

# Fayoum University

Faculty of Engineering

Department of Civil Engineering



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## **CE 402: Part D**

**Lecture No. (17)**

## **Slope Stability Analysis**

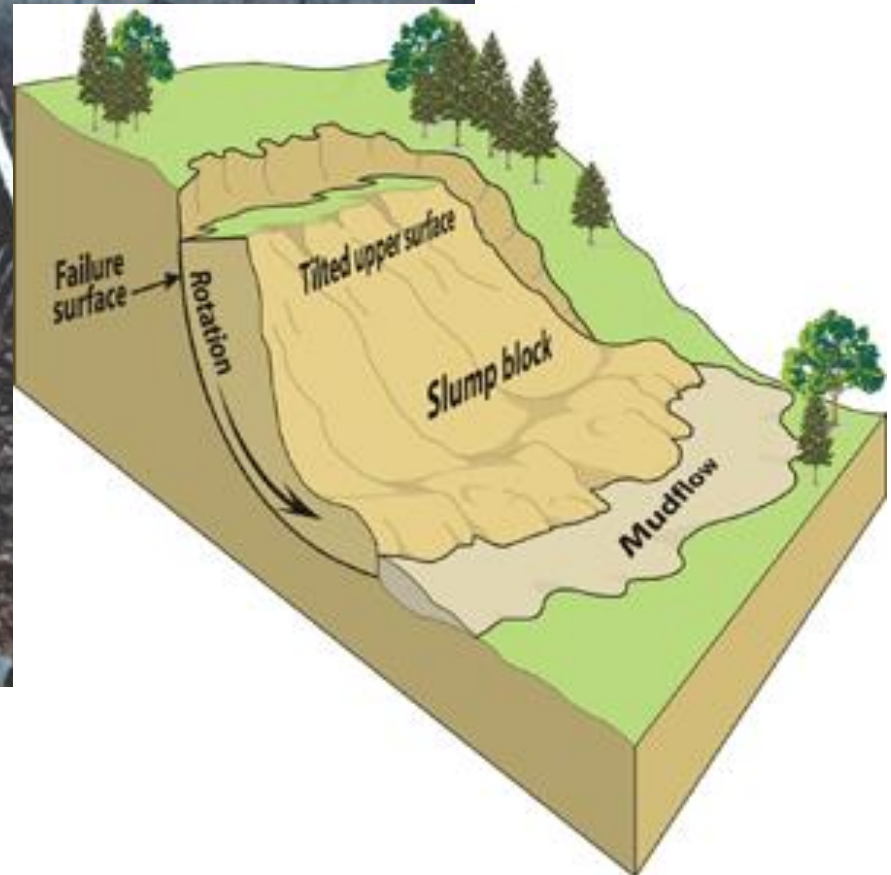
**Dr.: Youssef Gomaa Youssef**

# Definition

- It is an earth body which provides a gradual change in ground surface elevation between two different levels .



# Slope Stability



# Types Of Slopes

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## **Natural slopes**

(Mountains, Valley sides,.....)

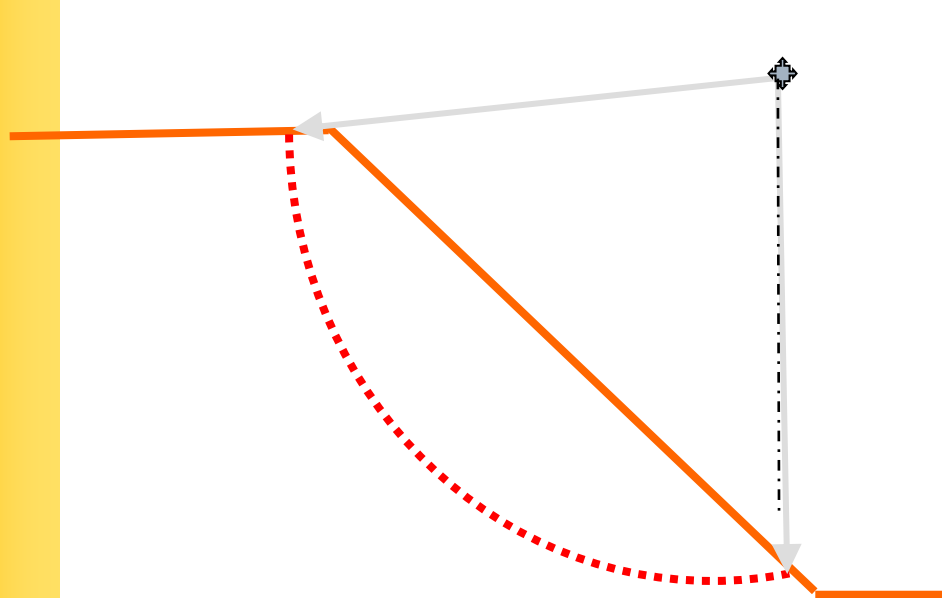
## **Man-made slopes**

(Canal banks, Earth dams, Road cuts,.....)

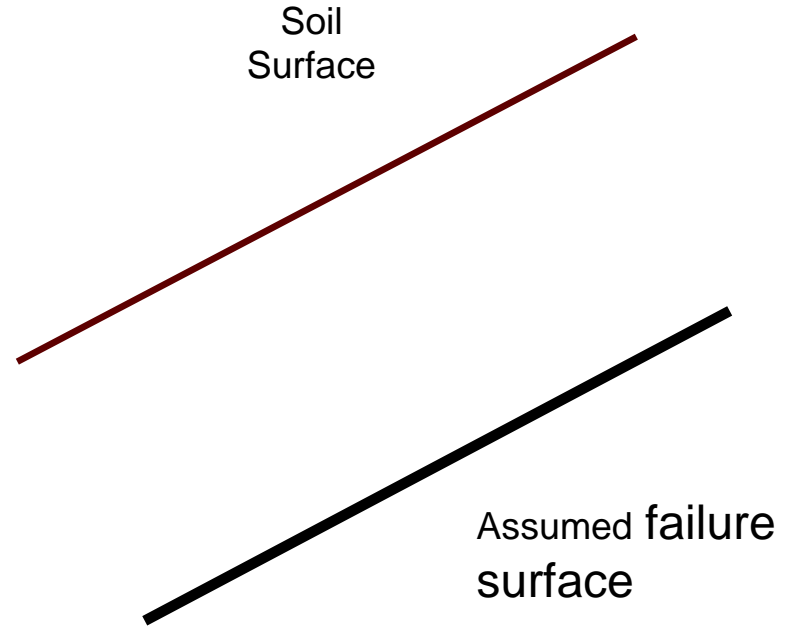
# Types Of Slopes Failures

1. Infinite slopes
2. Finite slopes
  - Rotational slip. ( Circular, non-circular)
  - Transitional slip.
  - Compound slip.

