بسم الله الرحمن الرحيم

Vibration

Fayoum University

Faculty of Engineering
Mechanical Engineering Dept.

Lecture (5)

on

The Vibrations of Systems Having Single Degree of Freedom- Response to Harmonic and Periodic Excitations

By

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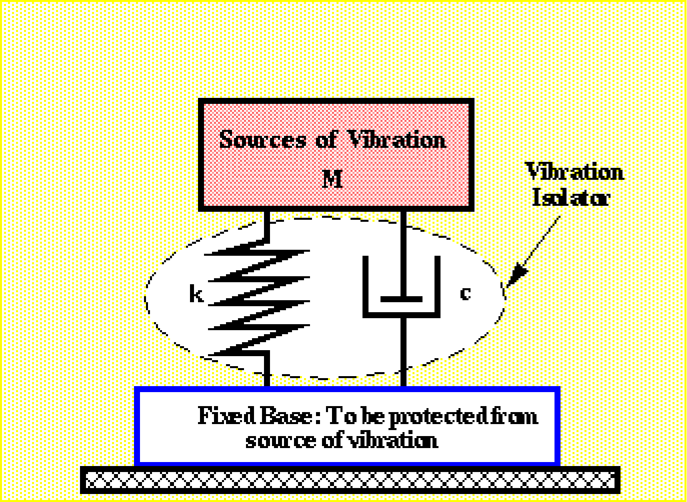
Transmissibility

The coefficient of transmissibility is

$$\tau = \frac{F_{tr,\max}}{F_{0,\max}}$$

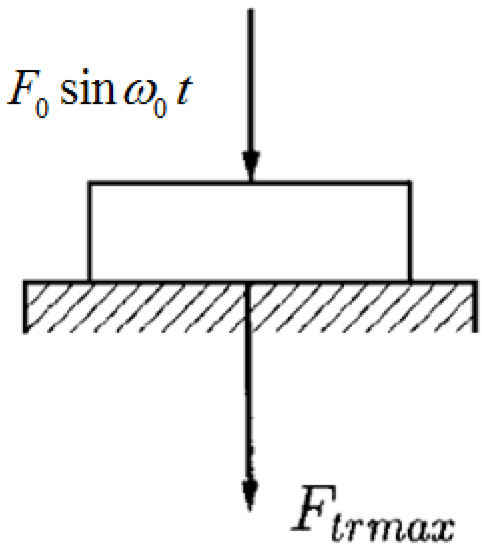
Where $F_{tr,\max}$ is the maximum transmitted force and $F_{0,\max}$ is the maximum perturbation force.



Transmissibility

Machine directly on a foundation

In this case (Figure (3.10)), the perturbation force is transmitted to the foundation. The transmissibility coefficient is $\tau = 1$ and the machine is not isolated.



Mechanical model of transmissibility in the case of a machine directly on a foundation.

Transmissibility

Machine on a foundation with an elastic element and a damper

$$m\ddot{x} + c\dot{x} + kx = F_0 \sin \omega_0 t$$

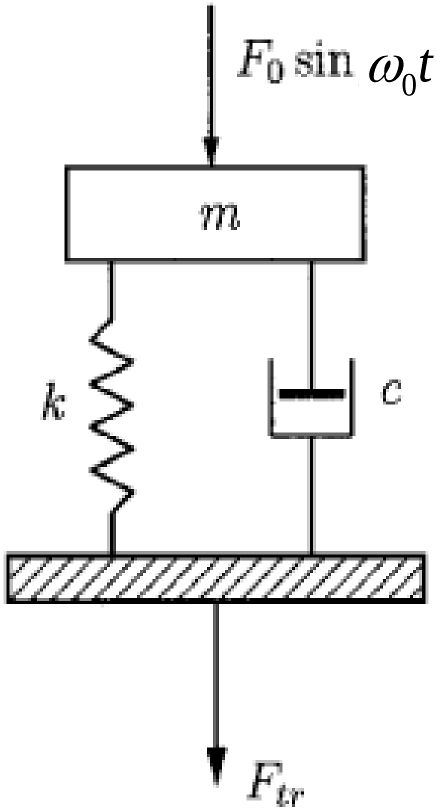
$$x(t) = \frac{F_0}{\sqrt{(k - m\omega_0^2)^2 + c^2\omega_0^2}} \sin(\omega_0 t - \phi)$$

