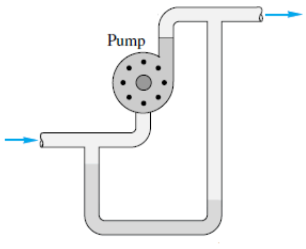
بسم الله الرحمن الرحيم

Pervious Lecture Contents

1. Characteristics of Pressure
2. Pressure of fluid at rest
3. Measurement of Pressure
   - Simple manometers
   - Differential manometers
   - Mechanical gauge

Mechanical Engineering (2)



Fayoum University

Faculty of Engineering
Mechanical Engineering Dept.

Lecture (3)

on

Fluid Statics

By

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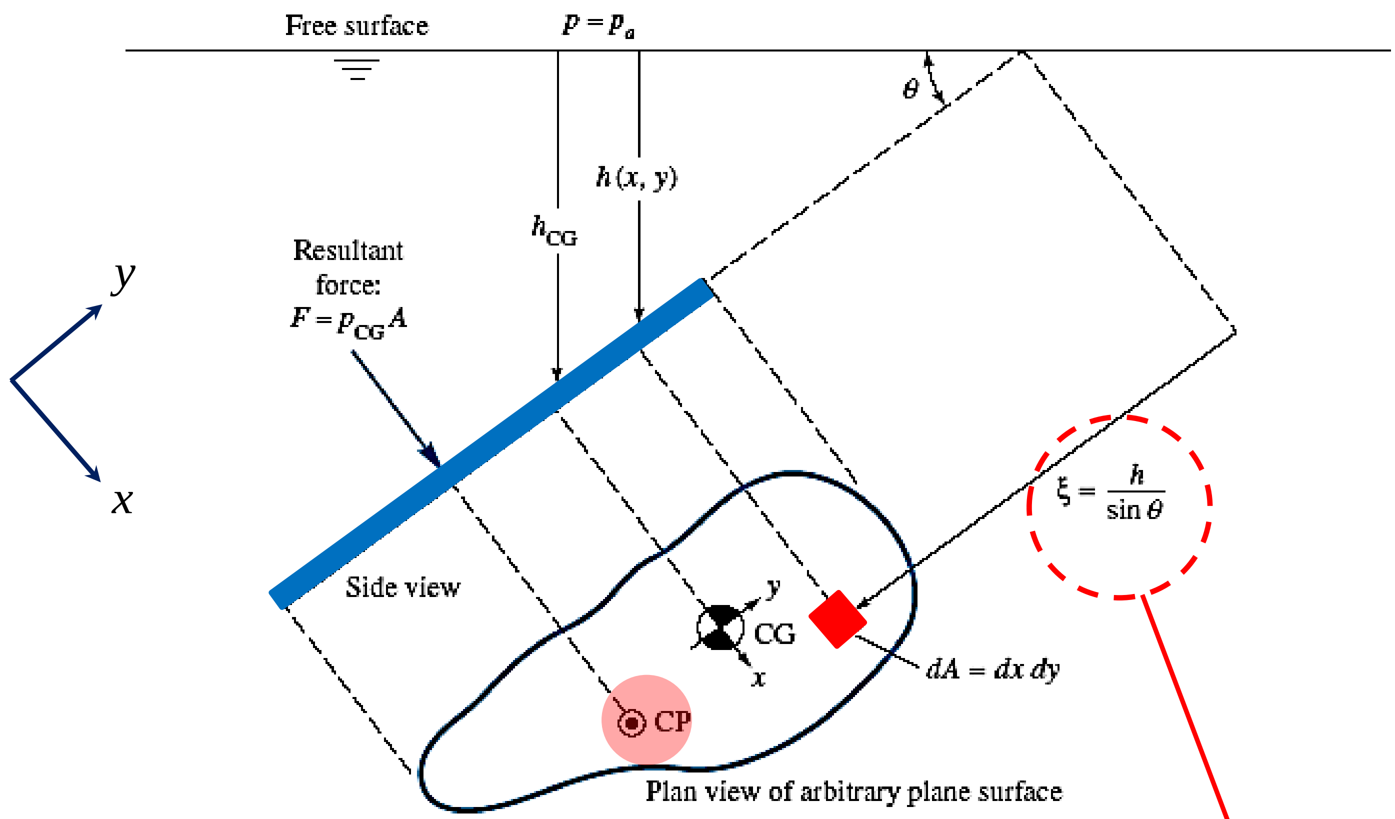
2015 - 2016

Hydrostatic Forces on Plane Surfaces

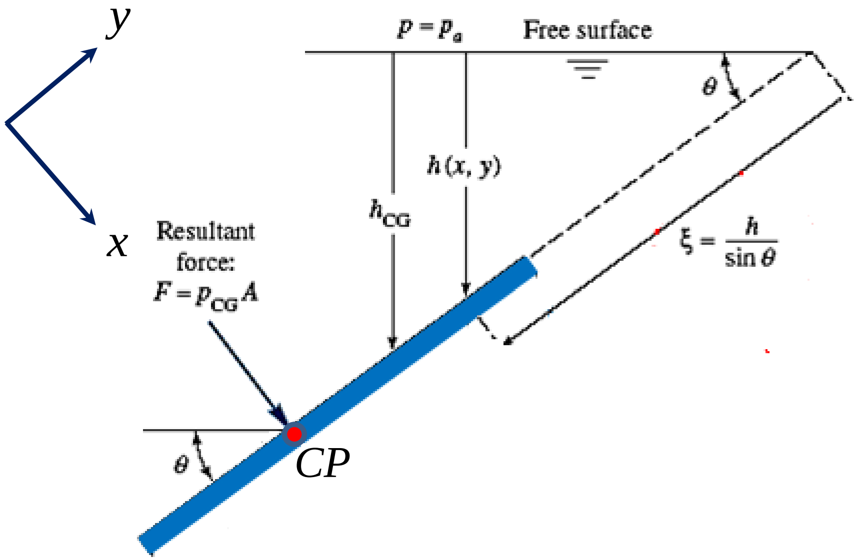


$$F = \int_A p dA = \int_A (p_a + \gamma h) dA = p_a A + \gamma \int_A h\, dA$$

from Figure $h = \xi \sin\theta$

Hydrostatic forces on plane surfaces

By definition $\xi_{CG} = \frac{1}{A}\int_A \xi \, dA$



$$F = p_a A + \gamma \sin\theta \int_A \xi \, dA = p_a A + \gamma A \xi_{CG} \sin\theta$$

$$\xi_{CG} \sin\theta = h_{CG}$$

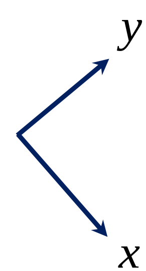
$$\boxed{\therefore\ F = p_a A + \gamma h_{CG} A = \left(p_a + \gamma h_{CG}\right) A = p_{CG} A}$$