# Types Of Slopes Failures



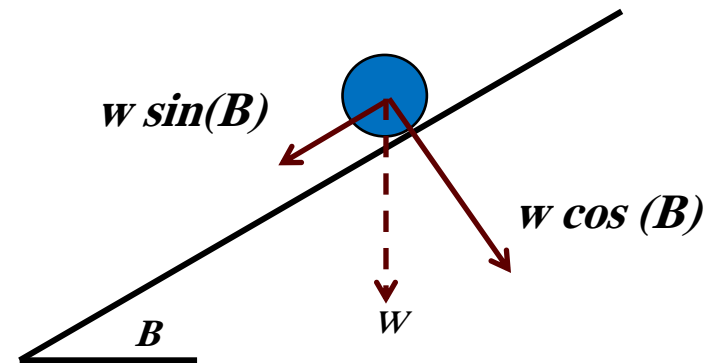
**Rotational slip**



**Infinite slopes**

# Infinite Slopes In Sand

Factor of safety in sandy slopes is **independent** of the height of the slope.



$$\text{Driving forces} = w \sin B$$

$$\text{Resisting forces} = N * \mu = w \cos (B) * \tan (\varphi)$$

$$F.S = w \cos (B) \tan(\varphi) / w \sin(B) = \tan(\varphi) / \tan (B)$$

$$F.S = \tan (\varphi) / \tan (B)$$

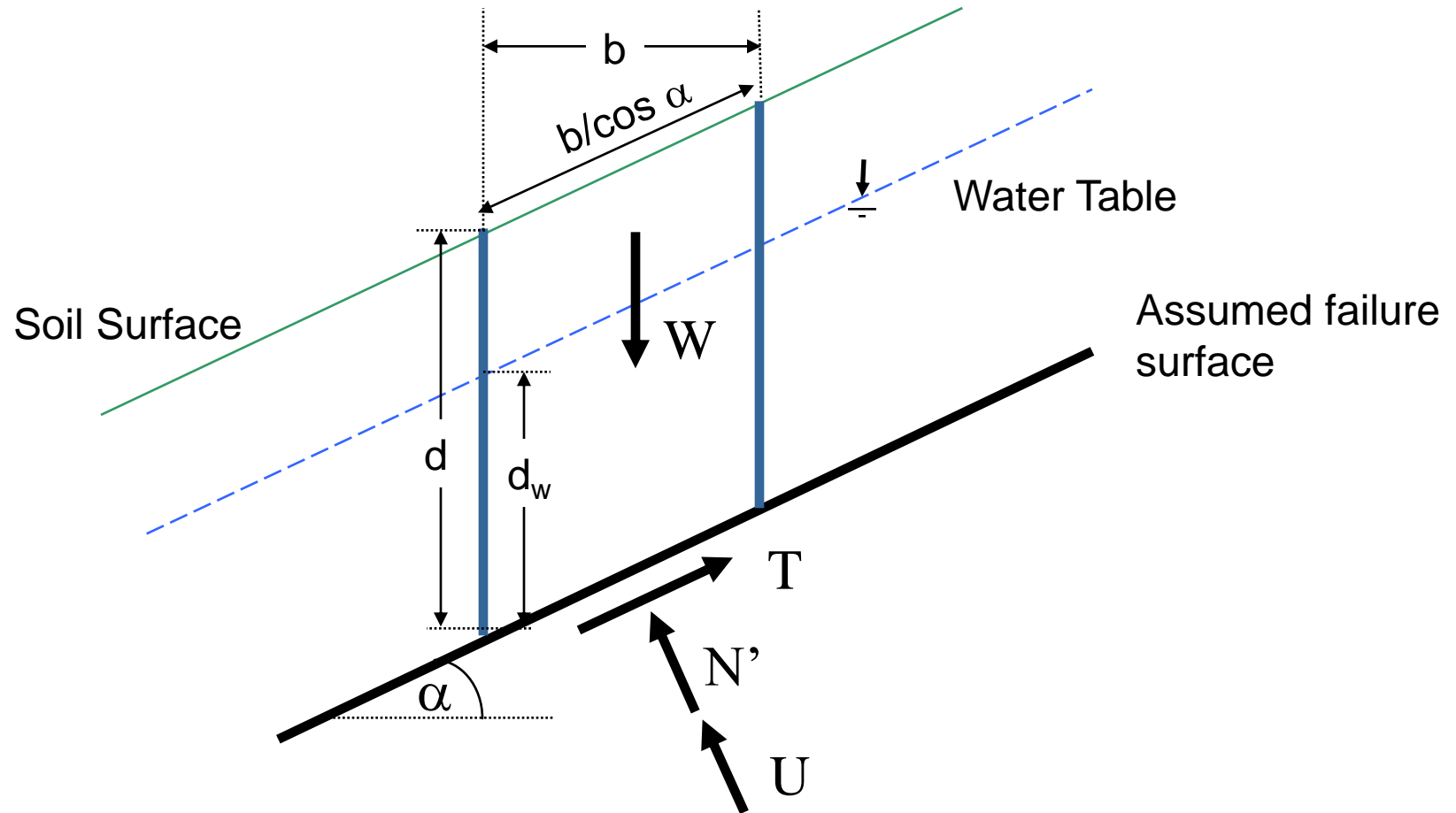
# Example (1):

Find the factor of safety of a slope of infinite extent having a slope angle =  $25^\circ$ . The slope is made of cohesionless soil with  $\phi = 30^\circ$ .

