

Answer the following FIVE questions:

Question 1 [15 points]:

1.1 Consider the following matrix:

$$A = \begin{bmatrix} 0 & 0 & 2 & 0 \\ 2 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 1 \end{bmatrix}$$

Which of the following statement is True (Mention clearly the reasons):

- 1.1.1 The Columns are linearly dependent.
- 1.1.2 The Matrix isn't invertible.
- 1.1.3 The Matrix has determinant -2.
- 1.1.4 None of the above.

1.2 The Matrix Q has orthonormal columns $\underline{q}_1, \underline{q}_2, \underline{q}_3$

$$Q = \begin{bmatrix} 0.1 & 0.5 & a \\ 0.7 & 0.5 & b \\ 0.1 & -0.5 & c \\ 0.7 & -0.5 & d \end{bmatrix}$$

- 1.2.1 What equations must be satisfied by the numbers a, b, c, d?
- 1.2.2 Is there a unique choice for those real numbers?
- 1.2.3 Suppose Gram-Schmidt starts with those same first two columns and with the third column $\underline{a} = (1 \ 1 \ 1 \ 1)^T$. What third column would it choose for \underline{q}_3 .

Question 2 [15 points]:

2.1 Suppose $\{\underline{v}_1, \underline{v}_2, \underline{v}_3\}$ is a linear independent set in \mathbb{R}^3 . Show that

$\{\underline{v}_1, \underline{v}_1 + \underline{v}_2, \underline{v}_1 + \underline{v}_2 + \underline{v}_3\}$ is also linearly independent.

2.2 For $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$; and assume that $\det(A) = -2$; Find:

2.2.1 $\det(-2A)$.

2.2.2 $\det(3(A^T)^{-1})$.

2.2.3 $\det \begin{bmatrix} a & g & d \\ b & h & e \\ c+2a & i+2g & f+2d \end{bmatrix}$

2.3 Obtain the inverse of the matrix $A = \begin{bmatrix} 7 & 4 & 0 & 0 & 0 \\ 5 & 3 & 0 & 0 & 0 \\ 0 & 0 & 5 & 4 & 0 \\ 0 & 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

Question 3 [15 points]:

Consider the linear system $A\underline{x} = \underline{b}$ where:

$$A = \begin{bmatrix} 1 & -1 & -2 & -2 & -2 \\ 3 & -2 & -2 & -2 & -2 \\ 4 & -3 & -4 & -4 & -4 \end{bmatrix}, \quad \underline{b} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$$

- 3.1 Find the rank of the matrix A .
- 3.2 Is the system of equations consistent? Why?
- 3.3 Solve the linear system $A\underline{x} = \underline{b}$.
- 3.4 Write the solution set for $A\underline{x} = \underline{0}$

Question 4 [25 points]:

4.1 Suppose A_n has eigenvalue λ with corresponding eigenvector \underline{v} :

- 4.1.1 If A is invertible, is \underline{v} still eigenvector A^{-1} ? If so, what is the corresponding eigenvalue? If not, explain why not.
- 4.1.2 Is $3\underline{v}$ an eigenvector of A ? If so, what is the corresponding eigenvalue? If not, explain why not.

4.2 For the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 4 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

- 4.2.1 Find the eigenvalues of matrix A ?
- 4.2.2 Evaluate $\sin(A t)$ by using minimal polynomial method.

Question 5 [15 points]:

Solve:

$$\dot{\underline{x}}(t) = A \underline{x}(t), \text{ where: } A = \begin{bmatrix} 4 & 0 & -2 \\ 2 & 5 & 4 \\ 0 & 0 & 5 \end{bmatrix}; \text{ and: } \underline{x}(0) = \begin{bmatrix} -2 \\ -2 \\ 1 \end{bmatrix}$$

Good Luck