To balance the bending-moment portion of the stress, the resultant force *F* does not act through the centroid. So, To find the coordinates (x_{CP}, y_{CP})

Hydrostatic forces on plane surfaces

To compute y_{CP}

$$F\,y_{cp} = \int_A y p\,dA = \int_A y\left(p_a + \gamma\xi\sin\theta\right)dA = \gamma\sin\theta\int_A y\xi\,dA$$



$$\xi = \xi_{CG} - y$$

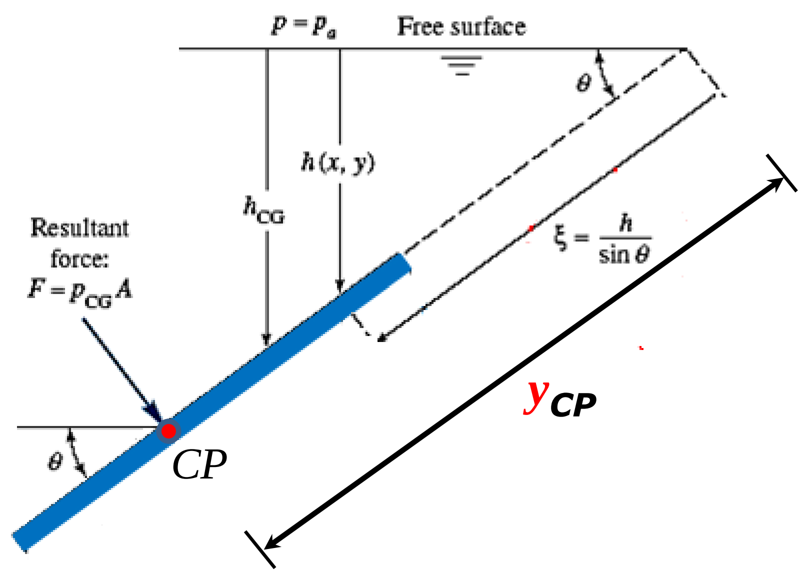
$$F\,y_{cp} = \gamma\sin\theta\left(\xi_{CG}\int_A y\,dA - \int_A y^2\,dA\right) = -\gamma\sin\theta I_{xx}$$

where $\int_A y\,dA = 0$

and $\boldsymbol{I_{xx}}$ is the area moment of inertia of the plate

area about its centroidal x axis

$$\boxed{y_{CP} = -\gamma\sin\theta\frac{I_{xx}}{p_{CG}A}}$$



Hydrostatic forces on plane surfaces

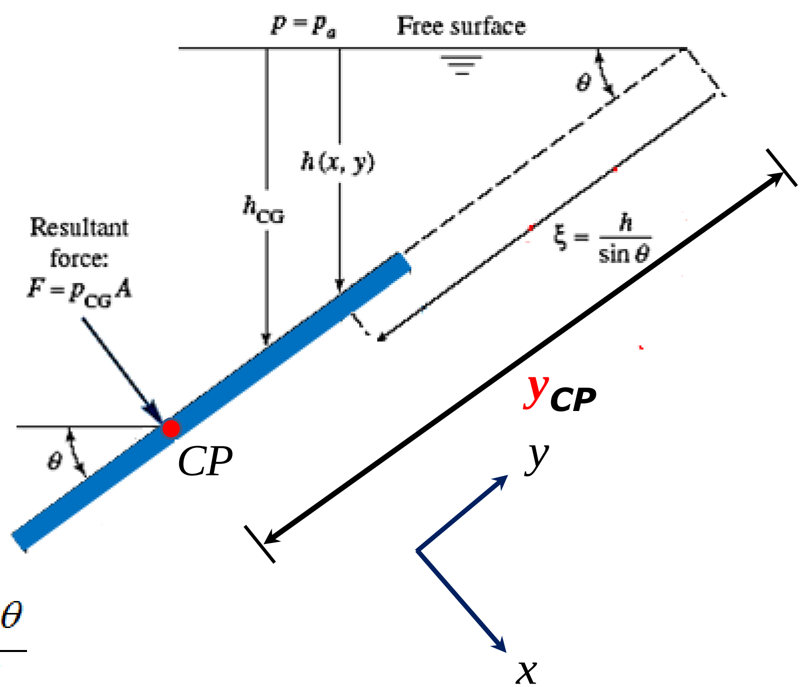
To compute x_{CP}

$$F\,x_{cp} = \int_A xp\,dA = \int_A x\left[p_a + \gamma\left(\xi_{CG} - y\right)\sin\theta\right]dA \quad = -\gamma\sin\theta\int_A xy\,dA = -\gamma\sin\theta\,I_{xy}$$

where I_{xy} is the product of inertia of the plate

$$\boxed{x_{CP} = -\gamma\sin\theta\,\frac{I_{xy}}{p_{CG}A}}$$

For positive I_{xy}, x_{CP} is negative because the dominant pressure force acts in the third or lower left, quadrant of the panel. If $I_{xy}= 0$, usually implying symmetry, $x_{CP} = 0$ and the center of pressure lies directly below the centroid on the y axis.

