

NUMERICAL SOLUTION OF THREE DIMENSIONAL FLOW AND HEAT  
TRANSFER FOR NON NEWTONIAN POWER-LAW FLUIDS IN THE  
ENTRANCE REGION OF  
A SQUARE DUCT

by

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## ABSTRACT

In this thesis, we study two problems for the flow and heat transfer of power law non-Newtonian fluids in the entrance region of a square duct. The flow is assumed to be laminar, steady and incompressible with constant thermophysical properties. The concept of the boundary layer assumptions is applied for the governing equations for flow and heat transfer.

In the first problem, the continuity and the non-linear momentum equations are solved iteratively by marching technique using finite difference method. Using this solution, we can obtain the velocity components, pressure in the flow field. The product of the apparent friction factor and the Reynolds number  $f_{app}R_e$  has been obtained from the pressure gradient. Second order accurate finite difference representations are used to obtain more accurate results for the secondary velocity components. The results are obtained at different values of axial position, and the flow index of the model. The comparisons of the present results have been made for some special cases of the model with the known previous results and are found to be in good agreement.

In the second problem, we consider simultaneously developing flow and heat transfer of power law fluids in the entrance region of a square duct. The energy equation is solved numerically by marching technique (depending on the values of the velocity components at each axial position in the entrance region of the square duct) using finite difference method to obtain temperature distributions. We consider two cases of thermal boundary conditions; Case(i) is called  $T$ -boundary condition. In this case, the wall temperature of the duct is uniform and constant both axially and peripherally. Case(ii) is called  $H2$ -boundary condition. In this case, the wall heat flux is constant both axially and peripherally. The local Nusselt numbers have been obtained for different values of axial position, Prandtl number, Graetz number, and the flow index behavior. For special cases of the model, the present results of heat transfer characteristics have been compared with the previous results and are found to be in good agreement.