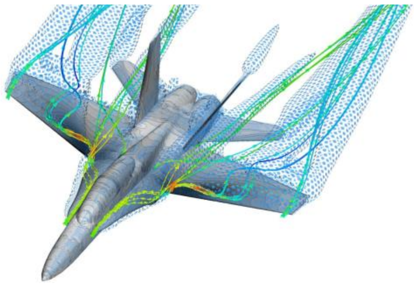
بِسْمِ اللَّهِ الرَّحْمَٰنِ الرَّحِيمِ

# Pervious Lecture Contents



1. Characteristics of Pressure
2. Pressure of fluid at rest
3. Measurement of Pressure
   - Simple manometers
   - Differential manometers
   - Mechanical gauge

**Fluid Mechanics I**



Fayoum University

Faculty of Engineering
Mechanical Engineering Dept.

## Lecture (3)
## on

# Fluid Statics

By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*Faculty of Engineering*
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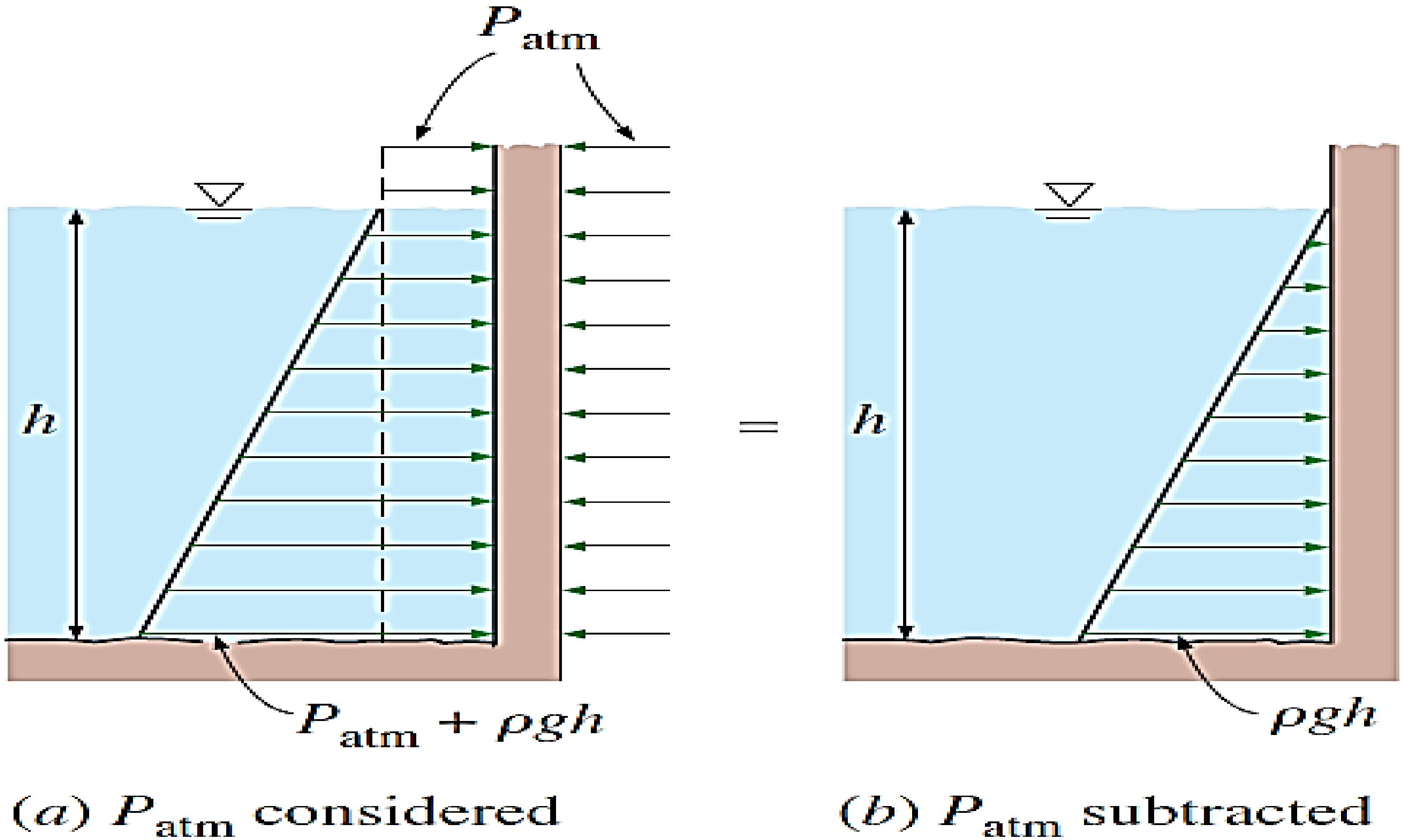
**2015 - 2016**

