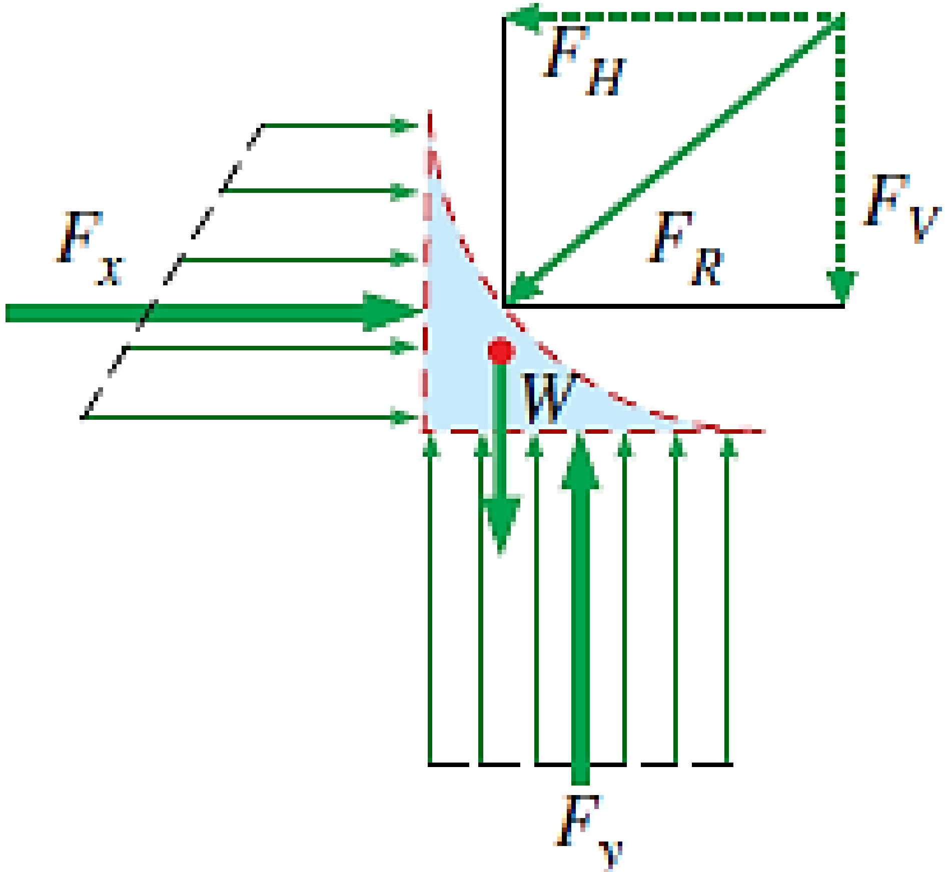
Ex2: Hydrostatic Forces on Completely Submerged Curved Surfaces



Ex2: Hydrostatic Forces on Completely Submerged Curved Surfaces

Horizontal force on vertical surface:

$$F_H = F_x = P_{\text{avg}}A = \rho g h_C A = \rho g(s + R/2)A$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(4.2 + 0.8/2\ \text{m})(0.8\ \text{m} \times 1\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= 36.1\ \text{kN}$$

Vertical force on horizontal surface (upward):

$$F_y = P_{\text{avg}}A = \rho g h_C A = \rho g h_{\text{bottom}} A$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(5\ \text{m})(0.8\ \text{m} \times 1\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= 39.2\ \text{kN}$$

Ex2: Hydrostatic Forces on Completely Submerged Curved Surfaces

Weight (downward) of fluid block for one m width into the page:

$$W = mg = \rho g V = \rho g(R^2 - \pi R^2/4)(1\text{ m})$$

$$= (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.8\text{ m})^2(1 - \pi/4)(1\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)$$

$$= 1.3\text{ kN}$$

Therefore, the net upward vertical force is

$$F_V = F_y - W = 39.2 - 1.3 = 37.9\text{ kN}$$

Then the magnitude and direction of the hydrostatic force acting on the cylindrical surface become

