Fayoum University
Faculty of Engineering
Dept. of Electrical Engineering (Communication Engineering Section)
Third Year

Mathematics
First Term of 2015/2016
Final Exam
Time: 3 Hours

Answer the following **SEVEN** questions:

1) Write down the general solution of the following differential equations:

a) $\left(1-x^2\right)y'' - 2xy' + \left(56 - \frac{81}{1-x^2}\right)y = 0$

b) $x^2y'' + xy' + \left(36x^2 - 4\right)y = 0$

c) $x^2y'' + xy' - \left(49x^2 + 9\right)y = 0$

d) $x^2y'' + xy' + \left(-ix^2 - 64\right)y = 0$

(12 points)

2) Let X be a Gaussian random variable with mean μ and standard deviation σ. Find the tenth central moment $E\left[(X-\mu)^{10}\right]$ of X. [Useful formula: $f(x) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$]

(12 points)

3) Let X and Y be random variables having the following means and variances:

$\mu_X = 2$, $\mu_Y = 3$, $\sigma_X^2 = 16$, $\sigma_Y^2 = 25$. The correlation coefficient of X and Y is $\rho = 0.5$. Find the best (in the least mean squared error sense) linear predictor $\hat{y}$ of y.

(8 points)

4) Two random processes U_t and V_t are defined by:

$U_t = X\sin t + Y\cos t$, $V_t = X\cos t + Y\sin t$

where X and Y are uncorrelated random variables having the following probability density functions:

$$f_X(x) = \begin{cases} 1-|x| & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases} \quad , \quad f_Y(y) = \begin{cases} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} < y < \frac{1}{\sqrt{2}} \\ 0 & \text{otherwise} \end{cases} .$$

a) Find the means μ_X and μ_Y of X and Y.

b) Find the variances σ_X^2 and σ_Y^2 of X and Y.

c) Find the means of U_t and V_t.

d) Find the autocorrelation function $R_U(t_1,t_2)$ of U_t.

e) Find the cross correlation function $R_{U,V}(t_1,t_2)$.

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