IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11, Issue 3 Ver. IV (May - Jun. 2015), PP 42-51
www.iosrjournals.org

Describing Pseudospherical Planes and Other Properties of Evolutionary SolitonEquations

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Abstract: *In thispaper we will derive Bäcklund transformations and conservation laws based on geometrical properities of evolution equations with more than two independent variables that describe pseudospherical surfaces.*
Keywords: *Evolution equations, Pseudospherical surfaces, Bäcklund transformations, conservation laws andSolitons.*

I. Introduction

In this paper we, interest in Bäcklund transformation[12], and its connection with some special equations and their associatedsoliton theory.Under this transformation.an infinite family ofconstant curvature surfaces can be produced from a given one.The notion of a differential equation for a function $u(x,t)$ that describes a pseudospherical surface (P.S.S.) was introduced in [1,6,7], where classifications for some equations oftypes

$$u_{xt} = \psi\left(u, u_x, u_{xx}, \dots\dots \frac{\partial^k u}{\partial x^k}\right) \quad \text{and} \quad u_t = \psi\left(u, u_x, \dots\dots \frac{\partial^k u}{\partial x^k}\right)$$

Were obtained. Furthermorecharacterizations of equations with more than two independent variables of types

$$u_{xt} = \psi\left(u, u_x, \dots\dots \frac{\partial^k u}{\partial x^k}, u_y, \dots, \quad \frac{\partial^{k'} u}{\partial x^{k'}}\right), u_t = \psi\left(u, u_x, \dots\dots \frac{\partial^k u}{\partial x^k}, u_y, \dots, \quad \frac{\partial^{k'} u}{\partial x^{k'}}\right)$$

$$\text{and} u_{tt} = \psi\left(u, u_x, \dots, \frac{\partial^k u}{\partial x^k}, u_y, \dots, \frac{\partial^{k'} u}{\partial y^{k'}}, u_t\right) \text{are given in [2,3,4].}$$

A systematic procedure to determine linear problems associated to non-linear equations of the abovetypes was also introduced in case of two independent variables.

In this work, we consider evolution equations for a function $u(x,y,t)$ that describes an(η,ξ) 3-dim. P.S.P. as given in [2,3,4] and we investigate an analogous method to derive Bäcklundtransformations and conservation laws based on geometrical properties of these 3- dimensionalpseudo spherical planes in R^5.

II. Local theory of constant negative curvature submanifolds of R^{2n-1}

Let M be an n-dimensional Riemannian manifold with constant curvature K isometrically immersed in $\overline{M}^{2n-1}$with constant curvature $\overline{K}$,with $K<\overline{K}$. Let $e_1, e_2, \dots, e_{2n-1}$be a moving orthonormal frame on an open set of $\overline{M}$,so that at points of M,$e_1, e_2, \dots, e_n$are tangents to M.Let $\omega_A$be the dual orthonormal coframe and consider ω_{AB} defined by [2]

$$de_A = \sum_B \omega_{AB} e_B$$

The structure equations of $\overline{M}$are

$$d\omega_A = \sum_B \omega_B \wedge \omega_{BA} \quad , \quad \omega_{AB} + \omega_{BA} = 0 \tag{1}$$

$$d\omega_{AB} = \sum_C \omega_{AC} \wedge \omega_{CB} - \overline{K}\omega_A \wedge \omega_B \quad \text{with} \quad 1 \le A, B, C \le 2n-1 \tag{2}$$

Restricting these forms to M we have $\omega_\alpha = 0$,so (1) gives with $n+1 \le \alpha, \beta, \gamma \le 2n-1$ and $1 \le I, J, L \le n$,

$$d\omega_\alpha = \sum_I \omega_I \wedge \omega_{I\alpha} = 0 \tag{3}$$

$$d\omega_I = \sum_J \omega_J \wedge \omega_{JI} \tag{4}$$

from (2) we obtain, Gauss equation