$$F.S. = \frac{\tan \phi'}{\tan \beta} = \frac{\tan 30}{\tan 25} = 1.238$$



# Infinite Slopes

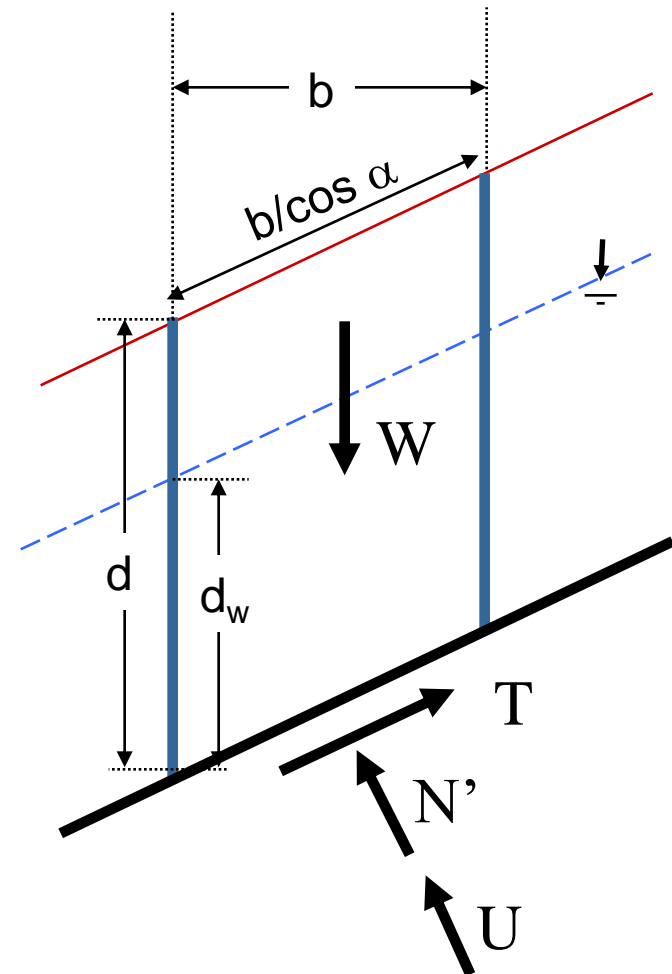


# Infinite Slopes

From equilibrium

$$N = W \cos \alpha = \gamma b d \cos \alpha$$

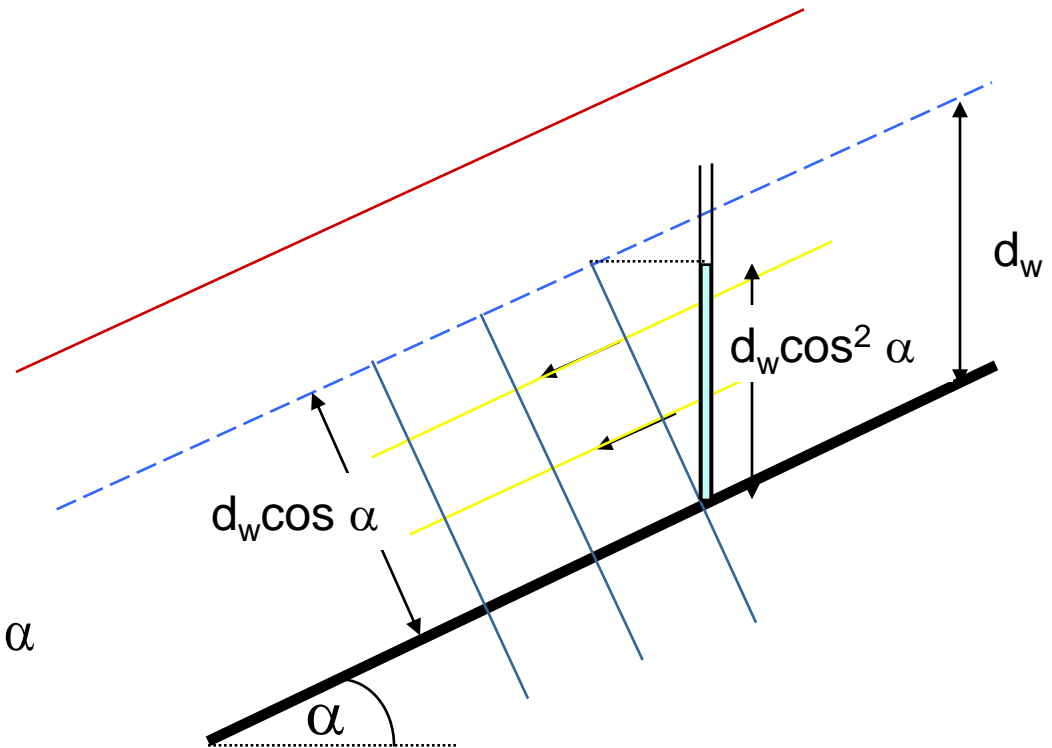
$$T = W \sin \alpha = \gamma b d \sin \alpha$$



# Infinite Slope

$$u = \gamma_w d_w \cos^2 \alpha$$

$$U = u \left( \frac{b}{\cos \alpha} \right) = \gamma_w b d_w \cos \alpha$$



# Infinite Slopes

On the failure plane  $\tau = c' + \sigma' \tan \phi'$

Defining the factor of Safety, F, by

$$F = \frac{\tau_f}{\tau} = \frac{\text{shear stress required for failure}}{\text{actual shear stress}}$$

$$F = \frac{c' + (\gamma d - \gamma_w d_w) \cos^2 \alpha \tan \phi'}{\gamma d \sin \alpha \cos \alpha}$$

# Infinite Slopes

If  $c' = 0$

$$F = \left( 1 - \frac{\gamma_w d_w}{\gamma d} \right) \frac{\tan \phi'_{cs}}{\tan \alpha}$$

$$F = \frac{\tan \phi'_{cs}}{\tan \alpha}$$

$$\alpha = \phi'_{cs}$$

# Infinite Slopes

If  $c' = 0$

$$F = \left( 1 - \frac{\gamma_w d_w}{\gamma d} \right) \frac{\tan \phi'_{cs}}{\tan \alpha}$$

If the water is at the soil surface,  $d = d_w$  and when  $F = 1$

$$\tan \alpha = \left( 1 - \frac{\gamma_w}{\gamma} \right) \tan \phi'_{cs}$$

For typical values it is found that  $\alpha$  is about  $0.5 \phi'$

Water reduces the stable angle of the slope by 50%

## Example (2):

Analyze the slope of infinite extent if it is made of clay having  $c' = 30 \text{ kN/m}^2$ ,  $\phi' = 20^\circ$ ,  $e = 0.65$  and  $G_s = 2.7$  and under the following conditions:

(i) when the soil is dry,

(ii) when water seeps parallel to the surface of the slope,

(iii) when the slope is submerged.

## Example (2):

$$\gamma_d = \frac{G_s}{1+e} \gamma_w = \frac{2.70}{1+0.65} * 9.81 = 16.05 \text{ kN/m}^2$$

$$\gamma_{sat} = \frac{G_s + e}{1+e} \gamma_w = \frac{2.70+0.65}{1+0.65} * 9.81 = 19.90 \text{ kN/m}^2$$

**(i) when the soil is dry,**

$$F_s = \frac{c' + (\gamma d - \gamma_w d_w) \cos^2 \alpha \tan \phi'}{\gamma d \sin \alpha \cos \alpha}$$

$$1.00 = \frac{30 + (16.05 * d) \cos^2 25 \tan 20}{16.05 * d \sin 25 \cos 25}$$

$$d = 22.23 \text{ m}$$



# Example (2):

**ii) when water seeps parallel to the surface of the slope,**

$$F_s = \frac{c' + (\gamma d - \gamma_w d_w) \cos^2 \alpha \tan \phi'}{\gamma d \sin \alpha \cos \alpha}$$

$$1.00 = \frac{30 + (19.90 - 9.81) * d \cos^2 25 \tan 20}{19.90 * d \sin 25 \cos 25}$$