$$\phi = \tan^{-1}\left[\frac{c\omega_0}{k - m\omega_0^2}\right]$$

Since $\omega_n = \sqrt{k/m}$ and $c_{crit} = 2m\omega_n$,

$$X = \frac{F_0/k}{\sqrt{\left[1 - (\omega_0/\omega_n)^2\right]^2 + \left[2(c/c_{crit})(\omega_0/\omega_n)\right]^2}}$$

$$\phi = \tan^{-1}\left[\frac{\left[2(c/c_{crit})(\omega_0/\omega_n)\right]}{1 - (\omega_0/\omega_n)^2}\right]$$



Transmissibility

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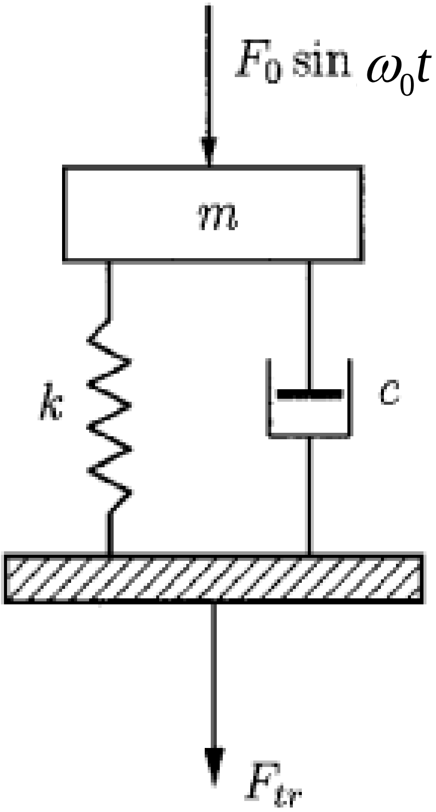


Transmissibility

Machine on a foundation with an elastic element and a damper

The transmitted force is not in the same phase with the perturbation force. In this case, the transmitted force is

$$F_{tr,\max} = [kx + c\dot{x}]_{\max}$$

The exciting vibration is

$$x(t) = X\sin(\omega_0 t - \phi)$$

and

$$\dot{x}(t) = X\omega_0 \cos(\omega_0 t - \phi)$$

Therefore, the force transmitted to the foundation is

$$\begin{aligned} F_{tr} &= kx + c\dot{x} \\ &= kX\sin(\omega_0 t - \phi) + cX\omega_0 \cos(\omega_0 t - \phi) \\ &= M\sin(\omega_0 t - \phi) \end{aligned}$$

The magnitude of the total transmitted force is given by

$$M = \sqrt{k^2 X^2 + c^2 X^2 \omega_0^2} = F_{tr,\max}$$

which represents the maximum transmitted force.

Transmissibility

Machine on a foundation with an elastic element and a damper

The transmissibility coefficient is

$$\tau = \frac{X\sqrt{k^2 + c^2\omega_0^2}}{F_0} = \frac{\sqrt{\dfrac{k^2}{m^2} + \dfrac{c^2\omega_0^2}{m^2}}}{\sqrt{\left(\omega_n^2 - \omega_0^2\right)^2 + 4\omega_0^2\left(\dfrac{c}{2m}\right)^2}} = \sqrt{\frac{\omega_n^4 + 4\omega_0^2\left(\dfrac{c}{2m}\right)^2}{\left(\omega_n^2 - \omega_0^2\right)^2 + 4\omega_0^2\left(\dfrac{c}{2m}\right)^2}}$$