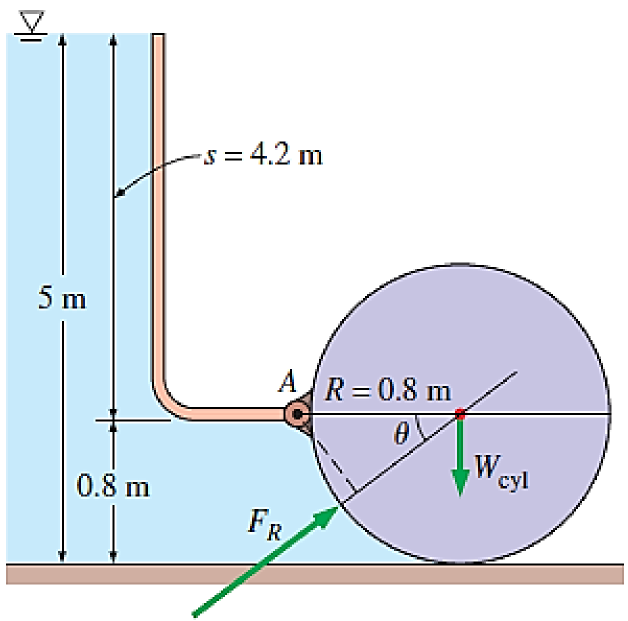
$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{36.1^2 + 37.9^2} = 52.3\text{ kN}$$

$$\tan\theta = F_V/F_H = 37.9/36.1 = 1.05 \rightarrow \theta = 46.4°$$

Ex2: Hydrostatic Forces on Completely Submerged Curved Surfaces

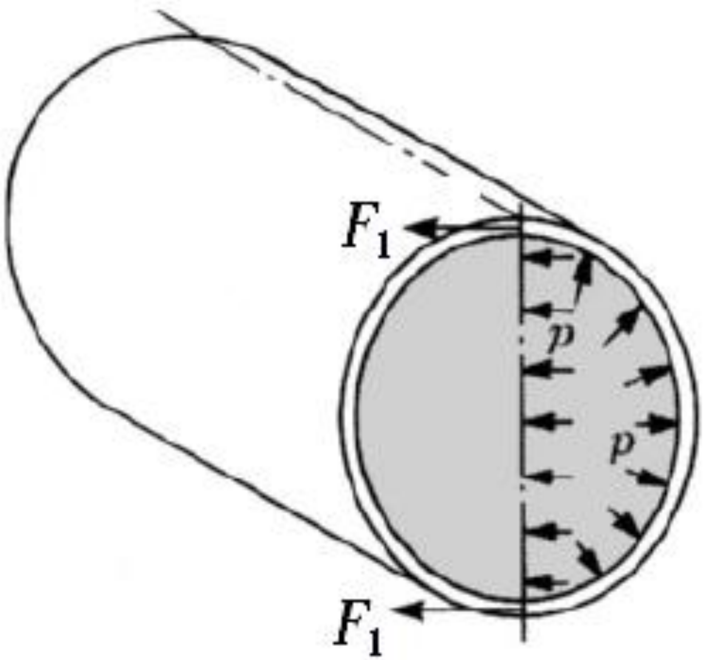
To find **the weight of the cylinder per m length of the cylinder.** Taking a moment about point A at the location of the hinge and equating it to zero gives

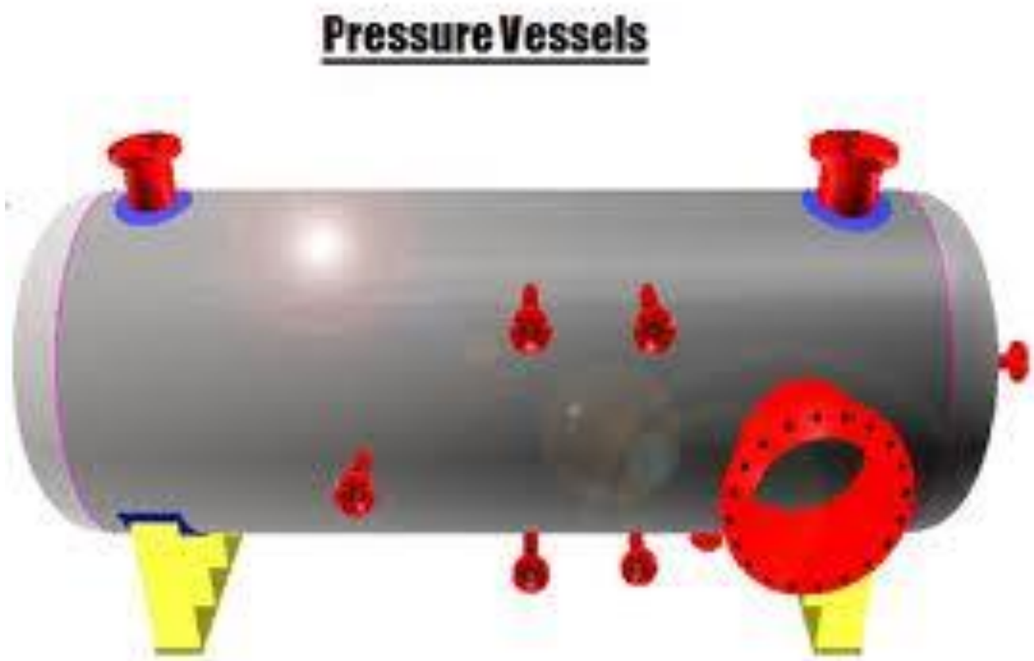
$$F_R R \sin\theta - W_{cyl} R = 0 \rightarrow W_{cyl} = F_R \sin\theta = (52.3 \text{ kN}) \sin 46.4° = 37.9 \text{ kN}$$