$$d = 6.52m$$

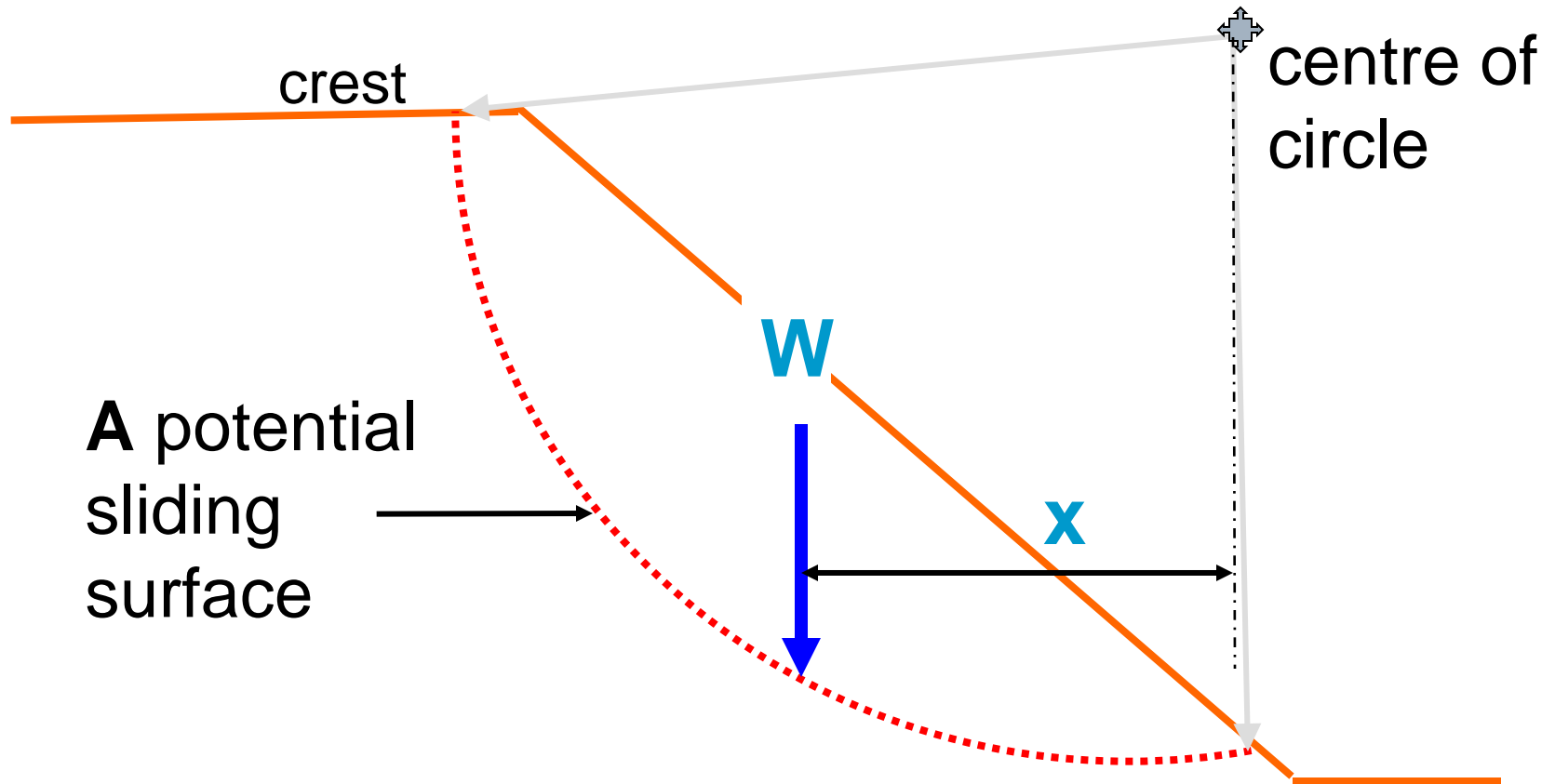
**iii) when the slope is submerged.**

$$F_s = \frac{c' + (\gamma d - \gamma_w d_w) \cos^2 \alpha \tan \phi'}{\gamma d \sin \alpha \cos \alpha}$$

$$1.00 = \frac{30 + (19.90 - 9.81) * d \cos^2 25 \tan 20}{(19.90 - 9.81) * d \sin 25 \cos 25}$$

