Nuclear Fission Energy

The mass of fission products on the right hand side of Eq. (2) is 0.207 amu less than mass on the left hand side. Thus this reaction means a mass defect of 0.207 amu which is equivalent to 0.207×931 *i.e.*, 193 MeV of energy. Generally it is assumed that 1 fission of U^{235} causes a release of 200 MeV of energy.

$$200 \text{ MeV} = 200 \times 1.6 \times 10^{-13} = 3.2 \times 10^{-11} \text{ watt secs. or joules}$$

Thus 1 watt (or 1 joule/sec) requires $\dfrac{1}{3.2 \times 10^{-11}}$ or 3.1×10^{10} fissions per second.

If all the atoms of 1 kg of pure U^{235} (containing 25.64×10^{23} atoms) were fissioned, the energy released would be equivalent to that contained in 3×10^{6} kg of coal with a calorific value of 6000 kcal/kg. Natural uranium contains only 0.7% U^{235}. If fission efficiency is 50% (*i.e.* only half of the total atoms take part in fission) fission of 1 kg of natural uranium would give energy equivalent to $3 \times 10^{6} \times \dfrac{0.7}{100} \times 5$ *i.e.* 10500 kg of coal.

Nuclear Energy Generation

Example 1. Find the energy equivalent of 1 gram of matter.

Solution. $m = 1 \text{ gm} = 0.001 \text{ kg}$

$$E = mc^2 = 0.001 \times (3 \times 10^8)^2 = 9 \times 10^{13} \text{ joules or watt-sec}$$

$$= \frac{9 \times 10^{13}}{1000 \times 3600} = 2.5 \times 10^7 \text{ kWh}$$

Example 2. Show that a mass defect of 1 amu is equivalent to about 931 MeV of energy.

Solution. $1 \text{ amu} = 1.66 \times 10^{-24} \text{ gram} = 1.66 \times 10^{-27} \text{ kg}$

$$\text{Energy} = mc^2 = 1.66 \times 10^{-27} \times (3 \times 10^8)^2 = 14.94 \times 10^{-11} \text{ joules}$$

Since $1 \text{ joule} = 6.242 \times 10^{12} \text{ MeV}$

$$\text{Energy} = 14.94 \times 10^{-11} \times 6.242 \times 10^{12} \simeq 931 \text{ MeV}$$

Nuclear Energy Generation

Example 3. If 0.190 amu. are converted to energy for every nucleus of U235 that undergoes the fission process, (a) show that the energy released is indeed approximately 0.9 MeV, (b) Show that the fission of 1 kg of U235 releases approximately a million times more energy than the combustion of 1 kg of coal if you know that the heating value of coal is of the order of 10,000 BTU per pound.

Nuclear Energy Generation

Example 3.

(a) $\Delta E = \Delta m\, c^2$

$$= (0.190 \text{ a.m.u.}) \left(\frac{1.66056\text{x}10^{-27} \text{ kg}}{1 \text{ a.m.u.}}\right) \left(3\text{x}10^{8} \frac{\text{m}}{\text{s}}\right)^2 =$$

$$= (2.84\text{x}10^{-11} \text{ J}) \left(\frac{6.242\text{x}10^{12} \text{ MeV}}{1 \text{ J}}\right) = 177 \text{ MeV/nucleus}$$

$$= \left(177 \frac{\text{MeV}}{\text{nucleus}}\right) \left(\frac{1 \text{ nucleus}}{235 \text{ nucleons}}\right) = 0.75 \text{ MeV/nucleon}$$

Nuclear Energy Generation

Example 3.

(b) We have seen that the heating value of coal is of the order of 10,000 BTU per pound. For every kg of coal, choosing the upper limit, this is

$$\left(\frac{10000\ \text{BTU}}{1\ \text{lb coal}}\right)\left(\frac{2.2\ \text{lb}}{1\ \text{kg}}\right) = 2.2\text{x}10^{4}\ \text{BTU/kg coal}$$

For fission of U-235, taking into account that there are $6\text{x}10^{23}$ atoms in one mole (Avogadro's number) and that the molar mass is 235 grams, we have:

$$\left(\frac{177\ \text{MeV}}{1\ \text{nucleus U-235}}\right) \text{x}$$

$$\text{x}\left(\frac{1\ \text{nucleus U-235}}{1\ \text{atom U-235}}\right)\left(\frac{6\text{x}10^{23}\ \text{atoms}}{1\ \text{mol}}\right)\left(\frac{1\ \text{mol}}{0.235\ \text{kg U-235}}\right)\left(\frac{1.52\text{x}10^{-16}\ \text{BTU}}{1\ \text{MeV}}\right) =$$

$$= 6.90\text{x}10^{10}\ \text{BTU/kg U-235}$$

So, the nuclear energy stored in 1 kilogram of radioactive uranium is equivalent to the chemical energy stored in approximately three million kilograms of coal. This represents a vast source of energy. How well it is harnessed depends on the efficiency of its conversion to useful energy.

