FINITE DIFFERENCE SOLUTIONS OF SOME NON-NEWTONIAN FLUIDS FLOW AND HEAT TRANSFER IN DUCTS

by

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ABSTRACT

In this thesis, four problems for the flow and heat transfer for non-Newtonian fluids in ducts using finite difference method are studied. The flow is assumed to be laminar, incompressible with constants physical properties. These studies are applicable in many engineering applications such as polymers processing solutions, heat exchangers, ground water pollution, fresh concrete, petroleum industry, micro-scale channels for cooling electronic components and MHD pumps.

In the first problem, the continuity and the non-linear momentum equations are solved iteratively by marching technique using finite difference method. Using this solution, we can obtain the velocity components, pressure in the entrance region of a rectangular duct. The Herschel-Bulkley model is used to characterize the fluid behavior. The product of the apparent friction factor and the Reynolds number is obtained from the pressure gradient. Second order accurate finite difference representations are used to obtain more accurate results for the secondary velocity components. The results are obtained at different parameters of the fluid and geometry. The comparisons of the present results are made for some special cases of the model with the known previous results and they are found to be in good agreement. This problem has been published in "**Al-Azhar University Engineering Journal, JAUES, Vol. 2, No. 11, 2007, pp. 17-31**".

In the second problem, we consider simultaneously developing flow and heat transfer of Herschel-Bulkley fluids in the entrance region of a rectangular duct. The energy equation is solved numerically by marching technique (depending on the values of the velocity components at each axial position in the entrance region of the rectangular duct) using finite difference method to obtain temperature distributions. We consider two cases of thermal boundary conditions. The local Nusselt numbers are obtained for different parameters of

the fluid, heat, geometry. For special cases of this problem, the present results of heat transfer characteristics are compared with the previous results and they are found to be in good agreement. This problem has been published in "**International Communications in Heat and Mass Transfer, Volume 35, Issue 8, October 2008, Pages 1007-1016**".

In the Third problem, we study a numerical solution of entrance heat transfer using the Papanastasiou model through porous medium in a rectangular duct considering viscous dissipation. The numerical methods are used to solve this problem because it is complex by the analytical methods. By using the regularized Papanastasiou model, we can overcome singularity in computations at using Bingham fluid. Thus, Papanastasiou model enables us to obtain results of high accuracy. Fluid flow, generally, requires approximations to overcome the nonlinearity of the differential equations due to presence of non-Newtonian apparent viscosity. The heat transfer is affected, substantially by presence of velocity and viscous dissipation. An excellent accurate cubic spline polynomials method is used for the Nusselt numbers that of excellent agreement with the available recent analytical results.

In the fourth problem, we investigate The unsteady MHD flow and heat transfer of an electrically conducting viscous incompressible Casson fluid bounded by two parallel porous plates. The Hall effect and Joule and viscous dissipations are taken into consideration. The fluid is acted upon by a decaying pressure gradient. An external magnetic field is applied perpendicular to the plates and the fluid motion is subjected to a uniform suction and injection. The lower plate is stationary and the upper plate is suddenly set into motion and suddenly heated to a temperature other than the lower plate temperature. Finite difference schemes of second order accurate and generalized Thomas algorithm are used to solve the governing equations. The effect of unsteady pressure gradient, the Hall term, the parameter describing the non-Newtonian behavior on both the velocities and temperature distributions are studied.