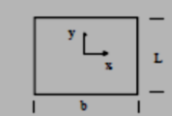
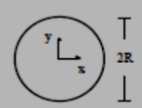
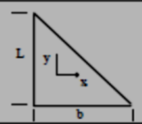
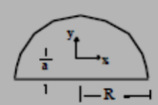
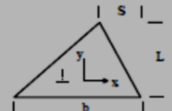
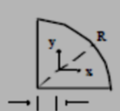
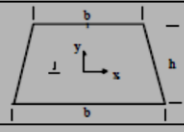


If ambient pressure p_a is neglected

$p_{CG} = \gamma h_{CG}$

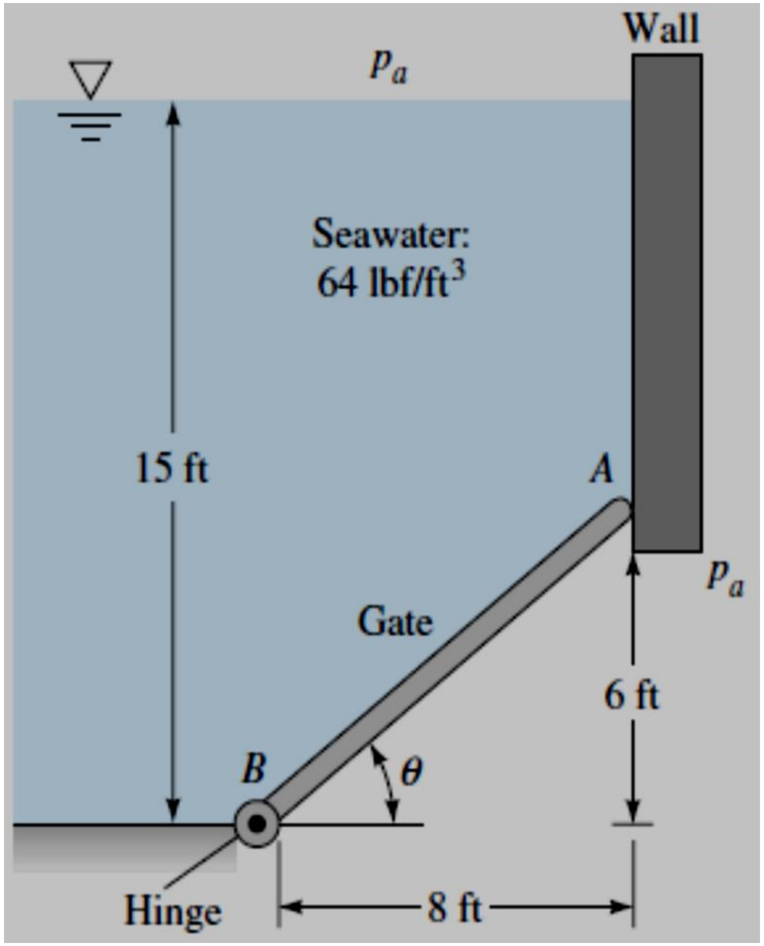
$$F = \gamma\, h_{CG}\, A \qquad y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}\,A} \qquad x_{CP} = -\frac{I_{xy}\sin\theta}{h_{CG}\,A}$$

Hydrostatic forces on plane surfaces

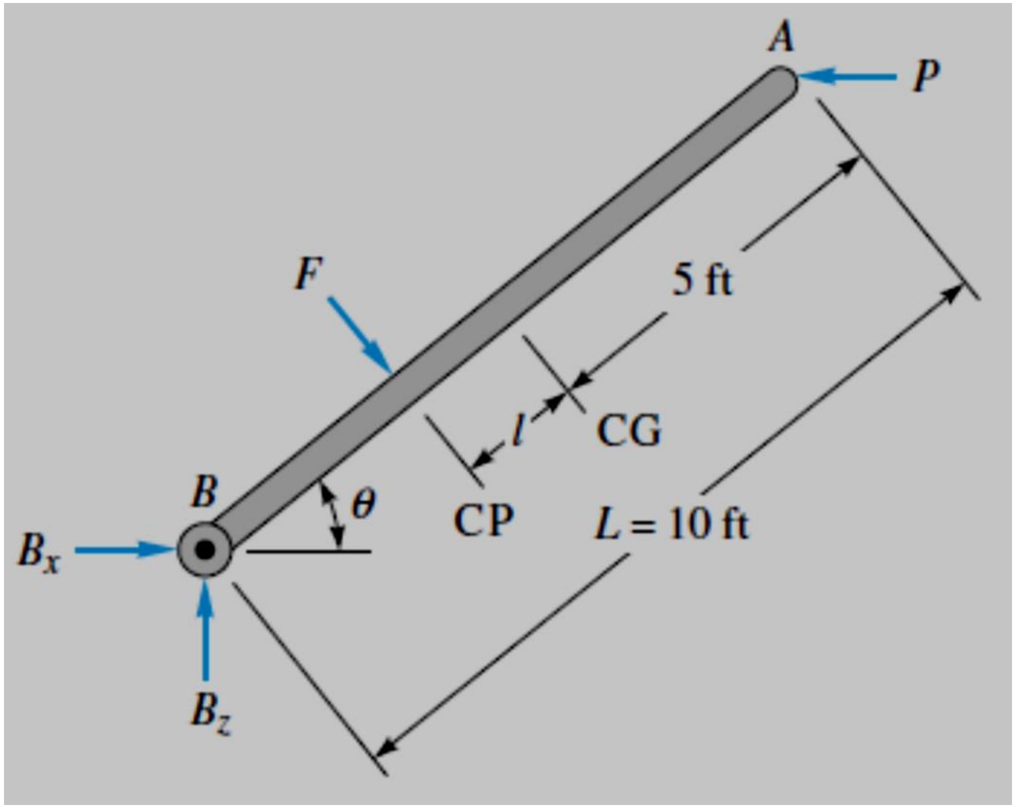
Geometry	Centroid	Moment of Inertia I_{xx}	Product of Inertia I_{xy}	Area
	$b/2,\ L/2$	$\frac{bL^3}{12}$	0	$b \cdot L$
	0, 0	$\frac{\pi R^4}{4}$	0	πR^2
	$b/3,\ L/3$	$\frac{bL^3}{36}$	$-\frac{b^2L^2}{72}$	$\frac{b \cdot L}{2}$
	$0, a = \frac{4R}{3\pi}$	$R^4\left(\frac{\pi}{8} - \frac{8}{9\pi}\right)$	0	$\frac{\pi R^2}{2}$
	$a = \frac{L}{3}$	$\frac{bL^3}{36}$	$\frac{b(b-2s)L^2}{72}$	$\frac{1}{2} b \cdot L$
	$a = \frac{4R}{3\pi}$	$\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)R^4$	$\left(\frac{1}{8} - \frac{4}{9\pi}\right)R^4$	$\frac{\pi R^2}{4}$
	$a = \frac{h(b+2b_1)}{3(b+b_1)}$	$\frac{h^3(b^2+4bb_1+b_1^2)}{36(b+b_1)}$	0	$(b+b_1)\frac{h}{2}$

