

**ON SOME FRACTIONAL
ORDER INTEGRAL EQUATIONS**

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ABSTRACT

Our aim here is to study the existence of solution of some coupled system of functional integral equations and coupled system of functional integral equations of fractional orders.

we study nonlinear coupled system of functional integral equations

$$x(t) = a_1(t) + \int_0^t f_1(t, s, y(\varphi_1(t))) ds \quad t \in [0, T] \quad (1)$$

$$y(t) = a_2(t) + \int_0^t f_2(t, s, x(\varphi_2(t))) ds \quad t \in [0, T] \quad (2)$$

and we study the integral equation

$$x(t) = a_1(t) + \int_0^t f_1(t, s, x(\varphi_1(t))) ds \quad t \in [0, T] \quad (3)$$

and also

$$x(t) = a_1(t) + \int_0^t f_1(t, s, I^{\beta_1} y(\varphi_1(t))) ds \quad t \in [0, T] \quad (6)$$

$$y(t) = a_2(t) + \int_0^t f_2(t, s, I^{\beta_2} x(\varphi_2(t))) ds \quad t \in [0, T] \quad (7)$$

and we study the coupled system of integro-differential equations

$$\frac{d}{dt} x(t) = a_1(t) + \int_0^t f_1(t, s, D^{\beta_1} y(\varphi_1(t))) ds \quad t \in [0, T] \quad (8)$$

$$\frac{d}{dt} y(t) = a_2(t) + \int_0^t f_2(t, s, D^{\beta_2} x(\varphi_2(t))) ds \quad t \in [0, T] \quad (9)$$

with the initial conditions

$$x(0) = x_0 \quad \text{and} \quad y(0) = y_0$$

where I^{β_1} and I^{β_2} are fractional orders integral operators.

And also we study

$$x(t) = a_1(t) + \int_0^1 f_1(t, s, I^{\beta_1} y(\varphi_1(t))) ds \quad t \in [0, 1] \quad (10)$$

$$y(t) = a_2(t) + \int_0^1 f_2(t, s, I^{\beta_2} x(\varphi_2(t))) ds \quad t \in [0, 1] \quad (11)$$

where I^{β_1} and I^{β_2} fractional orders integral operators

The coupled system of integro-differential equations

$$\frac{d}{dt} x(t) = a_1(t) + \int_0^1 f_1(t, s, D^{\beta_1} y(\varphi_1(t))) ds \quad t \in [0, 1] \quad (12)$$

$$\frac{d}{dt} y(t) = a_2(t) + \int_0^1 f_2(t, s, D^{\beta_2} x(\varphi_2(t))) ds \quad t \in [0, 1] \quad (13)$$

with the boundary conditions

$$x(0) = \gamma_1 x(1) \quad \text{and} \quad y(0) = \gamma_2 y(1) \quad , \gamma_1, \gamma_2 \neq 1.$$