$$d = 35.37m$$

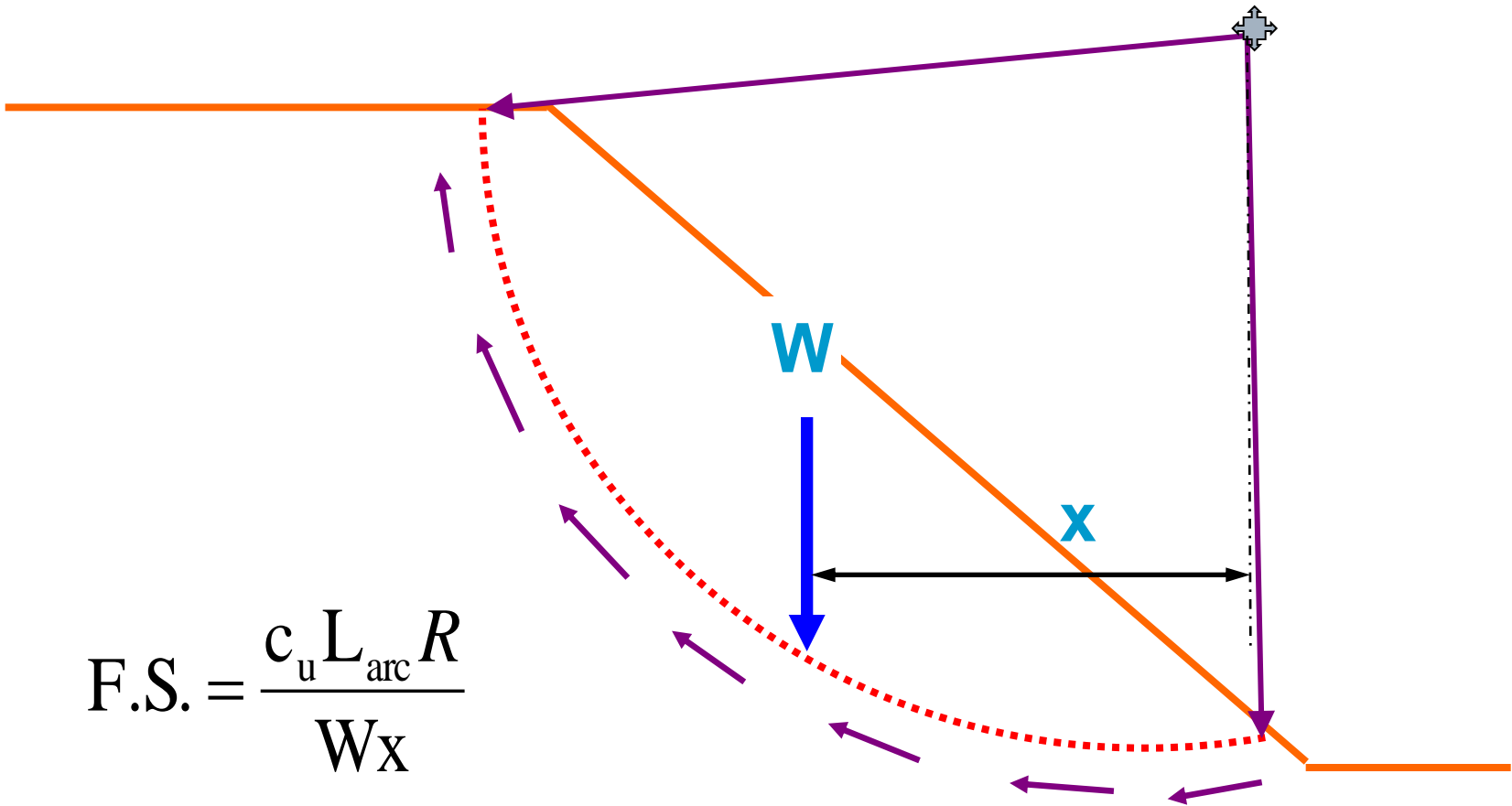
# Circular Slips



# CIRCULAR SLIPS Stability

## Case 1: $\phi' = 0$

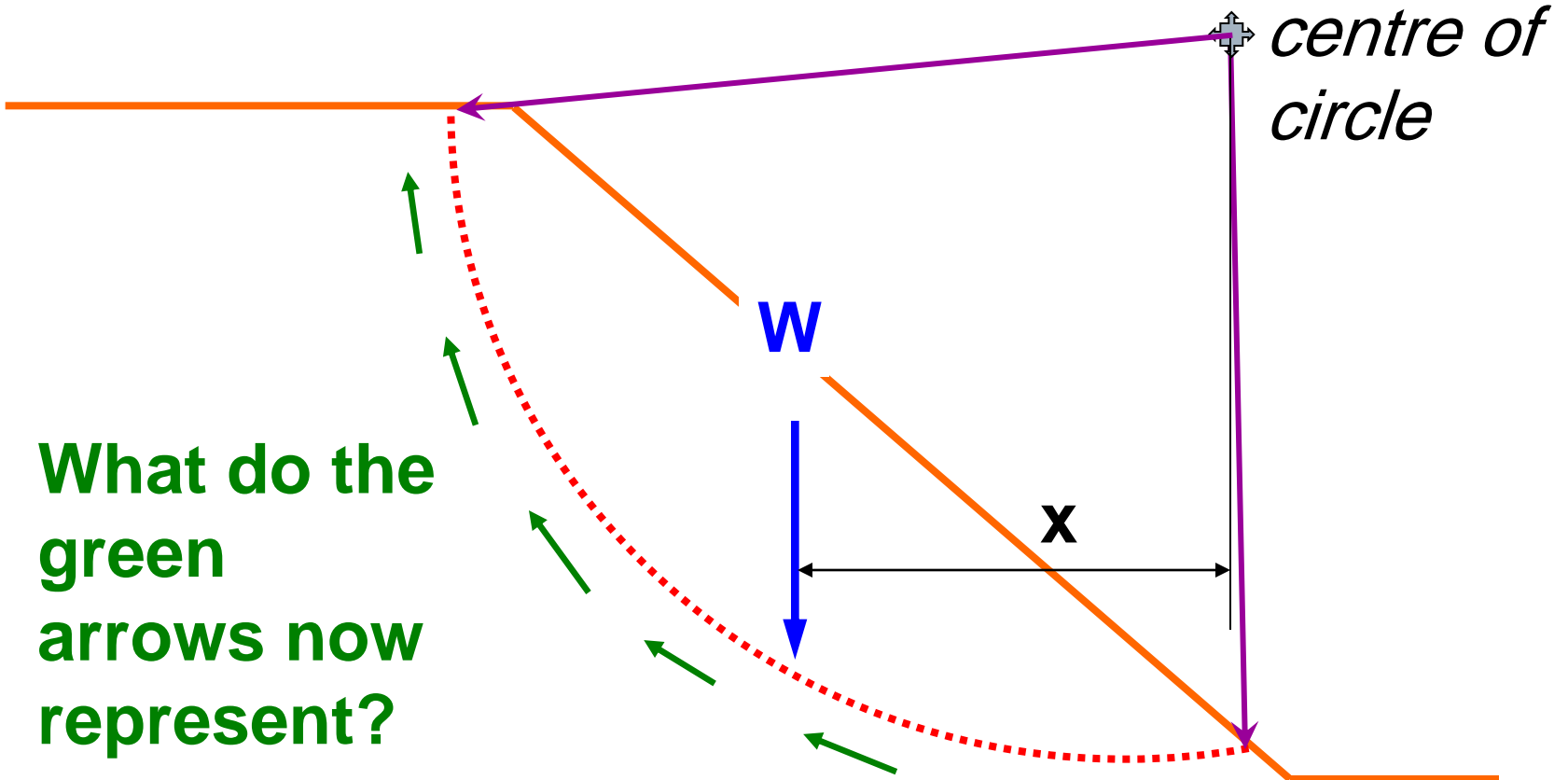
Stability? *Limit equilibrium*



$$F.S. = \frac{c_u L_{arc} R}{W_X}$$

# CIRCULAR SLIPS Stability

## Case 2: $\phi' \neq 0$



What do the green arrows now represent?



