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Research Article

Some Inclusion Relationships of Certain Subclasses of p-Valent Functions Associated with a Family of Integral Operators

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By making use of the new integral operator $\mathfrak{R}_{\beta,p}^{\alpha,\gamma}$, we introduce and investigate several new subclasses of p-valent starlike, p-valent convex, p-valent close-to-convex, and p-valent quasi-convex functions. In particular, we establish some inclusion relationships associated with the aforementioned integral operators. Some of the results established in this paper would provide extensions of those given in earlier works.

1. Introduction

Let $A(p)$ denote the class of functions of the form

$$f(z)=z^p+\sum_{k=1}^{\infty}a_{k+p}z^{k+p} \quad (p\in\mathbb{N}=\{1,2,3,\ldots\}), \tag{1}$$

which are analytic and p-valent in the unit disc $U=\{z: z\in\mathbb{C}, \text{and } |z|<1\}$ and let $A(1)=A$.

A function $f\in A(p)$ is said to be in the class $S_p^*(\lambda)$ of p-valent starlike functions of order λ in U if and only if

$$\operatorname{Re}\left(\frac{zf'(z)}{f(z)}\right)>\lambda \quad (z\in U;\ 0\le\lambda<p). \tag{2}$$

The class $S_p^*(\lambda)$ was introduced by Patil and Thakare [1].

Owa [2] introduced the class $K_p(\lambda)$ of p-valent convex of order λ in U if and only if

$$\operatorname{Re}\left(1+\frac{zf''(z)}{f'(z)}\right)>\lambda \quad (z\in U;\ 0\le\lambda<p). \tag{3}$$

It is easy to observe from (2) and (3) that

$$f(z)\in K_p(\lambda)\Longleftrightarrow\frac{zf'(z)}{p}\in S_p^*(\lambda). \tag{4}$$

We denote by $S_p^*=S_p^*(0)$ and $K_p=K_p(0)$ where S_p^* and K_p are the classes of p-valently starlike functions and p-valently convex functions, respectively, (see Goodman [3]).

For a function $f\in A(p)$, we say that $f\in C_p(\eta,\lambda)$ if there exists a function $g\in S_p^*(\lambda)$ such that

$$\operatorname{Re}\left(\frac{zf'(z)}{g(z)}\right)>\eta \quad (z\in U;\ 0\le\lambda,\eta<p). \tag{5}$$

Functions in the class $C_p(\eta,\lambda)$ are called p-valent close-to-convex functions of order η and type λ. The class $C_p(\eta,\lambda)$ was studied by Aouf [4] and the class $C_1(\eta,\lambda)$ was studied by Libera [5].

Noor [6, 7] introduced and studied the classes $C_p^*(\eta,\lambda)$ and $C_1^*(\eta,\lambda)$ as follows.

A function $f\in A(p)$ is said to be in the class $C_p^*(\eta,\lambda)$ of quasi-convex functions of order η and type λ if there exists a function $g\in K_p(\lambda)$ such that

$$\operatorname{Re}\left\{\frac{(zf'(z))'}{g'(z)}\right\}>\eta \quad (z\in U;\ 0\le\lambda,\ \eta<p). \tag{6}$$