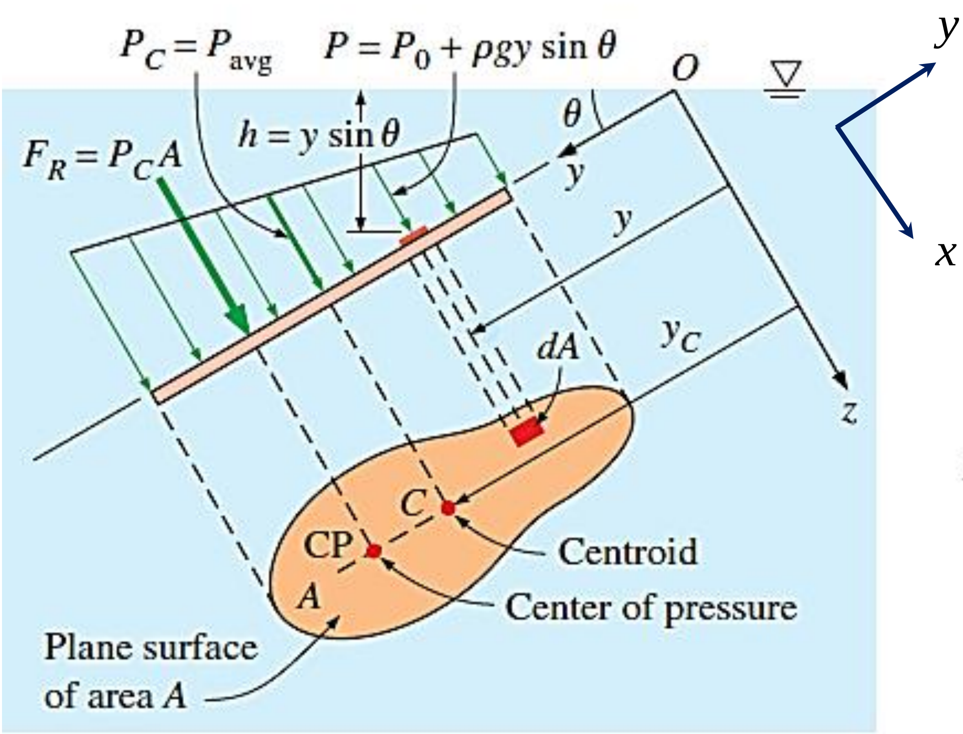
# Hydrostatic Forces on Completely Submerged Plane Surfaces