Example 1: Hydrostatic forces on plane surfaces

The gate in the following figure is 5 ft wide, is hinged at point B, and rests against a smooth wall at point A. Compute (a) the force on the gate due to seawater pressure, (b) the horizontal force *P* exerted by the wall at point A, and (c) the reactions at the hinge B.



Example 1: Hydrostatic forces on plane surfaces



Example 1: Hydrostatic forces on plane surfaces

By geometry the gate is 10 ft long from **A** to **B**, and its centroid is halfway between, or at elevation 3 ft above point **B**.

The depth h_{CG} is thus 15 - 3 = 12 ft.

The gate area is 5(10) = 50 ft^2.

Neglect p_a as acting on both sides of the gate.

From Eq. (2.20) the hydrostatic force on the gate is

$$F = p_{CG}A = \gamma\, h_{CG}A = \left(64\,(lbf/ft^3)\right)\left(12\,(ft)\right)\left(50\,(ft^2)\right) = 38400\,lbf \qquad \textit{Ans.}\ \textbf{(a)}$$

Example 1: Hydrostatic forces on plane surfaces

First we must find the center of pressure of F. A free-body diagram of the gate is shown in Figure (E2.4b). The gate is a rectangle, hence

$$I_{xy} = 0 = \quad and \quad I_{xx} = \frac{bL^3}{12} = \frac{(5\,(ft))(10\,(ft))^3}{12} = 417\, ft^4$$

The distance l from the CG to the CP is given by Eq. (2.26) since p_a is neglected.

$$l = -y_{CP} = +\frac{I_{xx}\sin\theta}{h_{CG}A} = \frac{(417\,(ft^4))(\frac{6}{10})}{(12\,(ft))(50\,(ft^2))} = 0.417\, ft$$

The distance from point **B** to force F is thus 10 - l - 5 = 4.583 ft. Summing moments counterclockwise about **B** gives

$$PL\sin\theta - F(5-l) = P(6\,(ft)) - [(38400\,(lbf))(4.583\,(ft))] = 0$$

or

$P = 29300\, lbf$ ***Ans.* (b)**

Example 1: Hydrostatic forces on plane surfaces

With $\boldsymbol{F}$ and $\boldsymbol{P}$ known, the reactions $\boldsymbol{B}_x$ and $\boldsymbol{B}_z$ are found by summing forces on the gate

$$\sum F_x = 0 = B_x + F\sin\theta - P = B_x + \big((38400)(0.6)\big) - 29300$$

or $B_x = 6300\,lbf$

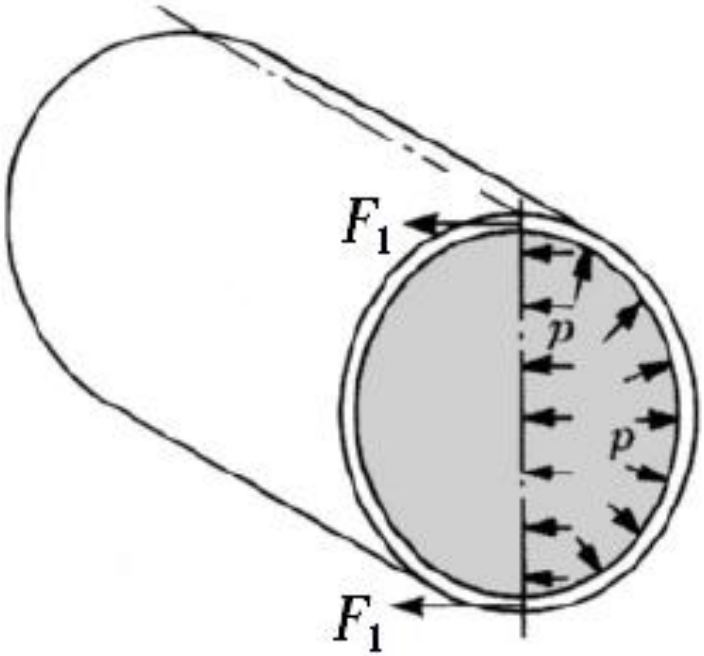
$$\sum F_z = 0 = B_z + F\cos\theta = B_z - \big((38400)(0.8)\big)$$