Force to pressure vessels

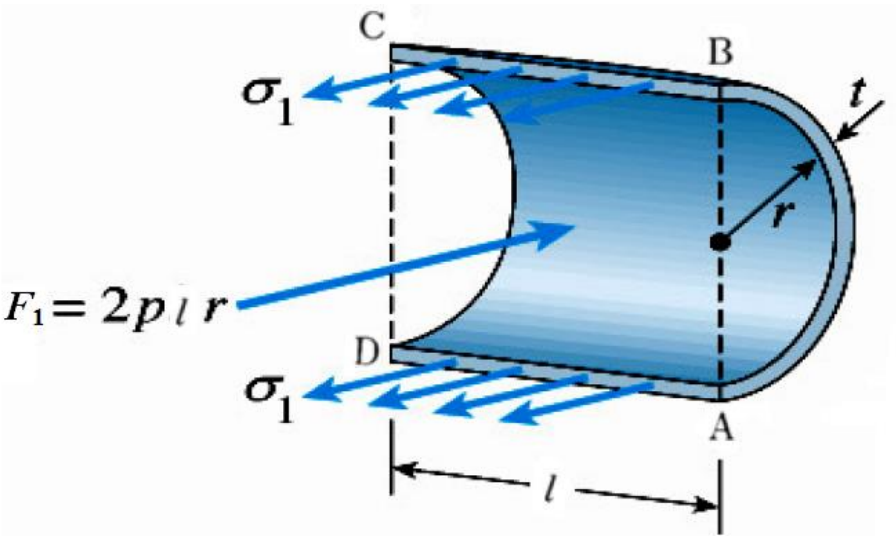
Cylindrical pressure vessel

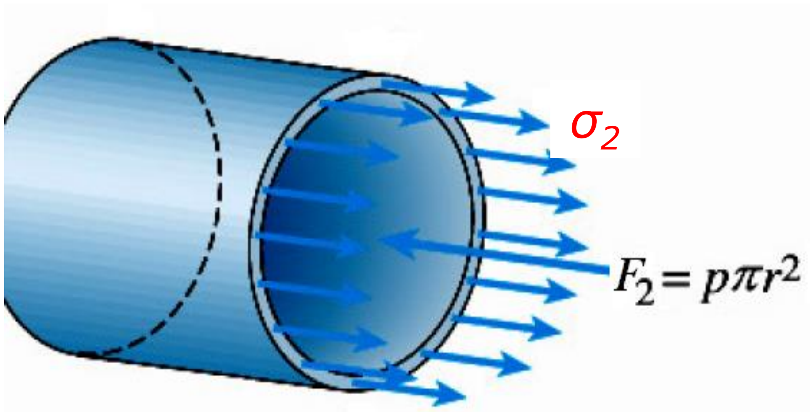




Force to pressure vessels

Cylindrical pressure vessel





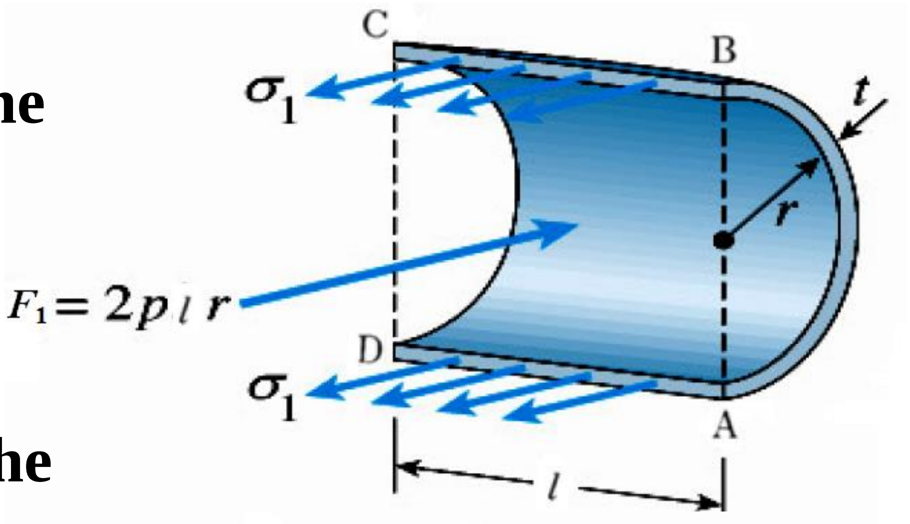
the stress σ_1 is called circumferential stressor the hoop stress, and the stress σ_2 is called the longitudinal stress or the axial stress.