# Overall Stability Method

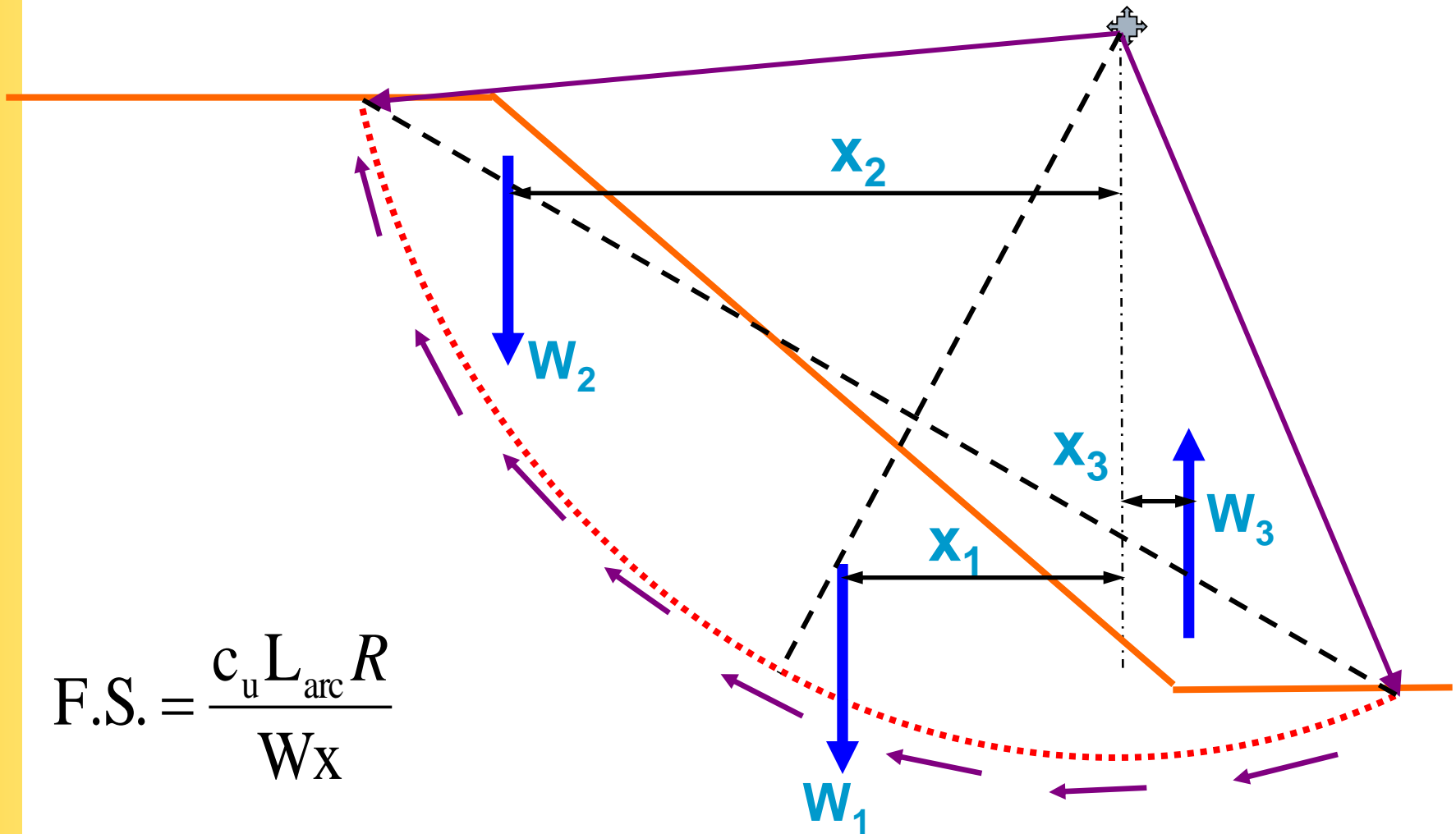
*For Cohesive Soil Only*

$$C_u = \checkmark$$

$$\phi = 0.0$$

$$F.S. = \frac{c_u L_{arc} R}{W_X}$$

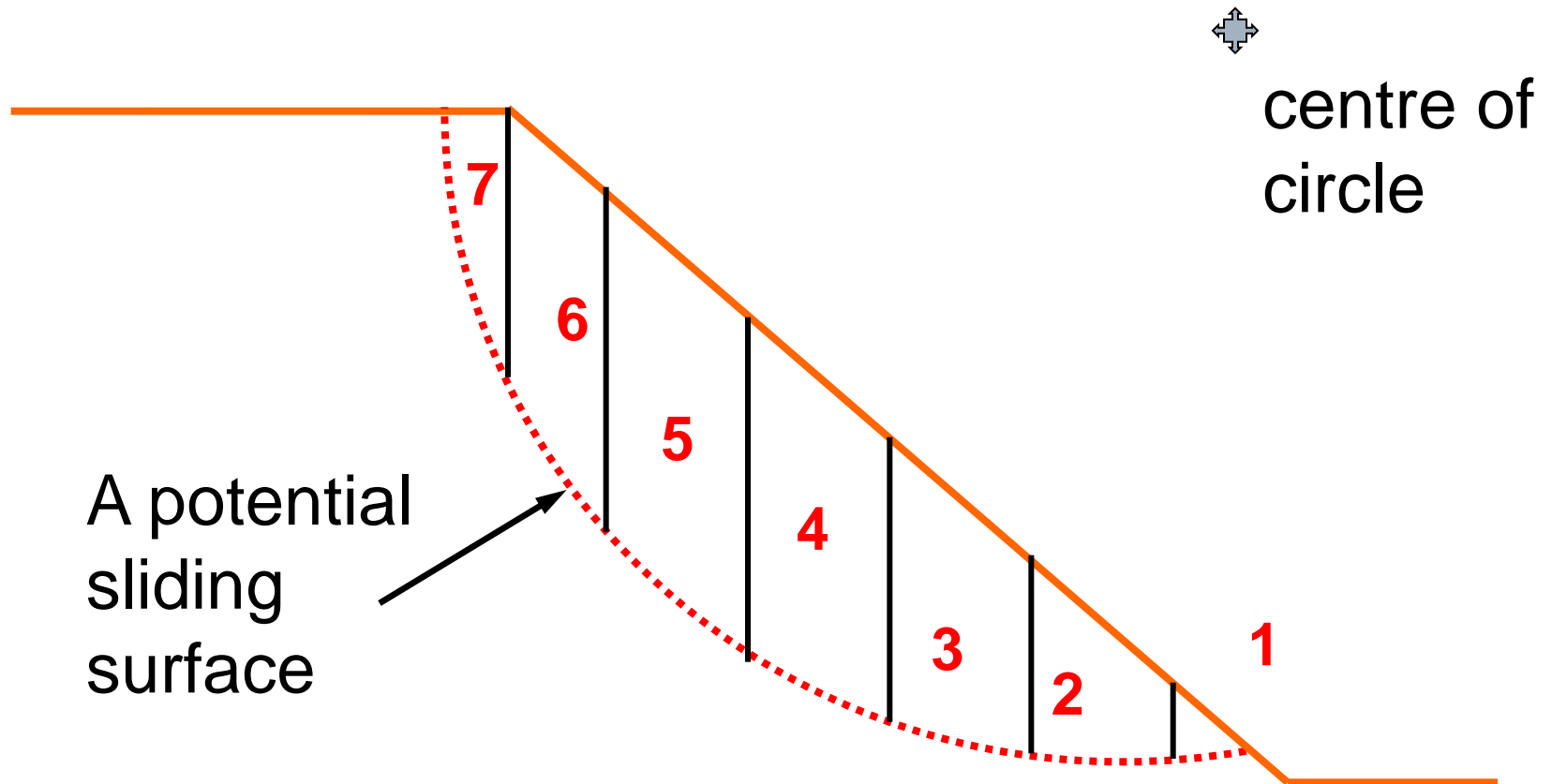
# Overall Stability Method



$$F.S. = \frac{c_u L_{arc} R}{W_X}$$

# CIRCULAR SLIPS

## "Method of Slices"

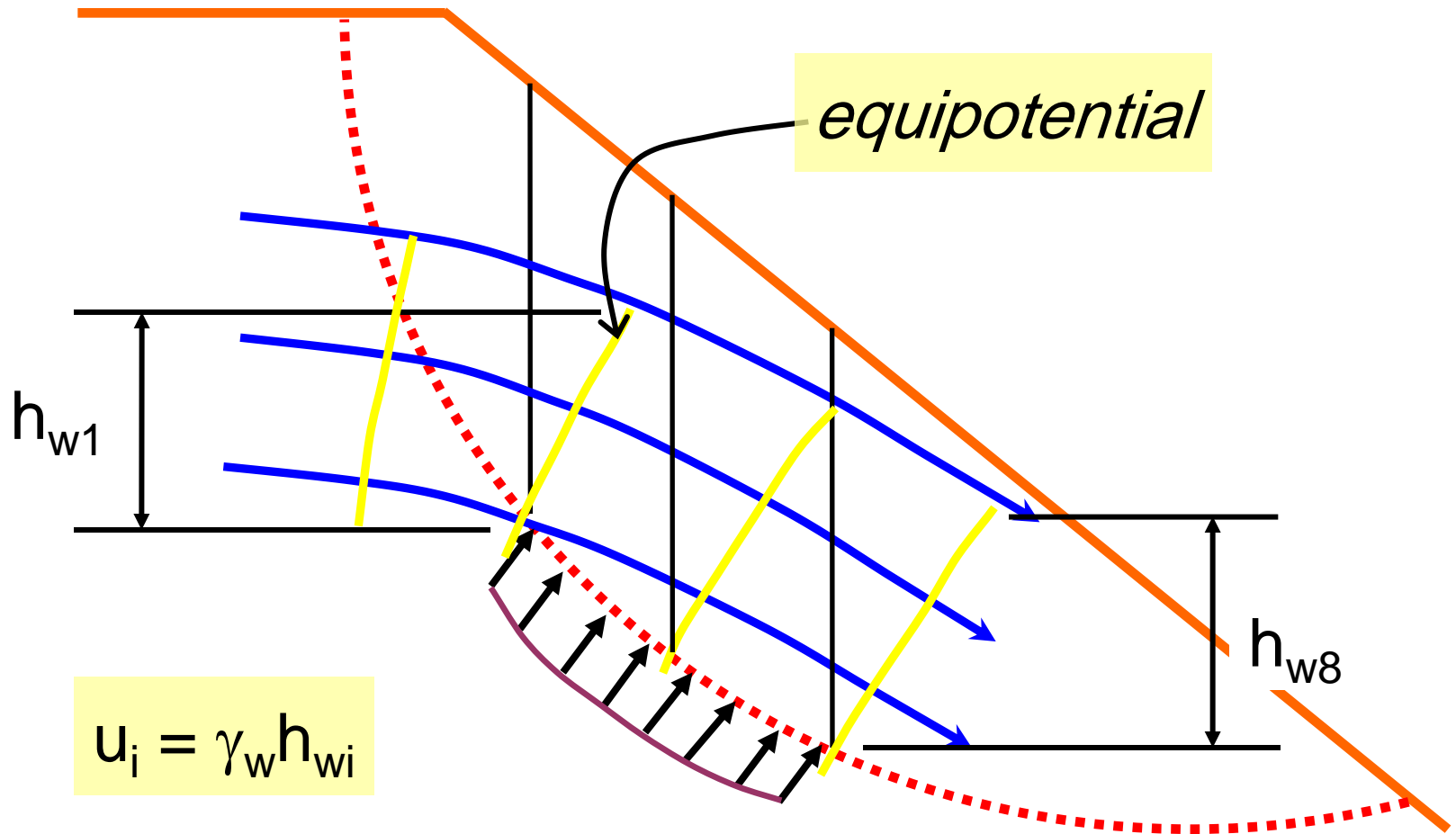




# Reasons for Slices

- Frictional shear resistance varies with both  $\sigma_N$  and  $\phi'$
- Varying cohesion with depth
- Non-uniform pwp's from seepage analysis