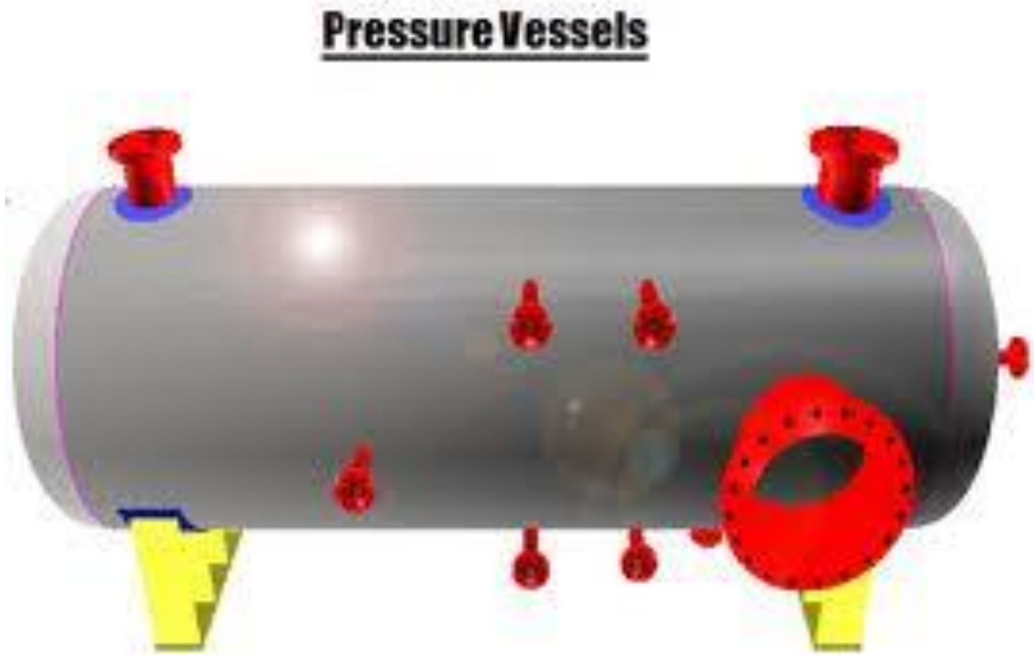
or $B_z = 30700\,lbf$ ***Ans.* (c)**

This example should have reviewed your knowledge of statics.

Force to pressure vessels

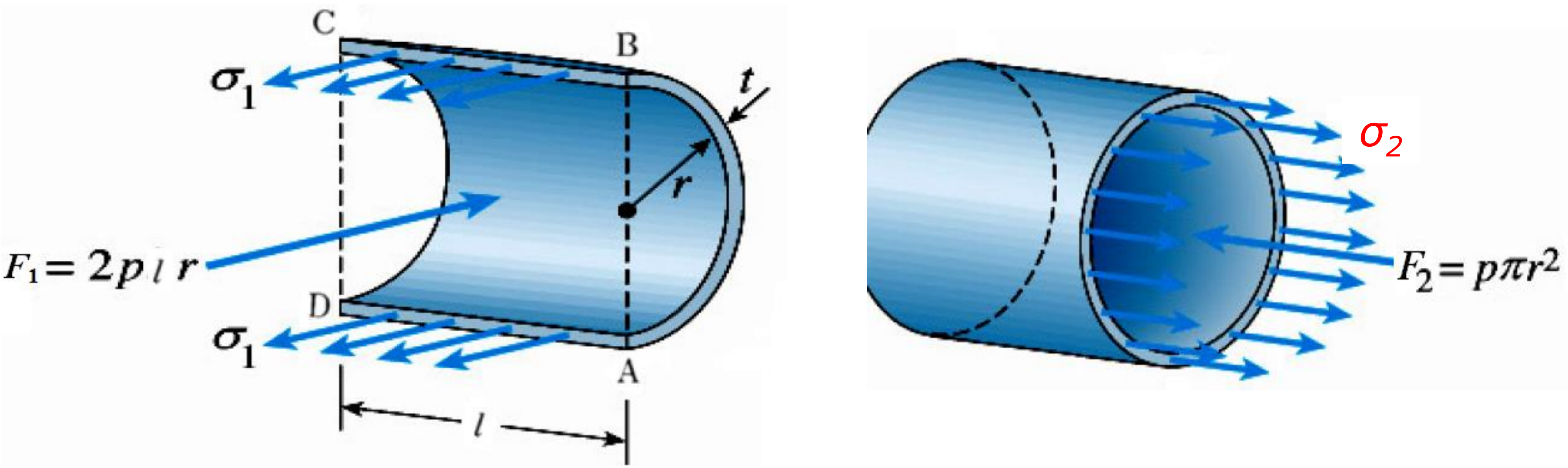
Cylindrical pressure vessel





Force to pressure vessels

Cylindrical pressure vessel



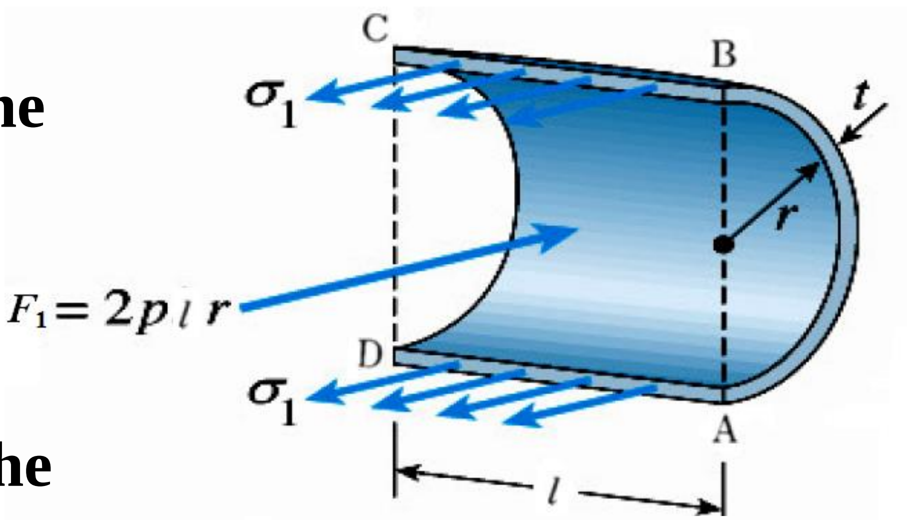
the stress σ_1 is called circumferential stressor the hoop stress, and the stress σ_2 is called the longitudinal stress or the axial stress.

