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Application of the Differential Quadrature Method to Some Elasticity Problems

By:

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1.1 Preamble

Nowadays, Studying the static and dynamic characteristics of elasticity problems grabbed scientists' attention as they can be employed in extensive applications such as structural components, tanks, aerospace industry, space crafts and deck plates in launch vehicles. Since circular plate is considered one of the most meaningful examples of elasticity problems through which we can review different elasticity theories and implementation of energy and variational principles, in this work, analysis of circular plates is considered.

From manufacturing and economic point of view, producing structural components that have high strength with low weight and can perform well under extraordinary conditions is one of the most important research fields. Consequently, the idea of composite materials arised to achieve that purpose. Recently, a new class of composite materials called functionally graded material (FGM), an inhomogeneous composite usually made from a mixture of ceramics and metal by gradually varying material properties through thickness direction. This type of composite materials is chosen to describe the inhomogeneity of the material of the plate. Furthermore, different parameters can influence the behavior of the plate, for instance, thickness profile, existence of elastic foundation and boundary conditions.

Dealing with such complicated problems that include different parameters reflects on formulation of the mathematical model. Employing conventional analytical methods in this case is not practical as it can be extremely hard to present an analytical solution. Instead, numerical approaches are the best choice to treat such problems. One of the most popular approaches is differential quadrature method (DQM), where the differential equation of the plate is discretized at grid points which result in a set of linear algebraic equations that can be solved together to present the solution. The merit of this method is the high level of accuracy with low computational effort in comparison with other numerical methods.

1.2 Problem Depiction

Most of research work considering the bending and free vibration analysis of circular plates only focus on simple types of plates, for example, isotropic plates with uniform thickness, circular plates resting on uniform one-parameter elastic foundation and circular plates fully resting on elastic foundation. But only few studied the effects of different parameters such as material gradient, variable thickness, two-parameters elastic foundation and the variation of soil subgrade modulus. However, no research work has been conducted to study all these parameters together deeply, besides investigating the real behavior of the soil underneath elastic foundation to present a formula describes the variation in the soil subgrade modulus.

1.3 Thesis Objectives

In this thesis, a numerical solution to the bending and free vibration analysis of FGM circular plate resting of Winkler-Pasternak foundation will be presented using differential quadrature method. The study will include investigation of different parameters on transverse displacements, radial stress, natural frequencies and modal shapes. Moreover, the behavior of soil will be demonstrated to develop an equation describes the variation of the soil subgrade modulus. The parametric study will provide a more-general overview about the behavior of circular plates resting on elastic foundation.

1.4 Procedure of Analyzing Circular Plates

1.4.1 Derivation of the Governing Equation

The governing equation is driven based on the classical plate theory as follows:

1. Determination of displacements field (u,v,w).
2. Find strain-displacement and stress relations.
3. Applying Hamilton's principle.

1.4.2 Implementation of DQM

The numerical solution of the governing equation of circular plate will be introduced using differential quadrature method as follow:

1. Computation of weighting coefficients.
2. Discretization of governing equation and boundary conditions.
3. Solve linear system of equations to obtain the solution.

1.4.3 Parametric Study

The analysis of deflection, radial stress, natural frequency and mode shapes will be discussed in terms of different parameters, such as

1. Material gradient index.
2. Non-uniform thickness.
3. Two-parameter elastic foundation with variable subgrade modulus.
4. Fully and partially supported area on elastic foundation.
5. Clamped and simply supported boundary conditions.

1.5 Thesis Organization

Organization of thesis chapters is listed as follow

Chapter (1): Introduction

This chapter consists of introduction, problem depiction, thesis objectives, procedure of analyzing circular plates.

Chapter (2): Literature Review

This chapter reviews past research work and efforts to analyze elasticity problems using various analytical and numerical methods.

Chapter (3): Classical Plate Theory

This chapter presents the derivation of the governing equation of circular plate based on the classical plate theory using Hamilton's principle.

Chapter (4): Differential Quadrature Method

This chapter reviews the differential quadrature method, computation of weighting coefficients, selection of grid points and the techniques used in the implementation of boundary conditions

Chapter (5): Axisymmetric Bending of FGM Circular Plate

This chapter investigates the static analysis of FGM circular plate with variable thickness fully or partially resting on non-uniform Winkler-Pasternak foundation using DQM.

Chapter (6): Axisymmetric Free Vibration of FGM Circular Plate

This chapter investigates the dynamic analysis of FGM circular plate with variable thickness resting on non-uniform Winkler-Pasternak Foundation using DQM.

Chapter (7): Distribution of Soil Subgrade Modulus

This chapter presents a formula describes the variation in the subgrade modulus of the soil underneath the circular plates based on regression analysis of different elastic foundation models.

Chapter (8): Conclusion and Recommendations for Future Work

This chapter concludes the main outcomes of the present work and focuses on the future research work.

References: The thesis contains 117 references

Appendix: Provides a MATLAB code to solve bending and free vibration of axisymmetric circular plates with variable thickness and resting on non-uniform two-parameter elastic foundation.

Arabic Summary: Provides a summary of the work in Arabic