جـــامعة الفـــيوم
Fayoum University

Name of Candidate: Raouth Refaat Shafik

Degree: Master of Science (M.Sc.)

Department: Engineering Mathematics and physics Department

Title of Thesis: The Jordan Canonical Form and Collatz matrix

Supervisors: 1- Prof. Magdy Tawfik Hanna

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Approval: --/--/--

ABSTRACT

The numerically challenging problem of generating the Jordan canonical form is investigated exclusively for a class of zero-one square matrices with the extra property that each column has at most one nonzero element. The approach is based on the generation and analysis of the directed graph having the matrix in question as its adjacency matrix. The computational investigation includes mainly the generation of the Jordan canonical form and modal matrix consisting of both eigenvectors and generalized eigenvectors. Additionally, the number of cycles and the number of vertices in all cycles of the associated graph are computed. The accuracy of the computation is assessed by computing the difference between the given matrix and that formed by combining the Jordan form and the modal matrix. Both the maximum element in absolute value and the Frobenius norm of the difference matrix, defined as the difference between the given and computed matrices, are used as error measures. The computational investigation is carried out by developing a set of MATLAB functions which can be viewed as a contributed experimental toolbox. Fortunately, it has been found that this toolbox outperformed the built-in MATLAB function “jordan” since the former successfully executed for square matrices of order up to 1000 while the latter lingered for matrices of order between 40 and 60.

Since the Collatz matrix generated by the Collatz function is a special case of the zero-one matrix investigated in this work, the obtained results are applicable to this important matrix. The open problems related to Collatz conjecture are computationally addressed and some empirical results emerged. More specifically, the number of cycles and the length of the longest chain in a proper partition of a Collatz graph are computed. Moreover, an empirical formula is devised for the number of connected components in the Collatz graph.

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