Force to pressure vessels

Cylindrical pressure vessel

Equilibrium of forces to find the circumferential stress:

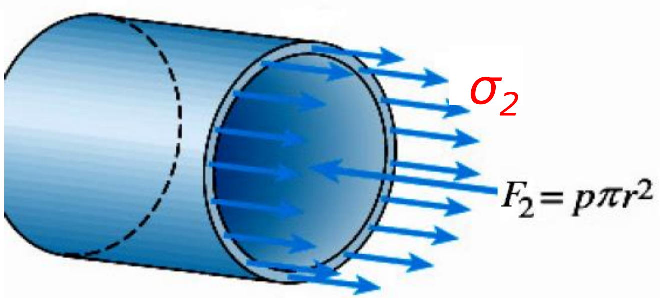
$$2lt\sigma_1 = 2lrp \Rightarrow \boxed{\sigma_1 = \frac{pr}{2t}}$$



Equilibrium of forces to find the longitudinal stress

$$2\pi rt\sigma_2 = \pi r^2 p \Rightarrow \boxed{\sigma_2 = \frac{pr}{t}}$$

$$\boxed{\sigma_2 = 2\sigma_1}$$



We note that the longitudinal welded seam in a pressure tank must be twice as strong as the circumferential seam.

Force to pressure vessels

Spherical pressure vessel

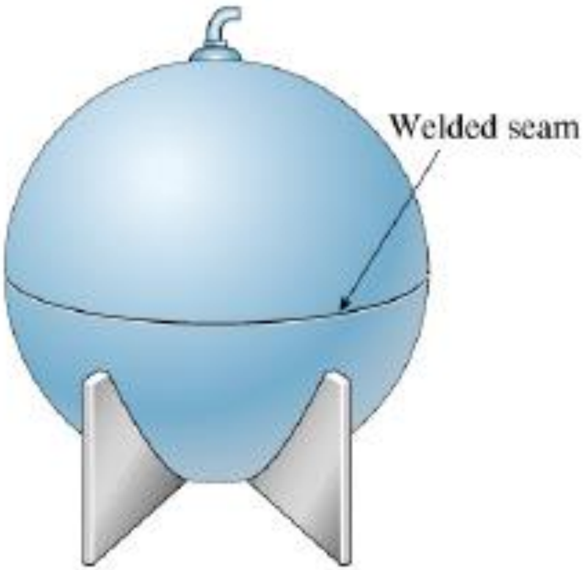
$$F = \pi\, p r^2$$

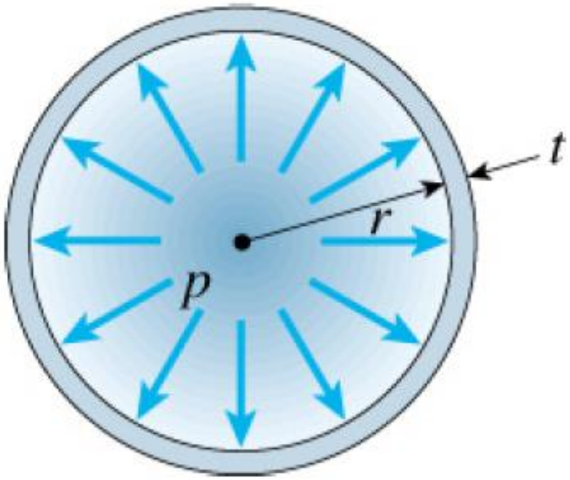
$$\text{Horizontal force} = \sigma\left(2\pi\, r_m\right)t$$

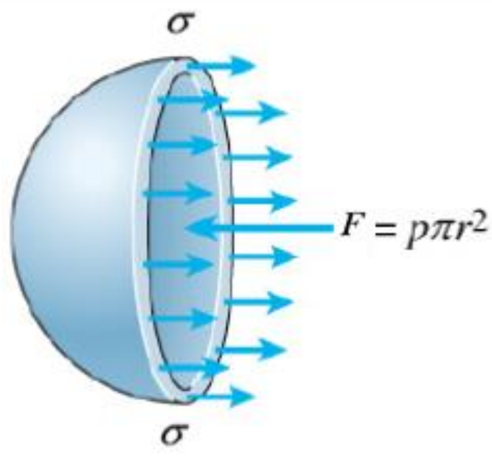
$$r_m = r + \frac{t}{2}$$

$$\sigma\left(2\pi\, r_m\right)t = p\pi\, r^2$$

$$\boxed{\sigma = \frac{p\,r}{2t}}$$

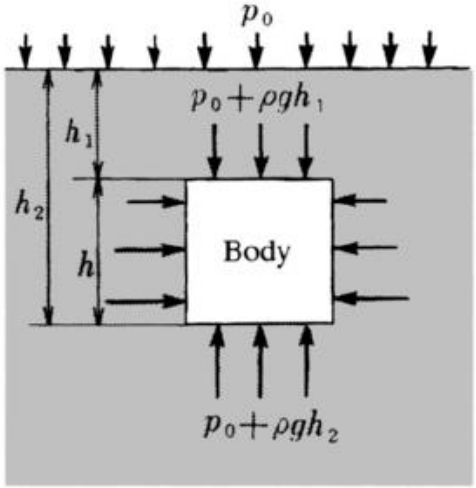






Buoyancy and Stability

Archimedes' Principle

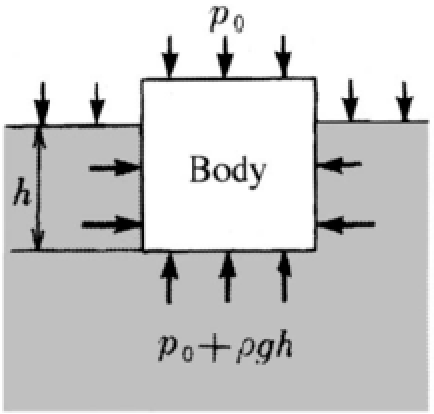


Fluid pressure acts all over the wetted surface of a body floating in a fluid, and the resultant pressure acts in a vertical upward direction. This force is called buoyancy.

The buoyancy of air is small compared with the gravitational force of the immersed body, so it is normally ignored.

