A Theory of Nonlinear Negative Imaginary Systems

Ahmed G. Ghallab

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School of Engineering & Information Technology
University of New South Wales at Australian
Defence Force Academy

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Abstract

Feedback control theory is aimed at controlling a system input to obtain a desired output and making the system robust in the face of unmodeled dynamics and external disturbances. In real-word applications, most physical and engineering systems exhibit nonlinear behaviour which in general makes controller design difficult. One of the most appealing tools in the field of nonlinear control design is the "classical" dissipativity and passivity theory which characterizes the dissipation of energy with respect to a supplied energy rate from the outside environment. However, many systems that dissipate energy in the physical sense don't fall into this classical framework. For instance, flexible structures with colocated force actuators and position sensors are passive (dissipative) from the input to the derivative of the output instead of the output as in the classical passivity theory. It is not always straightforward to analyze the system's performance when the supply rate involves derivatives of the input and output.

In this regard, negative imaginary systems theory has proven to be an effective tool in the analysis and control design of linear time invariant system which are passive from the input to the derivative of the output. Negative imaginary systems theory has become a well-established systems-theoretical tool where it has been employed in a wide variety of control applications including, for instance, robust vibration control of flexible structure, atomic force microscopy, and nano-positioning systems.

In this thesis, we aim to generalize the negative imaginary systems theory to a broad class of nonlinear systems. A formal definition will be given for the negative imaginary property in the nonlinear domain by invoking a new dissipativity notion with an appropriate work rate. This formula is considerably more general than the existing classical dissipativity framework. Flexible structures with colocated force actuators and position sensors are dissipative according to this new definition.

Having defined the nonlinear negative imaginary property in a time-domain dissipativity framework, we are able to extend some of the main existing results on negative imaginary systems from the linear to nonlinear domain. First, a Lyapunov-based approach will be used to establish the stability robustness of a positive feedback interconnection of negative imaginary systems in the linear case under a set of theoretical assumptions. Then, these assumptions will be adapted in the nonlinear setup to establish the stability robustness analysis of a positive feedback interconnection of nonlinear negative imaginary systems by making use of Lyapunov stability theory and dissipativity techniques. %The new stability results have been shown to reduce to that of linear case.

The applicability of this nonlinear stability result will be illustrated through a basic practical example of nonlinear mass spring damper system. Furthermore, the nonlinear negative imaginary systems theory will be extended to the case of free motion. It will be shown that, under suitable assumptions, a cascade connection of an affine nonlinear system and single integrator will lead to a nonlinear negative imaginary system (with integrator). Finally, this thesis is concluded by a summary of current progress and a discussion of possible future developments of the nonlinear negative imaginary systems theory.