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# SOME SANDWICH RESULTS FOR HIGHER-ORDER DERIVATIVES OF MULTIVALENT FUNCTIONS INVOLVING A GENERALIZED DIFFERENTIAL OPERATOR

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Abstract. In this paper, we obtain some applications of first order differential subordination, superordination and sandwich results for higher-order derivatives of $p-$valent functions involving a generalized differential operator. Some of our results improve and generalize previously known results.

## 1. Introduction

Let $H(U)$ be the class of analytic functions in the open unit disk $U = \{z \in \mathbb{C} : |z| < 1\}$ and let $H[a,p]$ be the subclass of $H(U)$ consisting of functions of the form:

$$f(z) = a + a_p z^p + a_{p+1} z^{p+1} ... \ (a \in \mathbb{C};\ p \in \mathbb{N} = \{1, 2, ...\}).$$

For simplicity $H[a] = H[a, 1]$. Also, let $\mathcal{A}(p)$ be the subclass of $H(U)$ consisting of functions of the form:

$$f(z) = z^p + \sum_{k=p+1}^{\infty} a_k z^k \quad (p \in \mathbb{N}), \tag{1}$$

which are $p-$valent in $U$. We write $\mathcal{A}(1) = \mathcal{A}$.

If $f,\ g \in H(U)$, we say that $f$ is subordinate to $g$ or $g$ is superordinate to $f$, written $f(z) \prec g(z)$ if there exists a Schwarz function $w$, which (by definition) is analytic in $U$ with $w(0) = 0$ and $|w(z)| < 1$ for all $z \in U$, such that $f(z) = g(w(z)),\ z \in U$. Furthermore, if the function $g$ is univalent in $U$, then we have the following equivalence, (cf., e.g., [10], [17] and [18]):

$$f(z) \prec g(z) \Leftrightarrow f(0) = g(0) \text{ and } f(U) \subset g(U).$$

Let $\phi : \mathbb{C}^2 \times U \to \mathbb{C}$ and $h$ be univalent function in $U$. If $\beta$ is analytic function in $U$ and satisfies the first order differential subordination:

$$\phi\left(\beta(z), z\beta'(z); z\right) \prec h(z), \tag{2}$$

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