Nuclear Energy Generation

Example 4. Find the U^{235} fuel used in one year in a 235 MW pressurised water reactor. Assume an overall plant efficiency of 33% and 100% load factor throughout the year.

Solution. Total energy output $= 235 \times 8760 = 2058600$ MWh

$$= 741.096 \times 10^{13} \text{ watt-sec}$$

$$\text{Energy Input} = \frac{741.096 \times 10^{13}}{0.33} = 2245.75 \times 10^{13} \text{ watt-sec}$$

Number of fissions required for 1 watt-sec $= 3.1 \times 10^{10}$

Total number of fissions required $= 2245.75 \times 3.1 \times 10^{23} = 6961.825 \times 10^{23}$ fissions

Number of atoms in 1 gm of $U^{235} = 2.563 \times 10^{21}$

$$\text{Fuel required in one year} = \frac{6961.825 \times 10^{23}}{2.563 \times 10^{21}} = 271.6 \times 10^{3} \text{ grams} = 271.6 \text{ kg}$$

Nuclear Energy Generation

Example 5. Calculate how much U235 is needed for the lifetime of a nuclear power plant that produces 1000 MW of electricity. Consider that the lifetime of the plant is 30 years and that its efficiency is 34%. Assume that the efficiency of conversion of nuclear energy to heat is 100% and that the power plant operates at 100% capacity.

Nuclear Energy Generation

Example 5.

In example (3) , we calculated that 1 kg of U-235 can be converted into approximately 10^{11} BTUs of kinetic energy. The representative power plant used as an example produces 1000 MW of electricity, or

$$\left(10^{9}\,\frac{\text{J}}{\text{s}}\right)\left(\frac{3.15\text{x}10^{7}\ \text{s}}{1\ \text{year}}\right)\left(\frac{1\ \text{BTU}}{1055\ \text{J}}\right) = 3.0\text{x}10^{13}\ \text{BTU/year}$$

At 34% efficiency, the thermal energy required to produce this amount of electricity is:

$$\left(\frac{3.0\text{x}10^{13}\ \text{BTU(electric)}}{\text{year}}\right)\left(\frac{1\ \text{BTU (thermal)}}{0.34\ \text{BTU(electric)}}\right) = 8.8\text{x}10^{13}\ \text{BTU/year}$$

Over a lifetime of 30 years, the quantity of thermal energy required for one plant will be:

$$\left(\frac{8.8\text{x}10^{13}\ \text{BTU}}{1\ \text{year}}\right)(30\ \text{years}) = 2.6\text{x}10^{15}\ \text{BTU}$$

Nuclear Energy Generation

Example 5.

Given the result of example (3), we finally have:

$$\left(\frac{2.6\text{x}10^{15}\ \text{BTU}}{1\ \text{plant}}\right)\left(\frac{1\ \text{kg U-235}}{10^{11}\ \text{BTU}}\right) = 26000\ \text{kg U-235/plant}$$

So about 26 tons of radioactive uranium are needed for a typical nuclear reactor.

Nuclear Energy Generation

Example 6.

What mass of uranium ore (in kg) enriched to 3% ^{235}U is required to produce 6132 GWh of electricity (equivalent to a 1 GW power plant running at 70% capacity factor)? Assume that each fission of ^{235}U produces 200 MeV (3.2×10^{-11} J), that all neutrons absorbed by ^{235}U cause fission, and that the nuclear power plant has a thermal efficiency of 33%.

Start with the energy requirement. Based on the efficiency, you know you'll need

$$\frac{6132 \text{ GWh}}{0.33} = 18582 \text{ GWh}$$

$$18582\ GWh \times \frac{3.6\ TJ}{GWh} = 66895\ TJ = 6.7 \times 10^{16}\ J$$

Nuclear Energy Generation

Example 6.

The fission of each atom (nucleus) of ^{235}U generates 3.2 x 10^{-11}J, so we know that we'll need

$$6.7 \times 10^{16} J \times \frac{1\ atom U235}{3.2 \times 10^{-11}\ J} = 2.1 \times 10^{27}\ atoms\, U235$$

^{235}U accounts for 3% of the total U, so

$$\frac{2.1 \times 10^{27}\ atoms\ U235}{0.03} = 7.0 \times 10^{28}\ atoms\ U$$

Assuming that the uranium ore is ^{238}U (i.e., having a molecular weight of 238 g/mol), we can calculate the mass of uranium required.

$$7.0 \times 10^{28}\ atoms\ U \times \frac{1\ mol}{6.02 \times 10^{23}\ atoms} \times \frac{238\ g}{1\ mol} \times \frac{1\ kg}{1{,}000\ g} = 2.8 \times 104\ kg\ uranium$$

Thank you