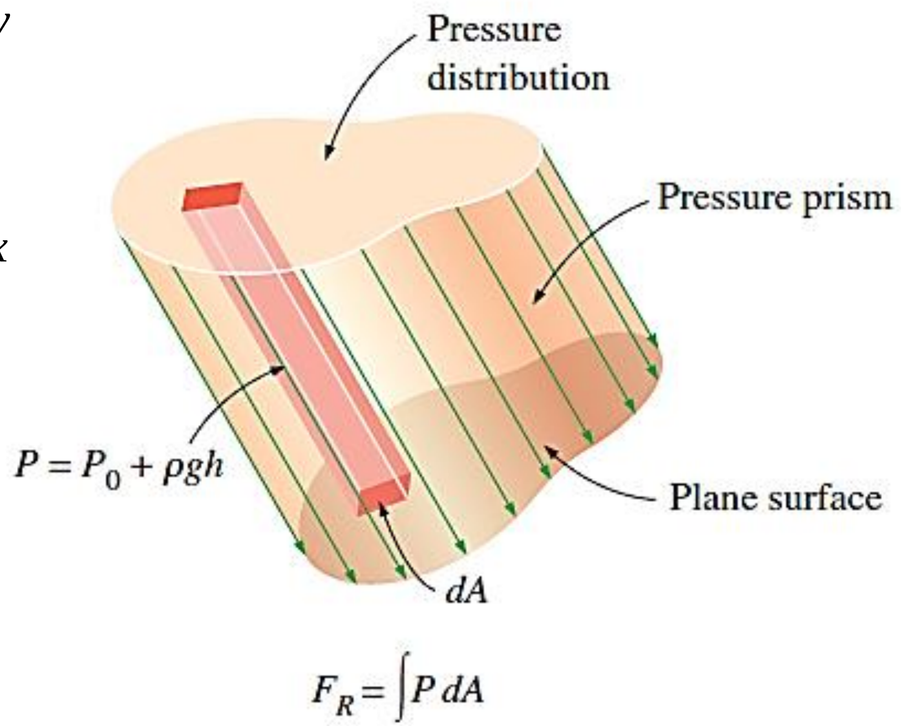




# Hydrostatic Forces on Completely Submerged Plane Surfaces





## Hydrostatic Forces on Completely Submerged Plane Surfaces



Then the absolute pressure at any point on the plate is

$$P = P_0 + \rho g h = P_0 + \rho g y \sin\theta$$

The resultant hydrostatic force $F_R$ acting on the surface is determined by integrating the force $P\,dA$ acting on a differential area $dA$ over the entire surface area,

$$F_R = \int_A P\,dA = \int_A (P_0 + \rho g y \sin\theta)\,dA = P_0 A + \rho g \sin\theta \int_A y\,dA$$

But the *first moment of area* $\int_A y\,dA$ is related to the *y*-coordinate of the centroid (or center) of the surface by

$$y_C = \frac{1}{A}\int_A y\,dA$$

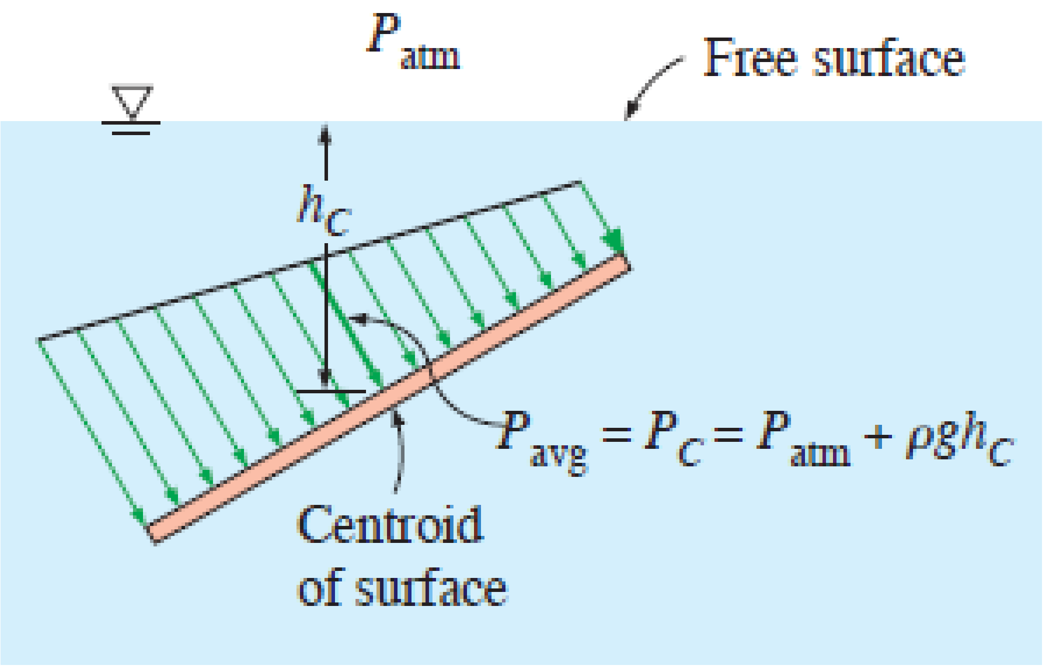
## Hydrostatic Forces on Completely Submerged Plane Surfaces



Substituting,

$$F_R = (P_0 + \rho g y_C \sin\theta)A = (P_0 + \rho g h_C)A = P_C A = P_{avg} A$$