The force F_1 acting on the upper surface is

$$F_1 = (p_0 + \rho g h_1)A$$



The force F_2 acting on the lower surface is

$$F_2 = (p_0 + \rho g h_2)A$$

So, when the volume of the body in the liquid is V, the resultant force F from the pressure acting on the whole surface of the body is

$$F = F_2 - F_1 = \rho g(h_2 - h_1)A = \rho g h A = \rho g V$$

Pressure Distribution in Rigid-Body Motion

Equiaccelerated straight-line motion

minute element of mass m on the liquid surface, where its acceleration is α,

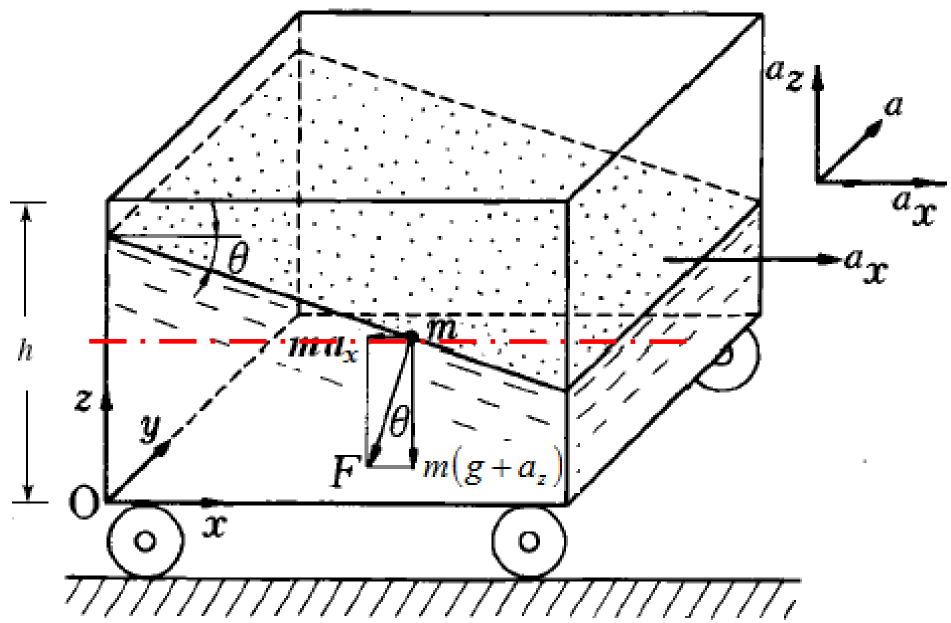
$$\theta = \tan^{-1}\frac{a_x}{g + a_z}$$

