جامعة الفيوم | اختبار المعادلات التفاضلية | الفرقة الرابعة رياضة+رياضة فيزياء
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قسم الرياضيات | | الزمن/3 ساعات

Answer the following questions:

1. a) Verify that the equation is hyperbolic then find a change of coordinates that reduce it to canonical form:

$$u_{xx} + 2u_{xy} - 8u_{yy} + u_x + 5 = 0$$

b) Obtain the general solution of the equation:

$$\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$$

c) Find Laplace Transform of the functions:

i) $e^{-t}(1 - H(t-2))$ ii) $\dfrac{1+2t}{\sqrt{t}}$ iii) $t \sinh \dfrac{t}{2}$

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2. a) Find the general solution of the problem:

$$u_{tt} - c^2 u_{xx} = h(x,t) \quad , \quad -\infty < x < \infty$$
$$u(x,0) = f(x) \quad , \quad u_t(x,0) = g(x) \quad , \quad t > 0$$

b) Use Laplace Transform to solve the problem:

$$y'' + 4y' + 13y = e^{-t} \sin t \ , \quad y(0) = 0 \quad , \quad y'(0) = 0$$

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3. a) Find:

i) $L^{-1}[\dfrac{s^2 + s - 2}{s(s+3)(s-2)}]$ ii) $L^{-1}[\dfrac{e^{-s}}{(s-2)(s+3)}]$

b) Solve the problem:

$$u_{tt} - 25u_{xx} = 0 \ , \quad 0 < x < \infty \ , \quad t > 0$$
$$u(x,0) = \cos x \ , \quad u_t(x,0) = x \ , \quad u(0,t) = 0$$

c) Transform the problem:

$$u_{tt} - u_{xx} = 6xt \ , \quad 0 < x < 1$$
$$u(x,0) = x^2 \ , \quad u_t(x,0) = 2x \ , \quad u(0,t) = 0 \quad , \quad u(1,t) = t^3$$

To a new problem with zero equation and boundary conditions, then find the solution.

Good Luck