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Research Article

New Subclasses of Biunivalent Functions Involving Dziok-Srivastava Operator

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We introduce two new subclasses of biunivalent functions which are defined by using the Dziok-Srivastava operator. Furthermore, we find estimates on the coefficients $|a_2|$ and $|a_3|$ for functions in these new subclasses.

1. Introduction

Let A denote the class of all functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \quad (1)$$

which are analytic in the open unit disc $U = \{z \in \mathbb{C} : |z| < 1\}$. Also let S denote the class of all functions in A which are univalent in U.

Some of the important and well-investigated subclasses of the univalent function class S include, for example, the class $S^*(\beta)$ of starlike functions of order β in U and the class $K(\beta)$ of convex functions of order β in U. By definition, we have

$$S^*(\alpha) = \left\{ f \in S : \operatorname{Re}\left(\frac{zf'(z)}{f(z)}\right) > \beta, \; 0 \le \beta < 1, z \in U \right\},$$
$$K(\alpha) = \left\{ f \in S : \operatorname{Re}\left(1 + \frac{zf''(z)}{f'(z)}\right) > \beta, \; 0 \le \beta < 1, z \in U \right\}. \quad (2)$$

Ding et al. [1] introduced the following class $Q_\lambda(\beta)$ of analytic functions defined as follows:

$$Q_\lambda(\beta) = \left\{ f \in A : \operatorname{Re}\left((1-\lambda)\frac{f(z)}{z} + \lambda f'(z)\right) > \beta, \; 0 \le \beta < 1, \lambda \ge 0 \right\}. \quad (3)$$

It is easy to see that $Q_{\lambda_1}(\beta) \subset Q_{\lambda_2}(\beta)$ for $\lambda_1 > \lambda_2 \ge 0$. Thus, for $\lambda \ge 1$, $0 \le \beta < 1$, $Q_\lambda(\beta) \subset Q_1(\beta) = \{f \in A : \operatorname{Re} f'(z) > \beta, 0 \le \beta < 1\}$ and hence $Q_\lambda(\beta)$ is univalent class (see [2–4]).

It is well known that every function $f \in S$ has an inverse f^{-1}, defined by

$$f^{-1}(f(z)) = z \quad (z \in U),$$
$$f(f^{-1}(w)) = w \quad \left(|w| < r_0(f); r_0(f) \ge \frac{1}{4}\right), \quad (4)$$

where

$$f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3) w^3 - (5a_2^3 - 5a_2 a_3 + a_4) w_4 + \cdots. \quad (5)$$

A function $f \in A$ is said to be bi-univalent in U if both $f(z)$ and $f^{-1}(z)$ are univalent in U. Let Σ denote the class of