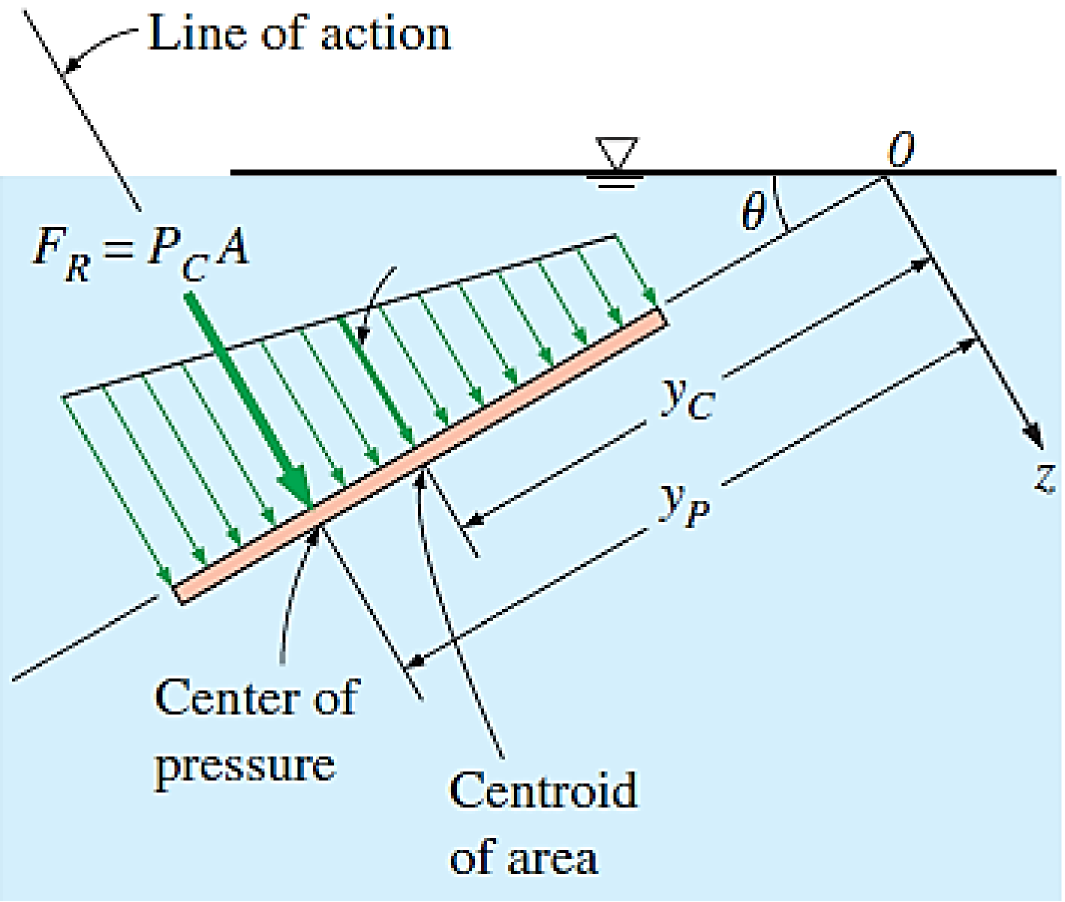
The magnitude of the resultant force acting on a plane surface of a completely submerged plate in a homogeneous (constant density) fluid is equal to the product of the pressure $P_C$ at the centroid of the surface and the area $A$ of the surface.



# Hydrostatic Forces on Completely Submerged Plane Surfaces



To balance the **bending-moment** portion of the stress, the resultant force $F_R$ does not act through the centroid. So, To find the coordinates ($x_P$, $y_P$)



## Hydrostatic Forces on Completely Submerged Plane Surfaces



The vertical location of the line of action is determined by equating the moment of the resultant force to the moment of the distributed pressure force about the *x*-axis:

$$y_P F_R = \int_A yP\, dA = \int_A y(P_0 + \rho g y \sin\theta)\, dA = P_0 \int_A y\, dA + \rho g \sin\theta \int_A y^2\, dA$$

**or**

$$y_P F_R = P_0 y_C A + \rho g \sin\theta\, I_{xx,O}$$

where $y_P$ is the distance of the center of pressure from the *x*-axis (point *O* in Figure and $I_{xx,O} = \int_A y^2\, dA$ is the *second moment of area* (also called the *area moment of inertia*) about the *x*-axis.

## Hydrostatic Forces on Completely Submerged Plane Surfaces



Fortunately, the second moments of area about two parallel axes are related to each other by the parallel axis theorem, which in this case is expressed as:

$$I_{xx,\,O} = I_{xx,\,C} + y_C^2 A$$

where $I_{xx,\,C}$ is the second moment of area about the $x$-axis passing through the centroid of the area and $y_C$ (the $y$-coordinate of the centroid) is the distance between the two parallel axes.