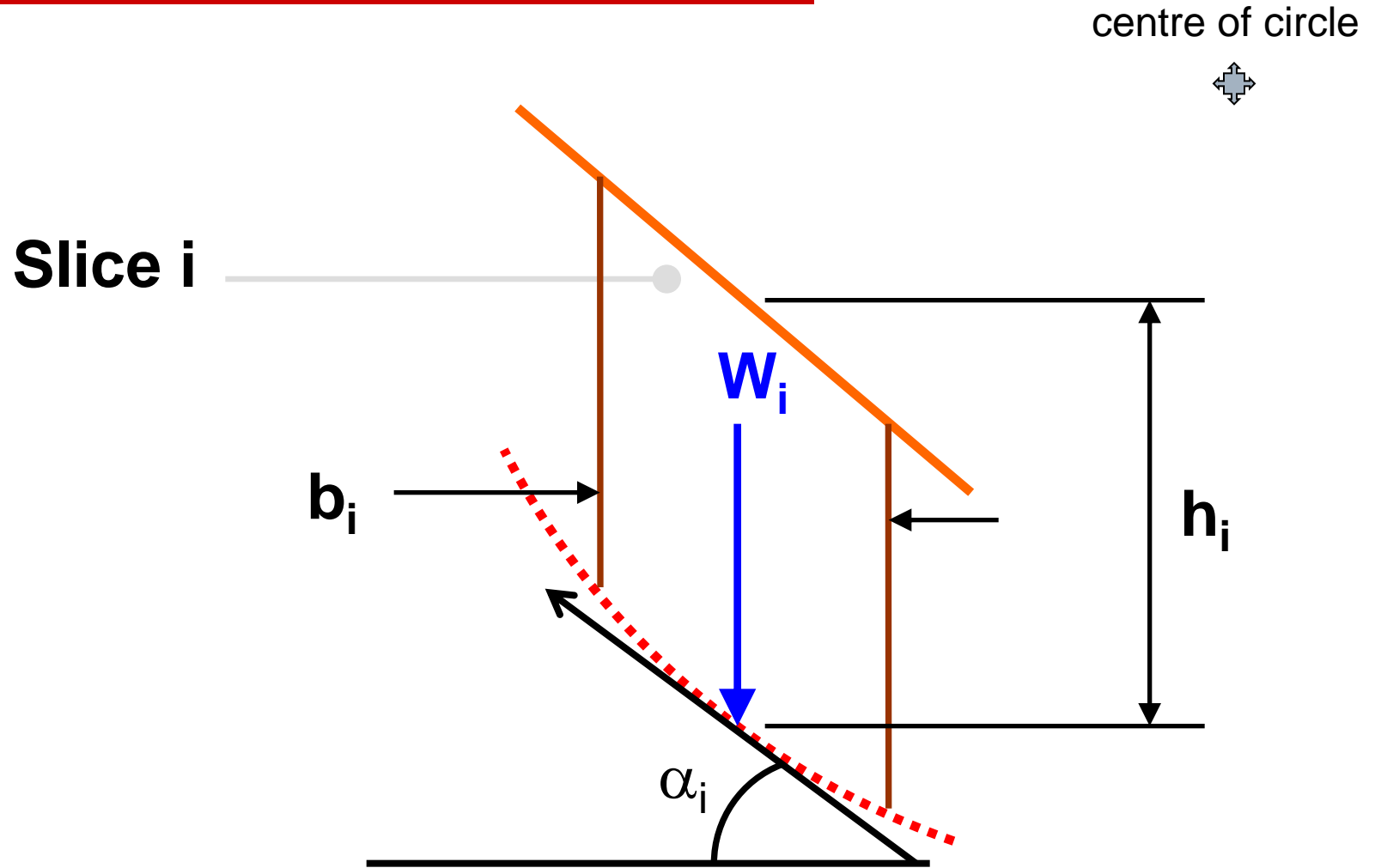
# PWP influence - "u" from flow net



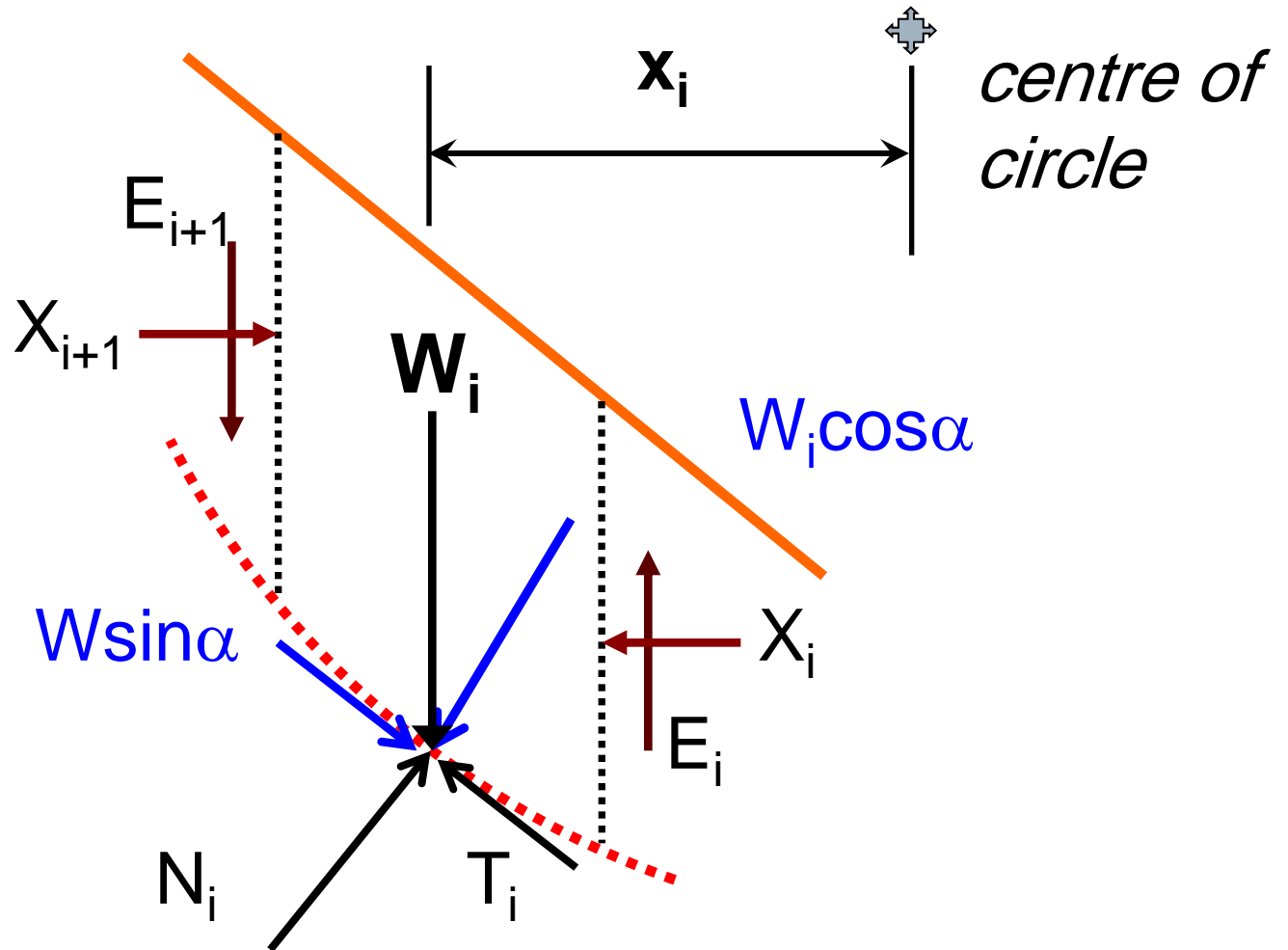
# General Method of Slices

- F.o.S by summation over **all slices** for trial failure surface
- 100's of trial surfaces evaluated
  - thank you for the pc!
  - XSLOPE and GALENA
- Lowest F.o.S  $\Rightarrow$  *the "critical failure surface"*