Force to pressure vessels

Cylindrical pressure vessel

Equilibrium of forces to find the circumferential stress:

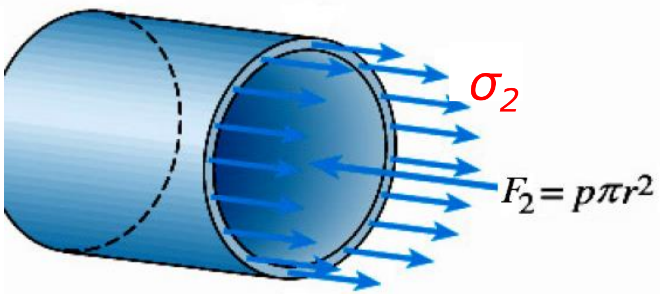
$$2lt\sigma_1 = 2lrp \Rightarrow \boxed{\sigma_1 = \frac{pr}{2t}}$$



Equilibrium of forces to find the longitudinal stress

$$2\pi rt\sigma_2 = \pi r^2 p \Rightarrow \boxed{\sigma_2 = \frac{pr}{t}}$$

$$\boxed{\sigma_2 = 2\sigma_1}$$



We note that the longitudinal welded seam in a pressure tank must be twice as strong as the circumferential seam.

Force to pressure vessels

Spherical pressure vessel

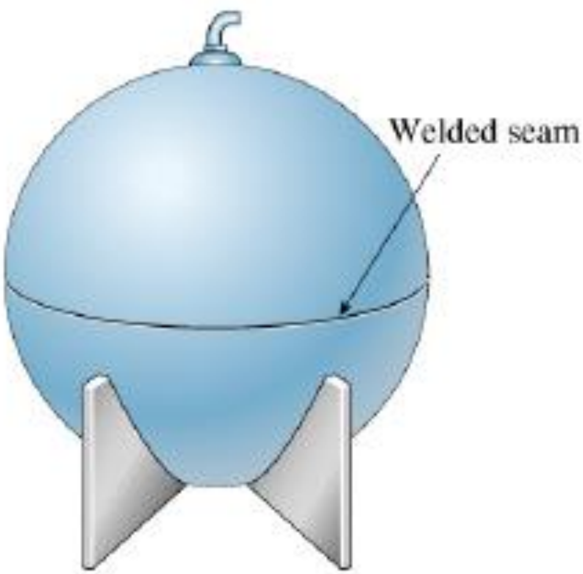
$$F = \pi\, p r^2$$

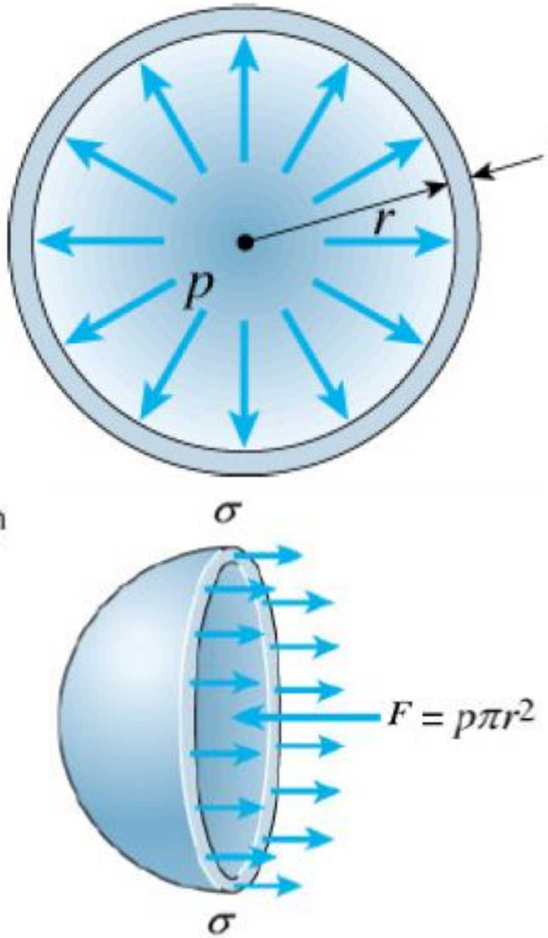
$$\text{Horizontal force} = \sigma\left(2\pi\, r_m\right)t$$