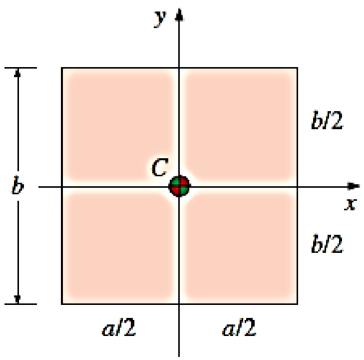
$$y_P = y_C + \frac{I_{xx,\,C}}{[y_C + P_0/(\rho g \sin\theta)]A}$$

For $P_0 = 0$, which is usually the case when the atmospheric pressure is ignored, it simplifies to

$$y_P = y_C + \frac{I_{xx,\,C}}{y_C A}$$

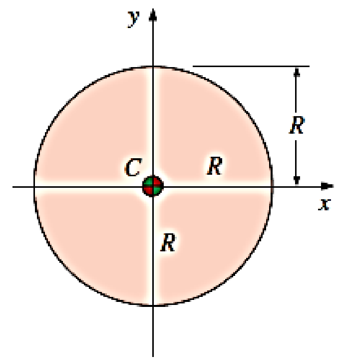
# Hydrostatic Forces on Completely Submerged Plane Surfaces





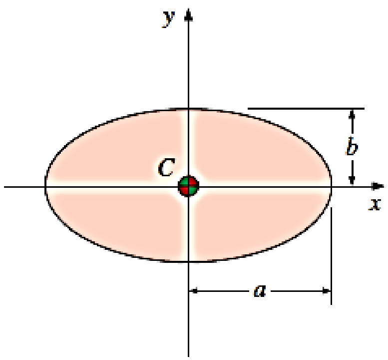

$A = ab,\ I_{xx,\,C} = ab^3/12$

(*a*) Rectangle



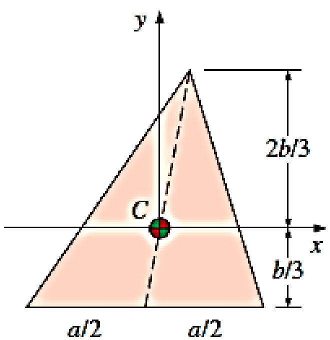

$A = \pi R^2,\ I_{xx,\,C} = \pi R^4/4$

(*b*) Circle



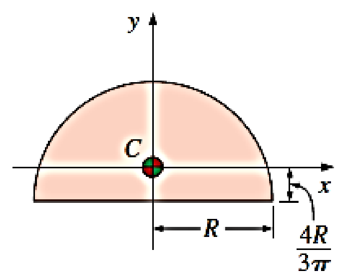

$A = \pi ab,\ I_{xx,\,C} = \pi ab^3/4$

(*c*) Ellipse



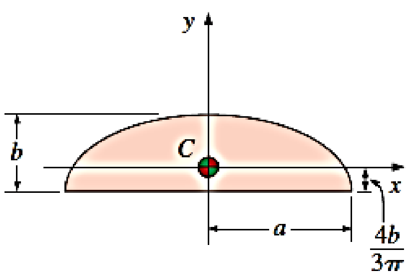

$A = ab/2,\ I_{xx,\,C} = ab^3/36$

(*d*) Triangle




$A = \pi R^2/2,\ I_{xx,\,C} = 0.109757R^4$

