delta_e}{\delta_i}\right)^{\frac{1}{2}}\left\langle\int \boldsymbol{B_0}\cdot\boldsymbol{u_{e1}}\times\boldsymbol{B_1}\, dV\right\rangle_{x,t} \cong \left\langle\int \boldsymbol{B}_1\cdot\boldsymbol{E_1}\, dV\right\rangle_{x,t} = -M\left\langle\int \boldsymbol{B}_1\cdot\frac{\partial\boldsymbol{A}_1}{\partial t_1}\, dV + \int \boldsymbol{B_1}\cdot\nabla\chi_1\, dV\right\rangle_{x,t} , \quad (8)$$

Equation (8) implies that, for the case of a strongly- magnetized, weakly-collisional fusion plasma, the MHD dynamo is indeed the dominant mechanism, the result which is consistent with the experimental observations [10]. The first term on the right hand side of equation (8) represents the electromagnetic contribution while the second term represents the electrostatic contribution. Thus, equations (6) reduce to

$$\left.\frac{\partial K_{mf}}{\partial t}\right|_{RDMHD} = \underbrace{-\frac{2\delta^3}{M(\Omega_i\tau_A)}\int\eta_2\,\mathbf{B_0}\cdot\mathbf{J_0}\, dV}_{\mathbf{A}} \underbrace{-\frac{2\delta^3}{M}\left\langle\int \mathbf{B_1}\cdot\frac{\partial\mathbf{A_1}}{\partial t_1}\, dV\right\rangle_{\mathbf{x,t}}}_{\mathbf{B}} \underbrace{-2\delta^3\left\langle\Psi_{t0}\frac{\partial\left(\Psi_p^h\right)_0}{\partial t_3}\right\rangle_{\mathbf{x,t}}}_{\mathbf{C}} \underbrace{-2\delta^3\left\langle\int(\mathbf{B_1}\chi_1)\cdot\mathbf{dS}\right\rangle_{\mathbf{x,t}}}_{\mathbf{D}} , \quad (9a)$$

$$\left.\frac{\partial K_{tf}}{\partial t}\right|_{RDMHD} = \underbrace{\frac{2\delta^3}{M}\left\langle\int \boldsymbol{B_1}\cdot\frac{\partial\boldsymbol{A_1}}{\partial t_1}\, dV\right\rangle_{x,t}}_{B} \underbrace{-\frac{\delta^3}{M}\left\langle\int\tilde{\mathbf{A}}_1'\times\frac{\partial\tilde{\mathbf{A}}_1'}{\partial t_1}\cdot\boldsymbol{dS}\right\rangle_{x,t}}_{E} \underbrace{-2\delta^3\left\langle\Psi_{t1}\frac{\partial\left(\Psi_p^h\right)_1}{\partial t_1}\right\rangle_{x,t}}_{F} . \quad (9b)$$

Where, term A on the right hand side of equation (9a) represents the helicity resistive dissipation by the mean fields. Term B, that appears on the right-hand side of equations (9), represents the amount of generated magnetic helicity due to the electromagnetic contribution of the MHD dynamo. Finite dissipation or wave-particle interactions can result in a finite term

B which converts helicity from the turbulent field to the mean field or vice versa [15]. Term C, represents the amount of helicity injection by means of an external applied loop voltage [4]. Now if the toroidal loop voltage is kept constant, in the outer region of the discharge where the resistivity becomes large, the helicity transport in this case counteract the resistive dissipation leading to the sustainment of the discharge. Thus, term D, in case of resistive boundary, can be interpreted as the rate of change of the mean-field magnetic helicity due to an outward helicity flow across the surface as a result of the electrostatic dynamo. Term E on the right-hand side of equation (9b), represents the transport of the magnetic helicity in the turbulent field by the propagation of electromagnetic waves possessing a finite helicity [15]. While term F, represents the helicity injection current due to the oscillating fields [13]. It is now obvious that for the case of strongly-magnetized, collisionless fusion plasmas the turbulent dynamo is dominantly due to MHD mechanisms. Finally, one concludes that, the role of turbulent dynamo in helicity transport depends on the nature of the turbulence. When the turbulence is electromagnetic, the dynamo α-effect, as seen from term B, generates the same amount of helicity in both the mean and turbulent fields but with opposite signs, leading to zero net effect on the overall magnetic helicity. When the turbulence is electrostatic, the dynamo α-effect, as seen from term D, does not affect the turbulent helicity but, transports the mean-field magnetic helicity across plasma boundary.

3. SUMMARY AND CONCLUSIONS

Based on a multiple time scales approach, the time evolution of the overall magnetic helicity on the resistive diffusion time scale (RDMHD) has been derived. It was shown that for the case of strongly magnetized weakly collisional fusion plasmas, dynamo α-effect is merely due to MHD mechanism in consistent with the experimental measurements. It was also concluded that the magnetic helicity is affected critically by the nature of the turbulence.

When the turbulence is due to electromagnetic fields, the dynamo α-effect converts helicity from turbulent, small-scale field to mean, large-scale field. When the turbulence is due to an electrostatic field, the dynamo α-effect transports the mean field helicity across space without dissipation. In all cases, it was shown that on the resistive time scale, the α-effect conserves the total magnetic helicity against the resistive effect, the results which explain the long discharge time of the RFP. These results are shown to be consistent with the experimental observations.

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