$$r_m = r + \frac{t}{2}$$

$$\sigma\left(2\pi\, r_m\right)t = p\pi\, r^2$$

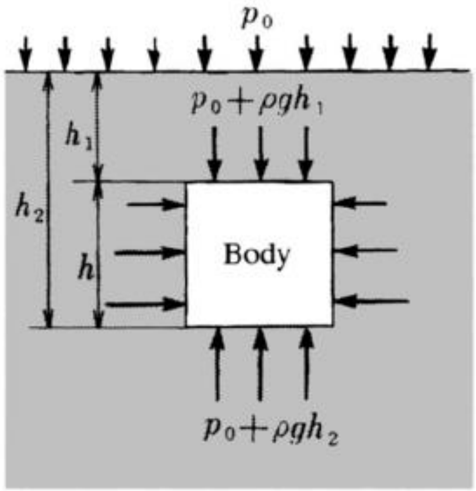
$$\boxed{\sigma = \frac{p\, r}{2t}}$$





Buoyancy and Stability

Archimedes' Principle



Fluid pressure acts all over the wetted surface of a body floating in a fluid, and the resultant pressure acts in a vertical upward direction. This force is called buoyancy.

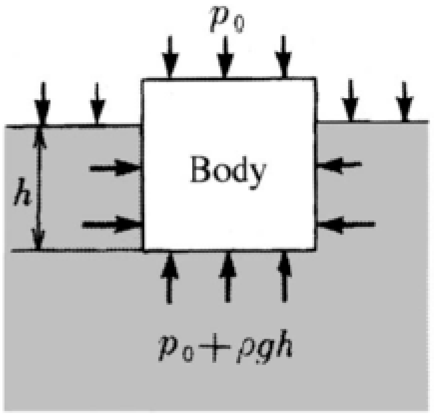
The buoyancy of air is small compared with the gravitational force of the immersed body, so it is normally ignored.

The force F_1 acting on the upper surface is

$$F_1 = (p_0 + \rho g h_1)A$$

The force F_2 acting on the lower surface is

$$F_2 = (p_0 + \rho g h_2)A$$



So, when the volume of the body in the liquid is V, the resultant force F from the pressure acting on the whole surface of the body is

$$F = F_2 - F_1 = \rho g(h_2 - h_1)A = \rho g h A = \rho g V$$

Pressure Distribution in Rigid-Body Motion

Equiaccelerated straight-line motion

minute element of mass m on the liquid surface, where its acceleration is α,

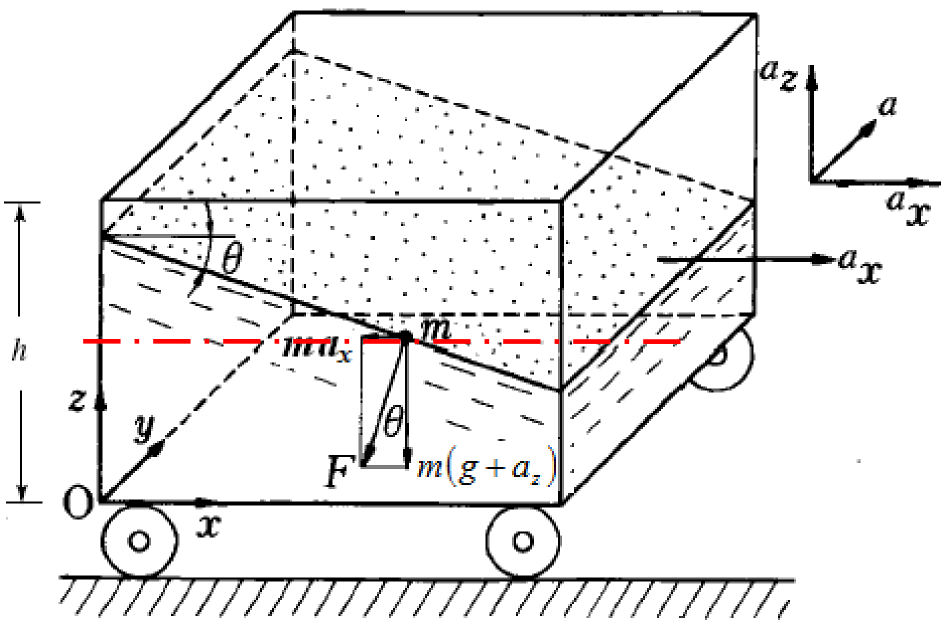
$$\theta = \tan^{-1}\frac{a_x}{g + a_z}$$