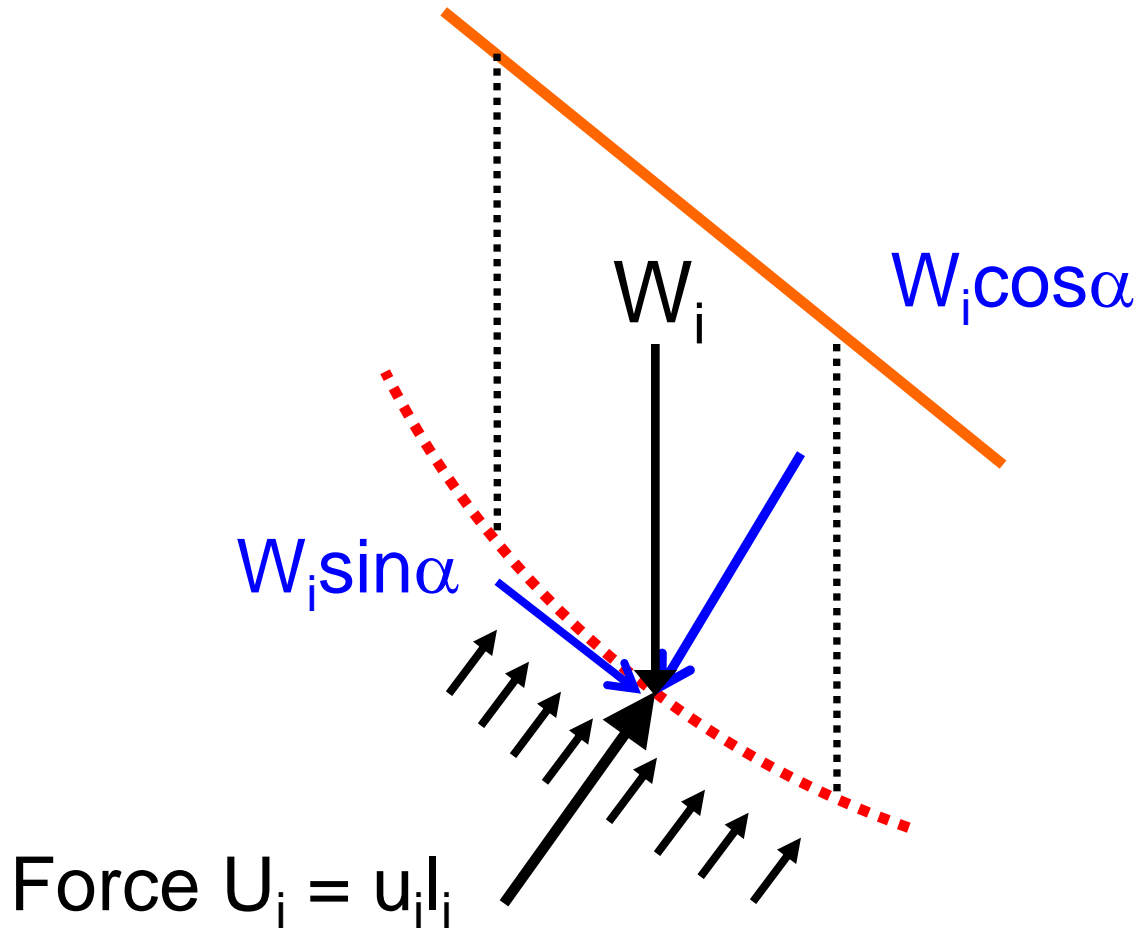
# Stability of a Vertical Slice



# Stability of a Slice (no pwp)



# PWP influence



# Different Methods of slices

**Slices - overall too many unknowns!**

**- need simplifying assumptions to  
get a solution!**

**Side Forces:**

- Assumptions for these forces  
= differences in methods  
**e.g. Fellenius v. Bishop's simplified method**

# Fellenius Method

Resultant of side forces = zero

i.e.  $X_i = X_{i+1}$  and  $E_i = E_{i+1}$

For homogeneous soil:

restoring shear force =  $c'L_{\text{arc}} + \tan\phi'\Sigma N'$

where,  $N'_i = W_i \cos\alpha_i - u_i l_i$

and  $l_i = \text{arc length of slice, } i$



# Factor of Safety - Fellenius

$$F = \frac{\text{restoring shear force}}{\text{sliding force on slip plane}}$$

$$F = \frac{c'L_{\text{arc}} + \tan\phi'\Sigma(W\cos\alpha - ul)}{\Sigma W\sin\alpha}$$

**Warning:** method regarded as simplistic and non-conservative

# Procedure of Analysis

•Fill the following table with these values

Slice No.	b	h	$\alpha$	$\gamma$	c	$\phi$	$W = \gamma hb$	$L = b / \cos \alpha$	cL	$w \cos \alpha \tan \phi$	$w \sin \alpha$
1											
2											
3											
4											
5											
6											
7											
									$\Sigma cL$	$\Sigma w \cos \alpha \tan \phi$	$\Sigma w \sin \alpha$

$$F.S = \Sigma C * L + \Sigma [W * \cos (\alpha) * \tan(\phi)] / \Sigma W * \sin (\alpha)$$

# Procedure of Analysis

- Draw the slope and the assumed slip surface with suitable scale.
- Divide the slope into slices with the following considerations:
  - 1.No. of slices = 5 : 7
  - 2.The slices have equal width ( as possible)
  - 3.Every slice base located in one soil layer.
- Determine these values for each slice:
  - 1.Width (b)
  - 2.Average height (h)
  - 3.Angle of inclination of the base of slice ( $\alpha$ )
  - 4.Average unit weight of the slice ( $\gamma$ )
  - 5.Shear parameters at the base (c,  $\phi$ )

# Simplified Bishop Method

## - a superior method

- Resultant of side forces acts horizontally
- Apply FoS (F) to restoring shear force
  - $T = [l(c' + \sigma_N' \tan \phi')]/F$
- Sum all vertical forces
  - $W = \Sigma [N' \cos \alpha + [(c'l + N' \tan \phi') \sin \alpha]/F]$
- Solve for N'
- Substitute in 
$$FoS = \frac{\Sigma (c'l + N' \tan \phi')}{\Sigma W \sin \alpha}$$

# The Bishop Equation

$$F = \frac{\sum \left[ \frac{(c'b_i + (W_i - u_i b_i) \tan \phi')}{M_i} \right]}{\sum W_i \sin \alpha_i}$$

Where

$$M_i = \cos \alpha_i \left( 1 + \frac{\tan \alpha_i \tan \phi'}{F} \right)$$

# Simplified Bishop Method

- Requires iteration
  - Assume initial  $F$ , then solve for  $F$
  - When trial  $F$  and determined  $F$  are equal, it's a solution
- Spreadsheet for simple slopes
- XSLOPE and GALENA otherwise
  - 1000 trial surfaces in 1 minute