

Answer the following questions:

1. a) Transform the equation:

$$e^{2x}u_{xx} + 2e^{x+y}u_{xy} + e^{2y}u_{yy} = 0$$

To its canonical form.

b) Find the Laplace Transform of the functions:

i)  $t \sin 3t$

ii)  $\frac{\sin t}{t}$

iii)  $t^2 e^{3t}$

2. a) Find the inverse Laplace Transform of the functions:

i)  $\frac{s}{(s-1)^2(s+2)}$

ii)  $\frac{s^2 + 2s - 4}{(s+1)(s-2)(s-4)}$

b) Use the convolution theorem to find  $L^{-1}\left[\frac{1}{s(s-2)^2}\right]$

3. a) Find the solution of the problem:

$$u_{tt} - c^2 u_{xx} = x, \quad -\infty < x < \infty$$
$$u(x,0) = \cos x, \quad u_t(x,0) = 1 + x$$

b) Transform the problem:

$$u_{tt} - u_{xx} = \sin \pi x \quad \text{for } 0 < x < 1$$
$$u(x,0) = u_t(x,0) = 0$$
$$u(0,t) = u_t(1,t) = 0, \quad 0 \leq x \leq t$$

To another problem with zero partial differential equation and then solve the problem.

4. a) Use the Laplace transform to solve the problem:

$$ty'' + 2y' + 4ty = 0, \quad y(0) = 1, \quad y'(0) = 0$$

b) Solve the problem:

$$u_{tt} = u_{xx} \quad \text{for } 0 < x < \infty, \quad 0 < t < \infty$$
$$u(x,0) = \cos x, \quad u_t(x,0) = x, \quad u(0,t) = 0$$

**Good Luck**