Fayoum University
Faculty of Engineering
1st year Electrical Engineering

كلية الهندسة
جامعة الفيوم

Engineering Mathematics
1st Term Exam, Jan., 2016
Time Allowed: 3 Hours

**Attempt all the following questions, maximum degree 85M**

## Question No. (1) [30 M]

a. Find equation of tangent plane and normal line to the surface $z = x^2 + y^2$ at the point (1,1,2) **[5 M]**

b. Find dimensions of rectangle of largest area with sides parallel to the coordinates that can be inscribed in the ellipse $\frac{x^2}{25} + \frac{y^2}{144} = 1$. **[5 M]**

c. If $z = f(\frac{y}{x})$, show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$. **[5 M]**

d. Examine the extremum of $f(x,y) = 25x - 25xe^{-y} - 50y - x^2$ **[5 M]**

e. If $A$ is constant vector and $r$ is position vector, show that $\nabla x(A \times r) = 2A$ and $\nabla.(A \times r) = 0$ **[5 M]**

f. if $\int_0^{\pi} \frac{1}{a-cosx}dx = \frac{\pi}{\sqrt{a^2-1}}$, a>1 find $\int_0^{\pi} \frac{1}{(2-cosx)^2}dx$ **[5 M]**

## Question No. (2): [30 M]

a. Evaluate $\iint_R \cos(x^2+y^2)dA$ ; R: $(x^2+y^2) \le (\frac{\pi}{2})^2$ , $x \ge 0$ **[5 M]**

b. Find mass and moment of inertia around origin for the region $R$ of a thin plate of constant density bounded by the quarter circle $x^2 + y^2 = 1$ in the first quadrant. **[9 M]**

c. Find the volume of the regior bounded by $7 + z = 4x^2 + y^2$ and $7 - z = 4x^2 + y^2$ **[5 M]**

d. Find $\iiint_R (x^2+y^2+z^2)dv$, where $R$ is upper half of sphere $x^2 + y^2 + z^2 = a^2, a \ge 0$. **[5 M]**

e. Find the area enclosed by the lemniscate $r^2 = 2a^2\cos(2\theta)$. **[6 M]**

## Question No.(3) [ 25 M]

a. A semi-circular wire has the equation $y = \sqrt{25 - x^2}$ and its mass density per unit length is given by $\rho(x, y) = 15 - y$. Find its mass **[5 M]**

b. Verify Green's Theory in the plane for $\oint_C ydx - xdy$, where C is the unit circle with center at (0,0). **[9 M]**

c. Find $\int_{(0,0)}^{(1,2)} (x^2 + y^2)dx + 2xydy$. **[5 M]**

d. Evaluate $\iint_S y^2 z^2 \, ds$, where S is the part of the cone $z = \sqrt{x^2 + y^2}$ lies between $z = 1$ and $z = 2$. **[6 M]**

## Question No.(4) [ 23 M]

a. Test the following series

$$\sum_1^\infty \frac{3^n - 2^n}{3^n + 2^n}, \quad \sum_1^\infty \left(\frac{n}{n+1}\right)^{-n^2}, \quad \sum_1^\infty \frac{1}{n^{(2+\frac{1}{n})}}, \sum_1^\infty \frac{\ln(n)}{n}$$

**[8 M]**

b. Examine for conditional and absolute convergence of:

$$\sum_2^\infty \frac{(-1)^n}{n^2 + 1}, \sum_1^\infty \frac{(-3)^n}{n!}$$

**[6 M]**

c. Find convergence interval for

$$\sum_1^\infty \frac{(2x+1)^n}{n^2}, \quad \sum_1^\infty \frac{n!}{n^{2n}}(x)^n, \quad \sum_1^\infty \frac{1}{n^{|\ln(x)|}}$$

**[9 M]**

*"Best Regards"*
*Dr. Sherif E. Nasr*