(*e*) Semicircle



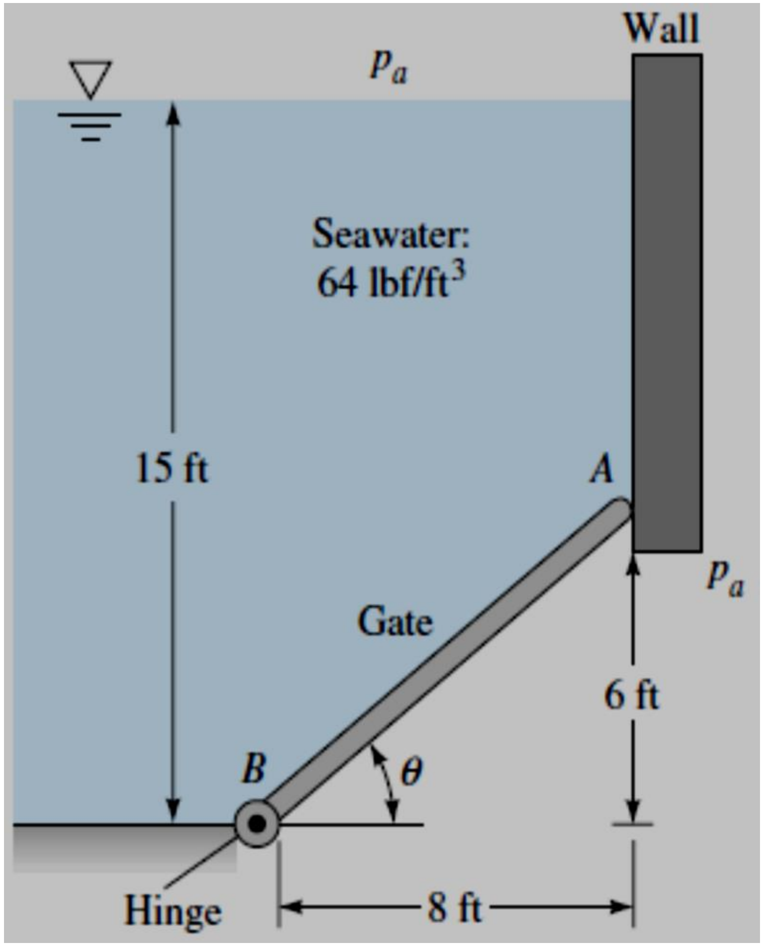

$A = \pi ab/2,\ I_{xx,\,C} = 0.109757ab^3$

(*f*) Semiellipse

# Ex1: Hydrostatic Forces on Completely Submerged Plane Surfaces

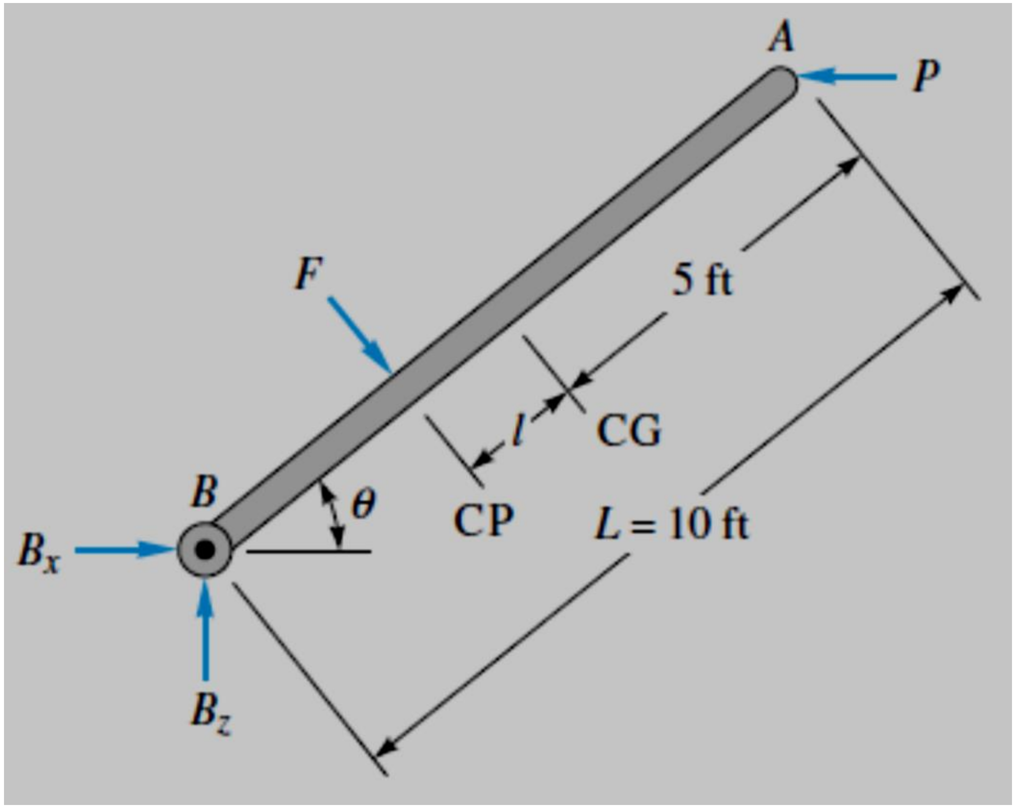


The gate in the following figure is 5 ft wide, is hinged at point *B*, and rests against a smooth wall at point *A*. **Compute (a) the force on the gate due to seawater pressure, (b) the horizontal force P exerted by the wall at point *A*, and (c) the reactions at the hinge *B*.**



# Ex1: Hydrostatic Forces on Completely Submerged Plane Surfaces





## Ex1: Hydrostatic Forces on Completely Submerged Plane Surfaces



By geometry the gate is 10 ft long from **A** to **B**, and its centroid is halfway between, or at elevation 3 ft above point **B**.