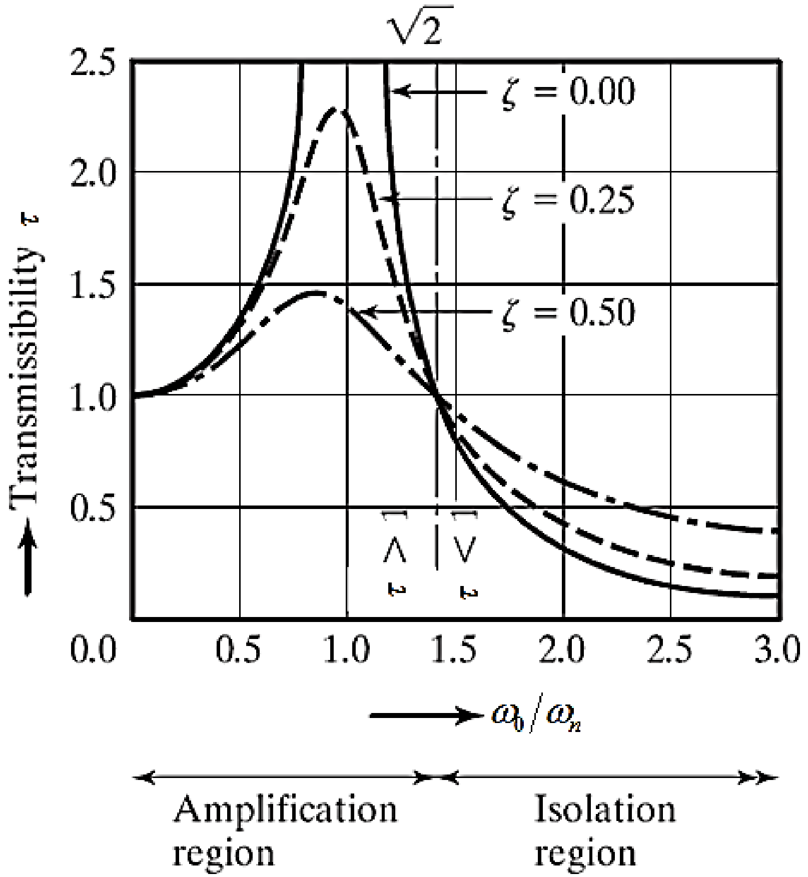
$$= \sqrt{\frac{1 + \left[4\left(\dfrac{c}{2m\omega_n}\right)^2\left(\dfrac{\omega_0}{\omega_n}\right)^2\right]}{\left(1 - \dfrac{\omega_0^2}{\omega_n^2}\right)^2 + \left[4\left(\dfrac{c}{2m\omega_n}\right)^2\left(\dfrac{\omega_0}{\omega_n}\right)^2\right]}}$$

Transmissibility

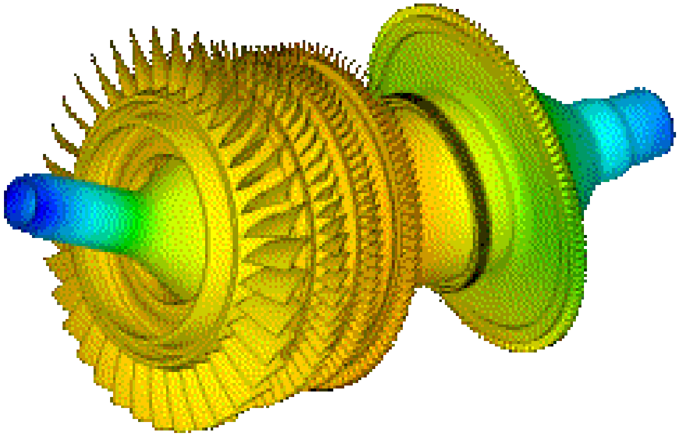
Machine on a foundation with an elastic element and a damper

