ABSTRACT

In this thesis we investigate some of the qualitative properties of dynamic integral equations on arbitrary time scale $\mathbf{T}$. A time scale is nonempty closed subset of the real numbers. First, we investigate the wellposedness of some kinds of nonlinear integral equations of *Volterra-Fredholm* type. We consider the following two different types

$$x(\mathrm{t}) = f(\mathrm{t}, \mathrm{x}(\mathrm{t}), \int_a^t h\,(\mathrm{t},\mathrm{s},\mathrm{x}(\mathrm{s}))\,\Delta \mathrm{s}, \int_a^b g(\mathrm{t},\mathrm{s},\mathrm{x}(\mathrm{s}))\,\Delta \mathrm{s}), \qquad \mathrm{t} \in I_{\mathbf{T}} := [a,\infty) \cap \mathbf{T} \qquad (1)$$

$$x^{\Delta}(\mathrm{t}) = f(\mathrm{t}, \mathrm{x}(\mathrm{t}), \int_a^t h\,(\mathrm{t},\mathrm{s},\mathrm{x}(\mathrm{s}))\,\Delta \mathrm{s}, \int_a^b g(\mathrm{t},\mathrm{s},\mathrm{x}(\mathrm{s}))\,\Delta \mathrm{s}), \qquad \mathrm{t} \in I_{\mathbf{T}} := [a,\infty) \cap \mathbf{T} \qquad (2)$$

where : $f : I_{\mathbf{T}} \times \mathbf{X} \times \mathbf{X} \times \mathbf{X} \to \mathbf{X}$, $h : I_{\mathbf{T}}^2 \times \mathbf{X} \to \mathbf{X}$,

$g : I_{\mathbf{T}}^2 \times \mathbf{X} \to \mathbf{X}$ **and** $\mathbf{X}$ **is a Banach space.**

Secondly, we apply the method of upper and lower solutions to the equation

$$x(\mathrm{t}) = f(\mathrm{t}) + \int_a^t \mathrm{k}(\mathrm{t},\mathrm{s},\mathrm{x}(\mathrm{s}))\,\Delta \mathrm{s}, \quad \mathrm{t} \in [\mathrm{a},\mathrm{b}]_{\mathbf{T}} = [\mathrm{a},\mathrm{b}] \cap \mathbf{T} \quad (3)$$

where: $f : [\mathrm{a},\mathrm{b}]_{\mathbf{T}} \to \mathbf{R}$ and $k : [\mathrm{a},\mathrm{b}]_{\mathbf{T}} \times [\mathrm{a},\mathrm{b}]_{\mathbf{T}} \times \mathbf{R} \to \mathbf{R}$.

Finally we study *Hyers-Ulam* stability and *Hyers-Ulam-Rassias* stability of a Volterra integral equation of the first kind

$$x(\mathrm{t}) = f(\mathrm{t}) + \int_a^t k(\mathrm{t},\mathrm{s})\,\mathrm{x}(\mathrm{s})\,\Delta \mathrm{s}, \quad \mathrm{t} \in I_{\mathbf{T}} \qquad (4)$$

where: $I_{\mathbf{T}}$ **is a time scale interval,** $f : I_{\mathbf{T}} \to \mathbf{R}$ **and**

$k : I_{\mathbf{T}} \times I_{\mathbf{T}} \to \mathbf{R}$**.**