The depth $h_{CG}$ is thus 15 - 3 = 12 ft.

The gate area is 5(10) = 50 ft$^2$.

Neglect $p_a$ as acting on both sides of the gate.

From Eq. (2.20) the hydrostatic force on the gate is

$$F = p_{CG} A = \gamma h_{CG} A = \left(64 (lbf/ft^3)\right)\left(12 (ft)\right)\left(50 (ft^2)\right) = 38400 lbf \qquad \textit{Ans.} \text{ (a)}$$

# Ex1: Hydrostatic Forces on Completely Submerged Plane Surfaces



First we must find the center of pressure of $F$. A free-body diagram of the gate is shown in Figure (E2.4b). The gate is a rectangle, hence

$$I_{xy} = 0 = \quad and \quad I_{xx} = \frac{bL^3}{12} = \frac{(5\,(ft))(10\,(ft))^3}{12} = 417\ ft^4$$

The distance $l$ from the CG to the CP is given by Eq. (2.26) since $p_a$ is neglected.

$$l = -y_{CP} = +\frac{I_{xx}\sin\theta}{h_{CG}A} = \frac{(417\,(ft^4))(\frac{6}{10})}{(12\,(ft))(50\,(ft^2))} = 0.417\ ft$$

The distance from point **B** to force $F$ is thus 10 - $l$ - 5 = 4.583 ft. Summing moments counterclockwise about **B** gives

$$PL\sin\theta - F(5-l) = P(6\,(ft)) - [(38400\,(lbf))(4.583\,(ft))] = 0$$

or

$$P = 29300\,lbf$$

***Ans.* (b)**

## Ex1: Hydrostatic Forces on Completely Submerged Plane Surfaces



With ***F*** and ***P*** known, the reactions $\boldsymbol{B_x}$ and $\boldsymbol{B_z}$ are found by summing forces on the gate

$$\sum F_x = 0 = B_x + F\sin\theta - P = B_x + \left((38400)(0.6)\right) - 29300$$

or $B_x = 6300\,lbf$

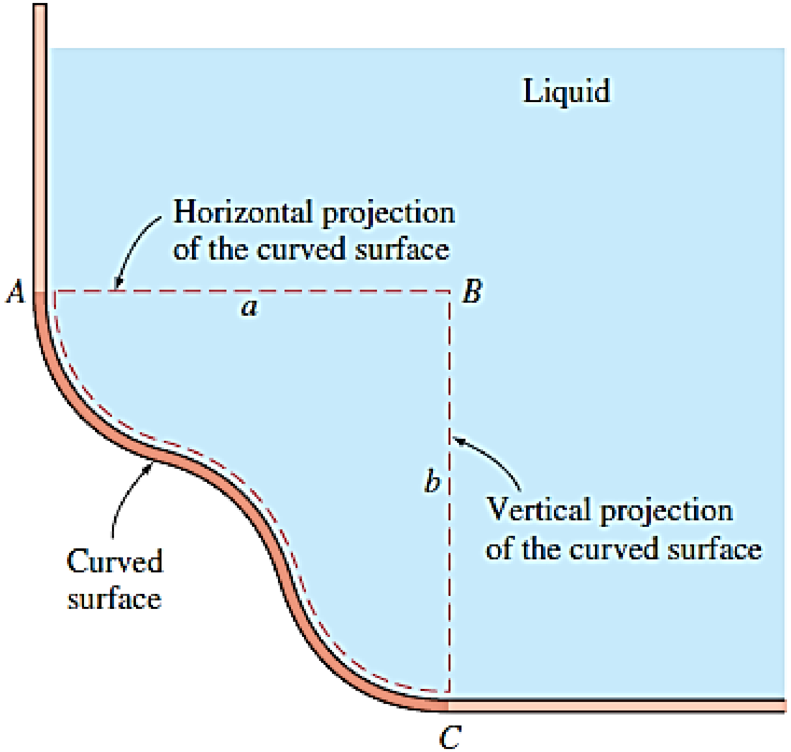
$$\sum F_z = 0 = B_z + F\cos\theta = B_z - \left((38400)(0.8)\right)$$