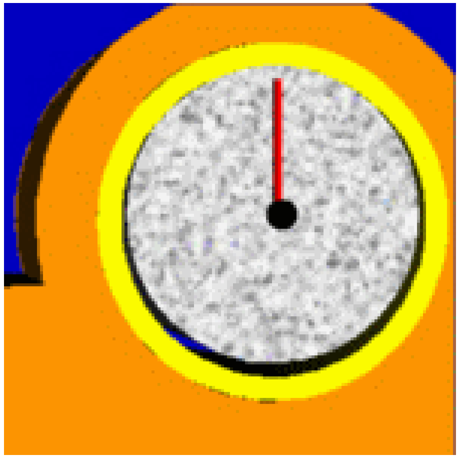
$$\tau = \sqrt{\frac{1+\left[4\left(\frac{c}{c_{crit}}\right)^2\left(\frac{\omega_0}{\omega_n}\right)^2\right]}{\left(1-\frac{\omega_0^2}{\omega_n^2}\right)^2+\left[4\left(\frac{c}{c_{crit}}\right)^2\left(\frac{\omega_0}{\omega_n}\right)^2\right]}}$$

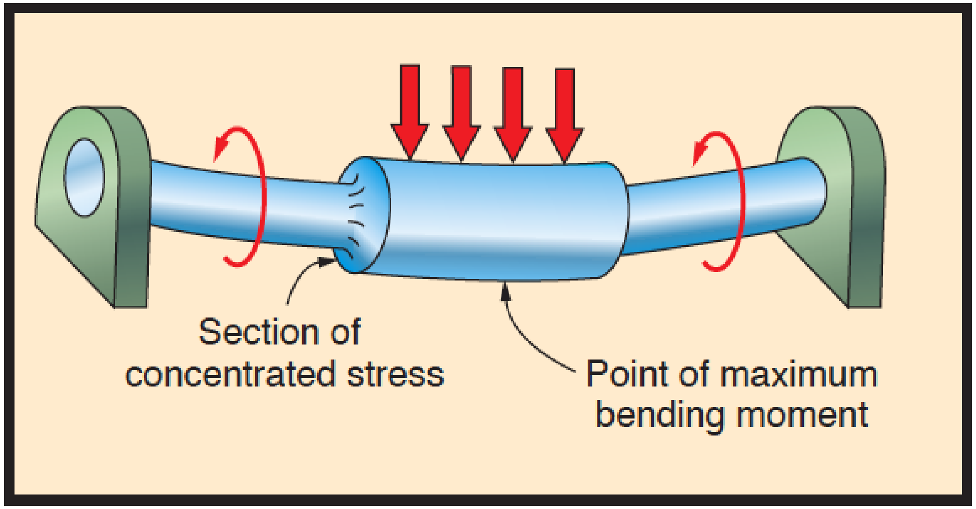
$$= \sqrt{\frac{1+\left(2\zeta\frac{\omega_0}{\omega_n}\right)^2}{\left(1-\frac{\omega_0^2}{\omega_n^2}\right)^2+\left(2\zeta\frac{\omega_0}{\omega_n}\right)^2}}$$



