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Some New Findings of Ordered Variables and their Concomitants Including Applications on Information Theory

A THESIS

Submitted to Mathematics Department, Faculty of Science
Al-AzharUniversity for the Degree of Doctor of Philosophy
in Mathematics (Mathematical Statistics)

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1447-2025

Summary

Information theory is a field of study that deals with the quantify action, storage, and communication of information. It was developed by Claude Shannon in the late of 1940. Since that data, information theory has become an important area of research in various fields, in clouding mathematics, computer science, and telecommunications. At its core, information theory seeks to understand how information can be efficiently represented, transmitted, and processed. It provides a mathematical framework for measuring the amount of information contained in a message or signal, known as entropy. Entropy is a mea sure of uncertainty or randomness in a random variable or probability distribution. It is derived from information theory and quantifies the average amount of information or uncertainty contained in a set of data. In information theory, there are several types of entropy, each with its own specific meaning and application. There are some important types of entropy such as Shannon entropy, residual, and past entropies. While entropy represents disorder and loss of energy, extropy represents order and growth. It embodies the pursuit of positive future possibili ties and the proactive shaping of a desirable future. Recently, Lad et al.[41] proved the complementary dual of entropy in information theory and an alternative measure of uncertainty, called extropy. The entropy and extropy measures relate as the positive and negative images of aphotographic film relate to each other. Extropy is used to score the forecasting distribution and in speech recognition.

On the other hand, concomitants of ordered random variables have a wide variety of applications in many areas such as selection problems, prediction analysis, double sampling plans, inference problems, and information theory. Their importance appears in many fields such as biological, biomedical, physical, industrial and economical disciplines. The aim of this thesis is to study the concomitants of ordered random variables arising from Morgenstern family (Lai and Xie extension). The thesis consists of five chapters:

Chapter 1 In this chapter, we give a brief introduction to the basic defined as the following: the different types of ordered random variables and its concomitants, Lai and Xie extension and measures of informs. In addition, a summary of previous studies is introduced.

Chapter 2 Lai and Xie extension is considered as extension of Morgenstern family and their concomitants for case-I of generalized order statistics and its dual are discussed in this chapter. Additionally, recurrence relation between moments is found for the recommended models. We have also derived the expression for the joint distribution of concomi tants for case-I of generalized order statistics and its dual.

Chapter 3 In this chapter, we obtain the concomitants of Lai and Xie ex tension for case-II generalized order statistics and its dual. In addition, recurrence relation between moments for the required models is obtained. Furthermore, the joint distribution between two concomitants is derived for case-II generalized order statistics and its dual.

Chapter 4 Our goal in this chapter presents the Shannon entropy and Fisher information with exponential distribution as an example from the Lai and Xie extension for concomitants of generalized order statis tics and its dual. Furthermore, we study the residual and past entropies from the Lai and Xie extension for concomitants of generalized order statistics and its dual.
Chapter 5 We obtain the extropy of concomitants from Lai and Xie ex tension of generalized order statistics and give some examples from well-known distributions. Furthermore, we study the residual and past extropy for its model. On the other hand, we consider the cu mulative residual extropy and negative cumulative extropy. Finally, non-parametric estimation is included.