$$\tan\theta = \alpha/g$$

$$p = \rho\beta h$$

$$\beta = F/m$$

$$\beta = [a_x^2 + (g + a_z)^2]^{1/2}$$



Pressure Distribution in Rigid-Body Motion

Rotational motion

$$\tan\phi = \frac{mr\omega^2}{mg} = \frac{r\omega^2}{g}$$

but also

$$\tan\phi = \frac{dz}{dr}$$

Therefore,

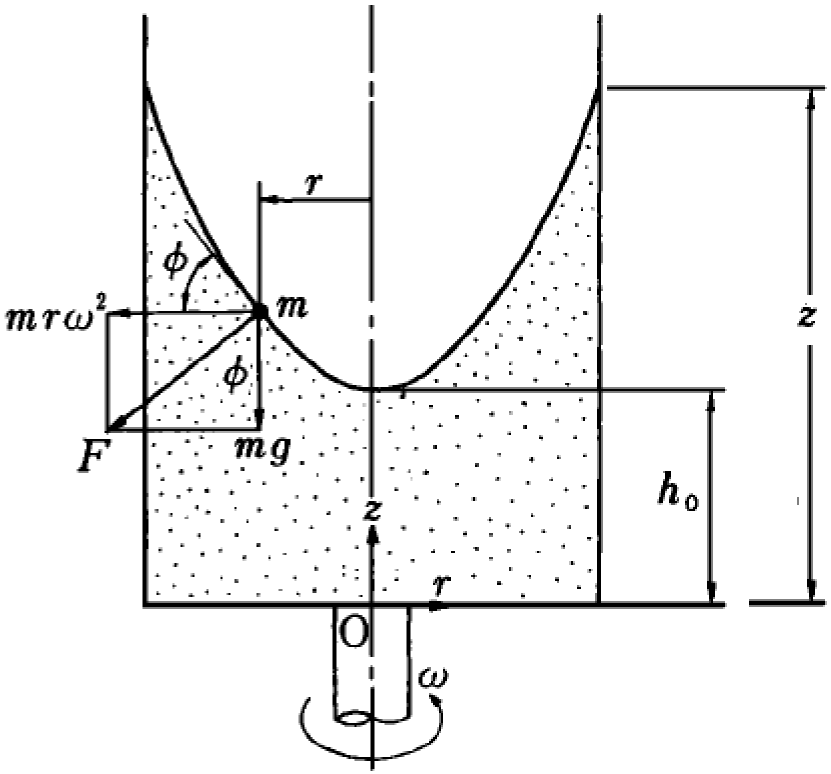
$$\frac{dz}{dr} = \frac{r\omega^2}{g}$$

Putting c as a constant of integration,

$$z = \frac{\omega}{2g}r^2 + c$$

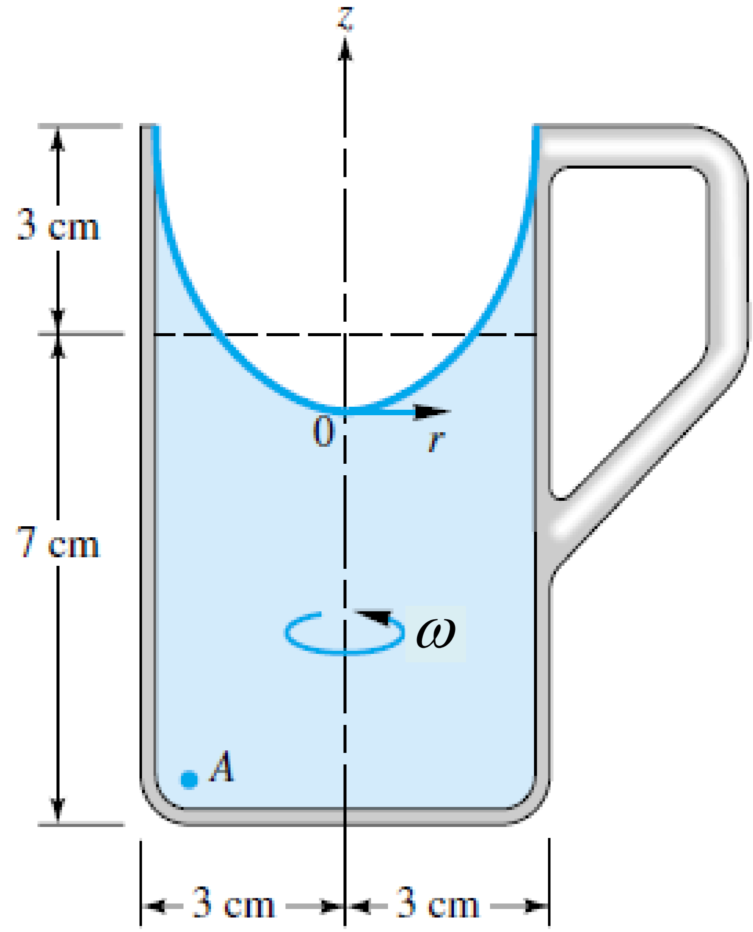
If $z = h_0$ at $r = 0$, $c = h_0$

$$z - h_0 = \frac{\omega^2 r^2}{2g}$$



Example 2: Pressure Distribution in Rigid-Body Motion

The coffee cup in the following figure is removed from the drag racer, placed on a turntable, and rotated about its central axis until a rigid-body mode occurs. Find (a) the angular velocity which will cause the coffee to just reach the lip of the cup and (b) the gage pressure at point A for this condition.



Example 2: Pressure Distribution in Rigid-Body Motion

The cup contains 7 cm of coffee. The remaining distance of 3 cm up to the lip must equal the distance $h/2$ in Figure (2.16). Thus from Eq. (2.43)

$$h = 2(0.03(m)) = \frac{\omega^2}{2g} r^2 = \frac{\omega^2 (0.03(m))^2}{2(9.81(m/s^2))}$$

Solving, we obtain

$$\omega^2 = 1308 \qquad or \qquad \omega = 36.2\ rad/s$$

Where $\omega = \dfrac{2\pi N}{60}$ (N: revolution per minute)

$$36.2 = \frac{2\pi N}{60} \rightarrow N = 345 rpm$$

***Ans.* (a)**

Example 2: Pressure Distribution in Rigid-Body Motion

To compute the pressure, it is convenient to put the origin of coordinates *r* and *z* at the bottom of the free-surface depression, as shown in Figure (E2.8). The gage pressure here is $p_0=0$, and point **A** is at (r, z) = (3 cm, -4 cm). Equation (2.44) can then be evaluated

$$p = \frac{1}{2}\rho r^2\omega^2 + p_1 + p_0 = \left[0.5\left(1010\,(kg/m^3)\right)\left(0.03\,(m)\right)^2\left(36.2\,(rad/s)\right)^2\right]$$
$$-\left[\left(1010\,(kg/m^3)\right)\left(9.81\,(m/s^2)\right)-\left(0.04\,(m)\right)\right]+0 = 990\,Pa$$

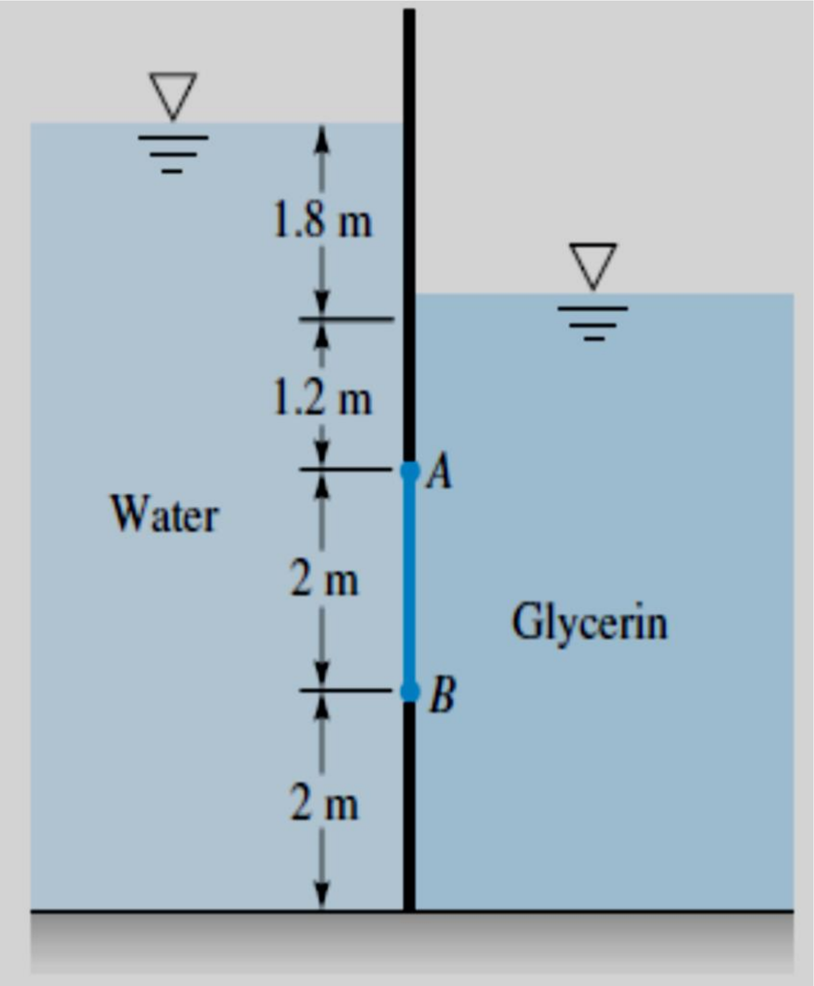
***Ans.* (b)**

This is about 43 percent greater than the still-water pressure p_A = 694 Pa.


QUIZ

Quiz

Find the net hydrostatic force per unit width on the rectangular gate **AB** in the following figure and its line of action.




Important
Announcement

Exam (1) on 04/11/2015

Thank you