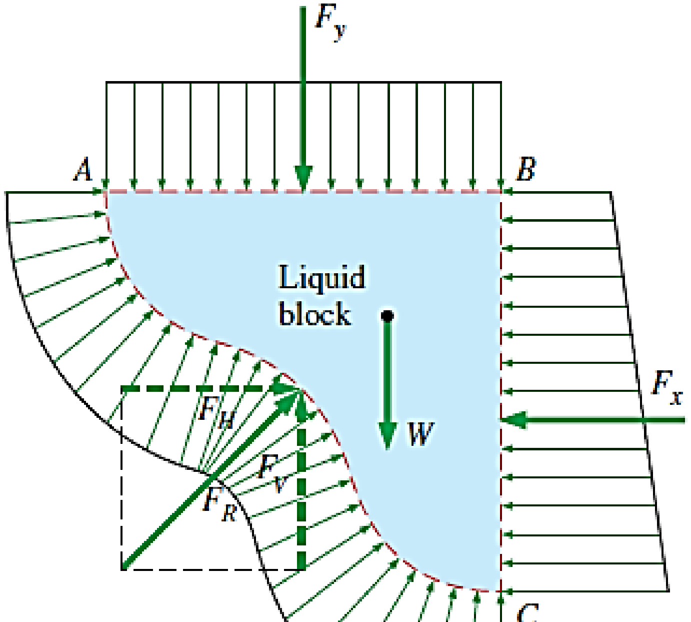
or $B_z = 30700\,lbf$ ***Ans.* (c)**

This example should have reviewed your knowledge of statics.

# Hydrostatic Forces on Completely Submerged Curved Surfaces





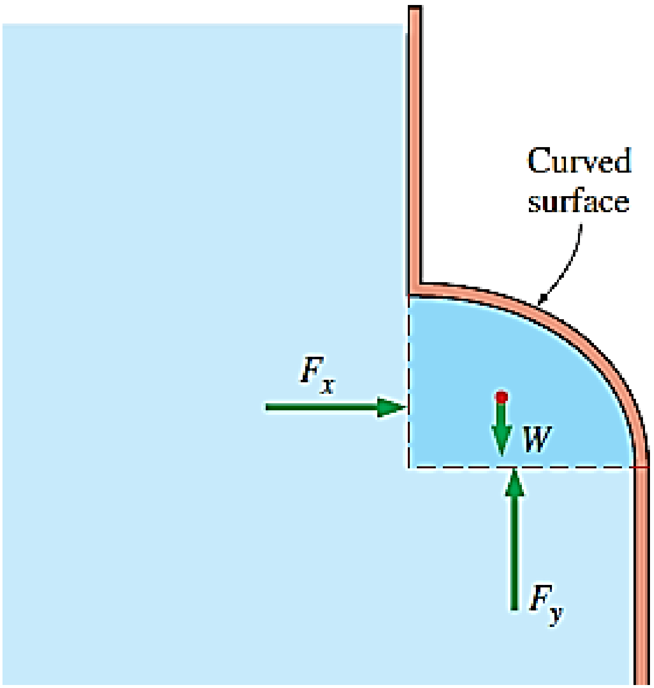

*Horizontal force component on curved surface:* $F_H = F_x$

*Vertical force component on curved surface:* $F_V = F_y \pm W$

## Hydrostatic Forces on Completely Submerged Curved Surfaces



where the summation $F_y \pm W$ is a vector addition (i.e., add magnitudes if both act in the same direction and subtract if they act in opposite directions). Thus, we conclude that



## Hydrostatic Forces on Completely Submerged Curved Surfaces



The magnitude of the resultant hydrostatic force acting on the curved sur-face is

$$F_R = \sqrt{F_H^2 + F_V^2}$$

and the tangent of the angle it makes with the horizontal is

$$\tan \alpha = F_V / F_H$$

# Ex2: Hydrostatic Forces on Completely Submerged Curved Surfaces



A long solid cylinder of radius 0.8 m hinged at point *A* is used as an automatic gate, as shown in the following figure. When the water level reaches 5 m, the gate opens by turning about the hinge at point *A*. **Determine (a) the hydrostatic force acting on the cylinder and its line of action when the gate opens and (b) the weight of the cylinder per m length of the cylinder.**