جامعة الفيـــــــوم	الفرقة الرابعة	المادة : تحليل دالـــى
كليــــة التربيــــة	عـــام	امتحان دور يناير ٢٠١١
قســــم الرياضيــات	شعبة رياضيات"حديث"	الزمن : ثلاث ساعة

(1)(a) Defind a normed space and Prove that ℓ_p^K is a Banach space.

(b) If $d(x,y)=\begin{cases}1 & x\neq y\\ 0 & x=y\end{cases}$

Prove that (X,d) is ametric space ; X is any set.

(2)(a) Prove that: If Y be a subspace of a normed space X and $f(x)$ is abounded linear functional on Y then there is a bounded linear functional $f(x)$ on X such that $F(x)=f(x)\ ;x\in Y$

and $\|F\|=\|f\|$.

(b) Prove that, If X be an inner product space and $x,y\in X$ then $\|x+y\|^2+\|x-y\|^2=2(\|x\|^2+\|y\|^2)$

(3)(a) state and Prove the Riesz's Lemma.

(b) Prove that, every convergent sequence is a Cauchy sequence.

(4)(a) Prove that, a linear operator $T:X\to Y\ ;X,Y$ normed space is continuous iff it is bounded .

(b) Prove that, C^n is an inner product space such that

$$\langle z,w\rangle=\sum_{i=1}^{n} z_i\overline{w_i}\ .$$

(5)(a) state the Hahn-Banach theorem and prove that If

$T\in B(X)$ *and* $T^2=I$ then $\sigma_p(T)=\{-1,1\}$

(b) Prove that : If Y be a closed subspace of a Hilbert space H, then for each $x\in H$ there is $v\in Y$ and w orthogonal to Y such that x=v+w and this decomposition is unique.

(مع تمنيـــــاتي بالنجـــاح)