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Fractional Order Modeling of Energy Systems

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ABSTRACT

Energy storage systems and components exist in our everyday life, from medical implanted sensors to mobile phones to electrified vehicles to large-scale energy management systems used near power plants. They are used to provide energy to mobile devices of different types and provide energy buffer when energy demand is more the production. Examples of energy storage elements are batteries, supercapacitors, compressed air, and pumped hydro. Models of supercapacitors and batteries are either PDE based, which are very complex and computationally exhaustive, or empirical and mathematical based, which are based on observations or intelligent like neural networks and state vector machines, or equivalent circuit models, which are the simplest and most commonly used for their real-time applicability. DC-DC converters are used to convert the voltage/current from one level to another. They are found in mobile devices and computers where voltage requirements are different for each subsystem. Bidirectional DC-DC converters are used, for example, to provide desired voltage levels for charging and discharging of supercapacitors and batteries when connected to a common DC bus, which is a typical situation in renewable energy systems and electrified vehicles.

Fractional calculus is a newly rediscovered branch of mathematics concerned with the study of non-integer order systems where integer-order models are a small part of the fractional-order space. The extra degrees of freedom in the differ-integral orders made these models able to provide a better estimation of actual system responses. Utilizing fractional-order concepts has opened, and is still opening, new research areas in different scientific and engineering fields. Almost every existing model and system can be re-analyzed using fractional calculus, and new and previously unattainable responses and behaviors are discovered.

In this thesis, the researcher aims to study different energy systems and devices from a fractional-order calculus perspective. First, in chapter two, the researcher surveys fractional calculus definitions, reviews methods for solving fractional-order systems and models, and provides an overview of recent engineering applications of fractional calculus. In the third chapter, the researcher investigates the modeling approaches for supercapacitors and proposes an identification method for estimating the parameters of fractional-order equivalent circuit models of supercapacitors. In chapter four, the researcher surveys the new concept of fractional-order calculus of variations and its analytical and numerical approaches. Then, these concepts are applied to solve the optimal charging problem of a supercapacitor modeled by a fractional-order equivalent circuit model. Two objective functions were presented and investigated in detail; the first is minimizing the energy loss in resistive elements, and the second is maximizing the charging efficiency. In the fifth chapter, the researcher reviews the modeling techniques of batteries and use two fractional-order impedance models to fit the Nyquist plots of commercial Panasonic Lithium-ion battery. In chapter six, the researcher investigates the modeling approaches for fractional-order DCDC converters and proposes a modeling approach for the three basic converters: buck, boost, and buck-boost. Also, a fractional-order model is introduced for the dynamic behavior of PV cells. The model parameters are extracted from experimental data of step loading.