$$\tan\theta = \alpha / g$$

$$p = \rho\beta h$$

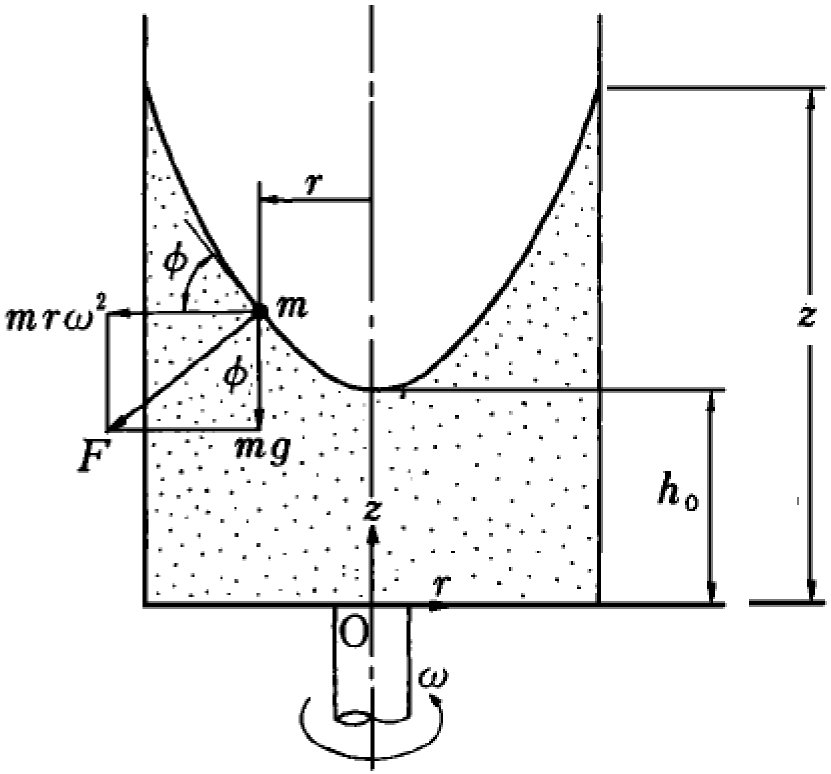
$$\beta = F / m$$

$$\beta = [a_x^2 + (g + a_z)^2]^{1/2}$$



Pressure Distribution in Rigid-Body Motion

Rotational motion



$$\tan\phi = \frac{mr\omega^2}{mg} = \frac{r\omega^2}{g}$$

but also

$$\tan\phi = \frac{dz}{dr}$$

Therefore,

$$\frac{dz}{dr} = \frac{r\omega^2}{g}$$

Putting c as a constant of integration,

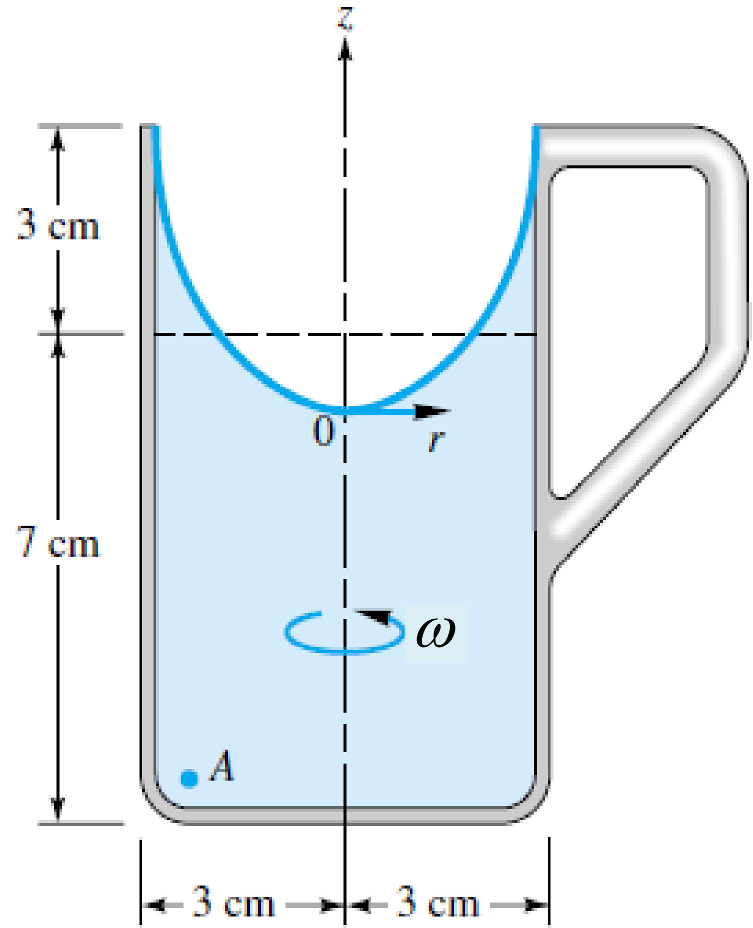
$$z = \frac{\omega}{2g}r^2 + c$$

If $z = h_0$ at $r = 0, c = h_0$

$$z - h_0 = \frac{\omega^2 r^2}{2g}$$

Example 2: Pressure Distribution in Rigid-Body Motion

The coffee cup in the following figure is removed from the drag racer, placed on a turntable, and rotated about its central axis until a rigid-body mode occurs. **Find (a) the angular velocity which will cause the coffee to just reach the lip of the cup and (b) the gage pressure at point *A* for this condition.**



Example 2: Pressure Distribution in Rigid-Body Motion

The cup contains 7 cm of coffee. The remaining distance of 3 cm up to the lip must equal the distance $h/2$ in Figure (2.16). Thus from Eq. (2.43)

$$h = 2(0.03(m)) = \frac{\omega^2}{2g}r^2 = \frac{\omega^2(0.03(m))^2}{2(9.81(m/s^2))}$$

Solving, we obtain

$$\omega^2 = 1308 \qquad or \qquad \omega = 36.2\ rad/s$$

Where $\omega = \dfrac{2\pi N}{60}$ (N: revolution per minute)

$$36.2 = \frac{2\pi N}{60} \rightarrow N = 345 rpm$$

***Ans.* (a)**

Example 2: Pressure Distribution in Rigid-Body Motion

To compute the pressure, it is convenient to put the origin of coordinates *r* and *z* at the bottom of the free-surface depression, as shown in Figure (E2.8). The gage pressure here is p_0=0, and point **A** is at (*r*, *z*) = (3 cm, -4 cm). Equation (2.44) can then be evaluated

$$p = \frac{1}{2}\rho r^2 \omega^2 + p_1 + p_0 = \left[0.5\left(1010\,(kg/m^3)\right)(0.03\,(m))^2\,(36.2\,(rad/s))^2\right]$$
$$-\left[\left(1010\,(kg/m^3)\right)\left(9.81\,(m/s^2)\right) - (0.04\,(m))\right] + 0 = 990\,Pa$$

***Ans.* (b)**

This is about 43 percent greater than the still-water pressure p_A = 694 Pa.


QUIZ

Q (2)

Two water tanks are connected to each other through a mercury manometer with inclined tubes, as shown in the following figure. If the pressure difference between the two tanks is 20 kPa, calculate a and θ.

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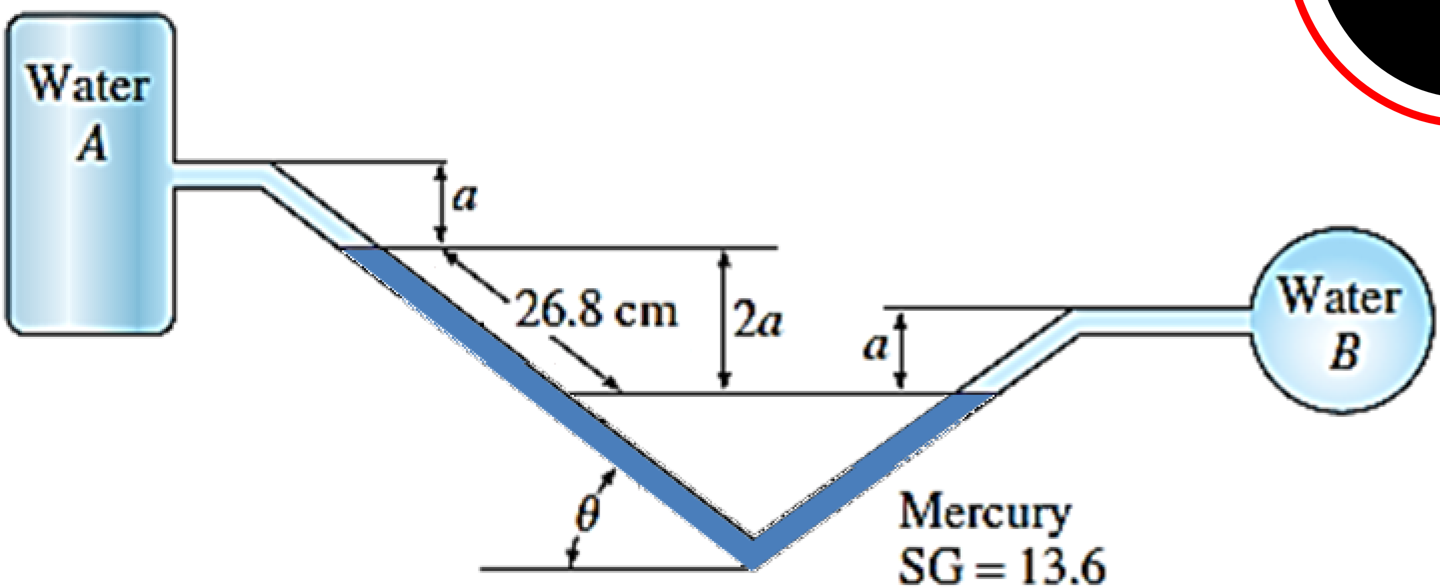
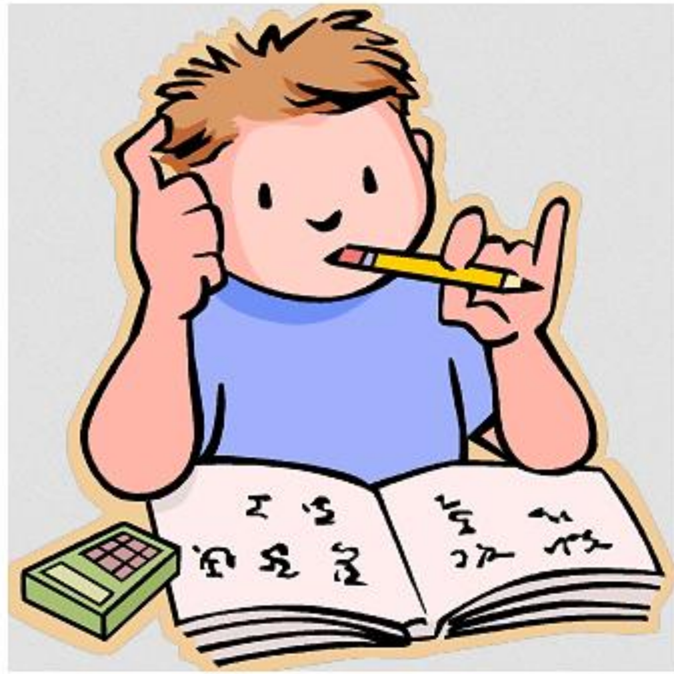
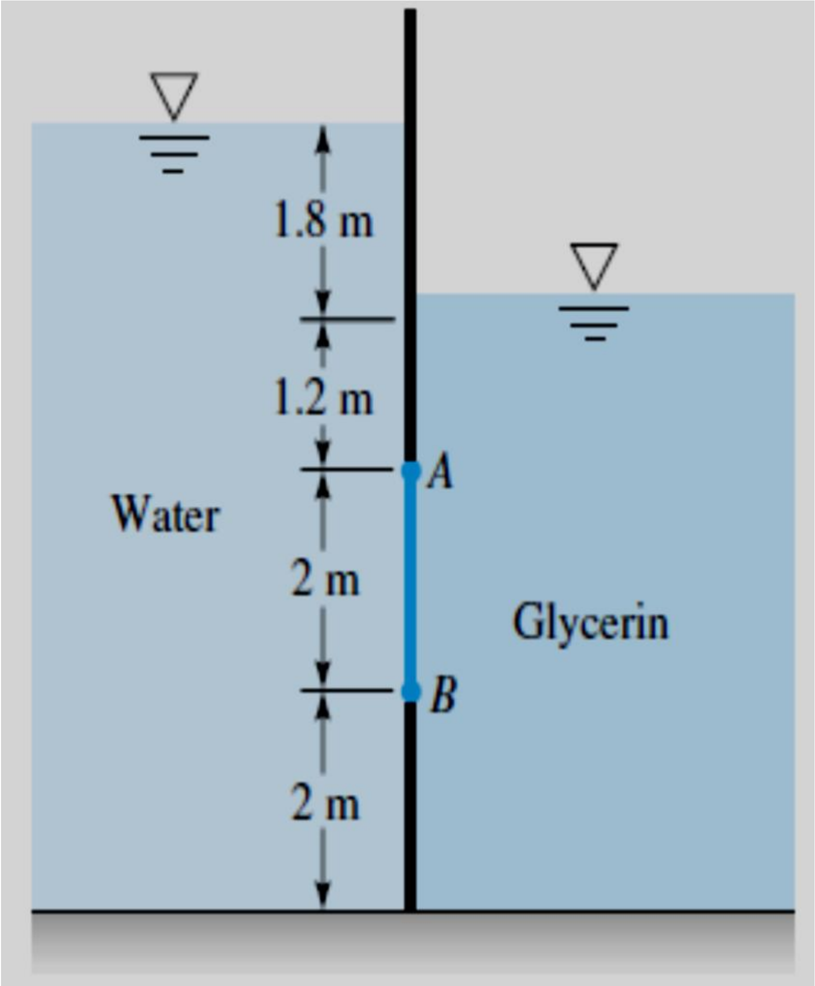




Homework

Hw (3)

Find the net hydrostatic force per unit width on the rectangular gate AB in the following figure and its line of action.



Thank you