The Whirling of Rotating Shafts







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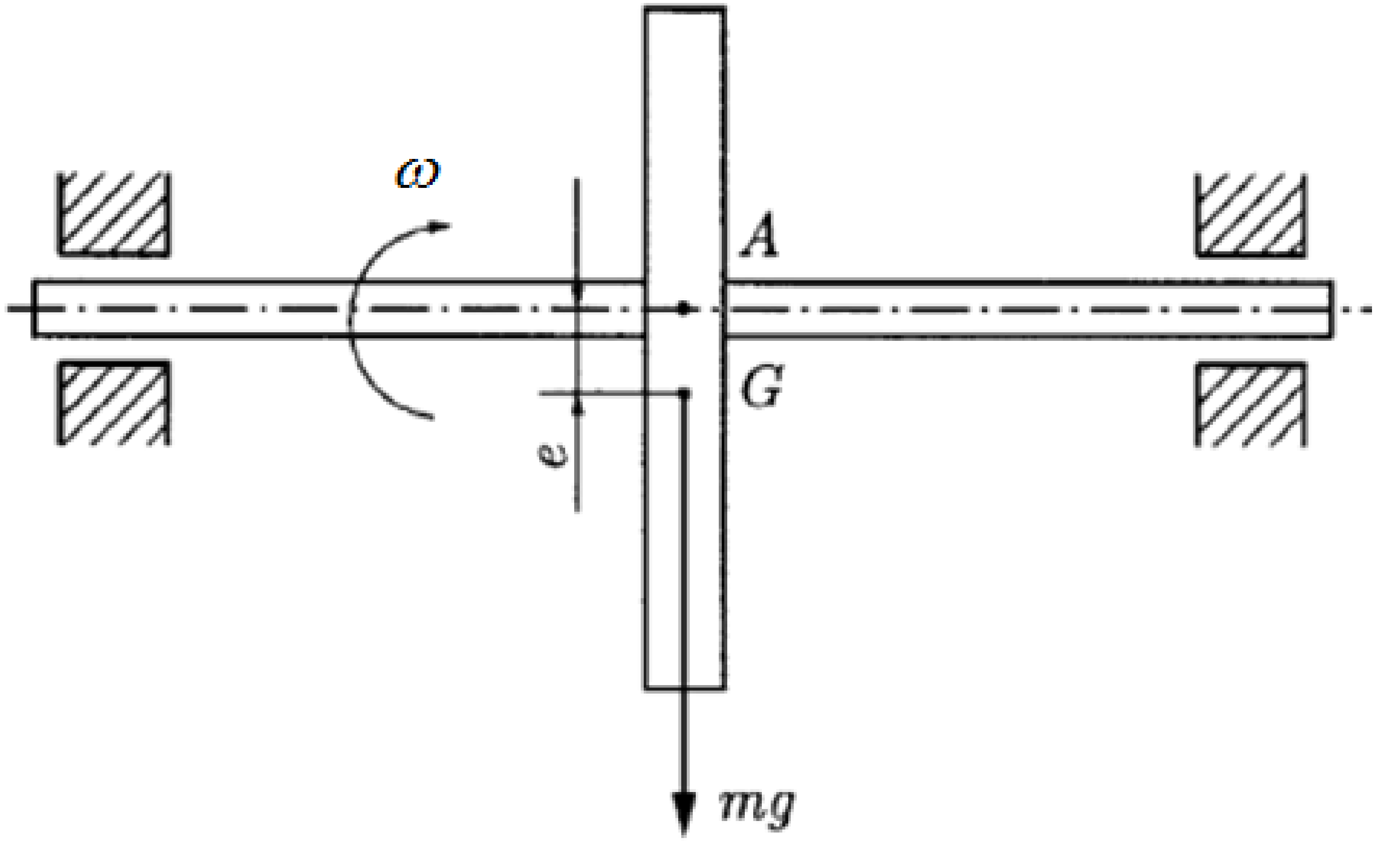

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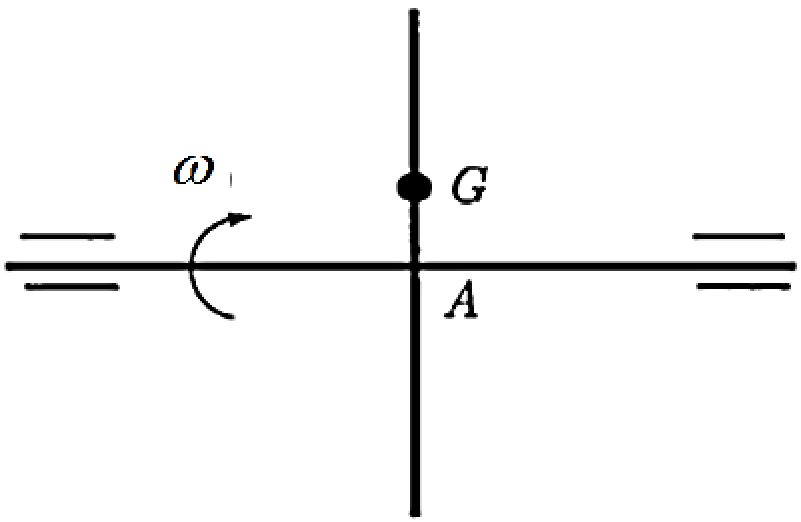
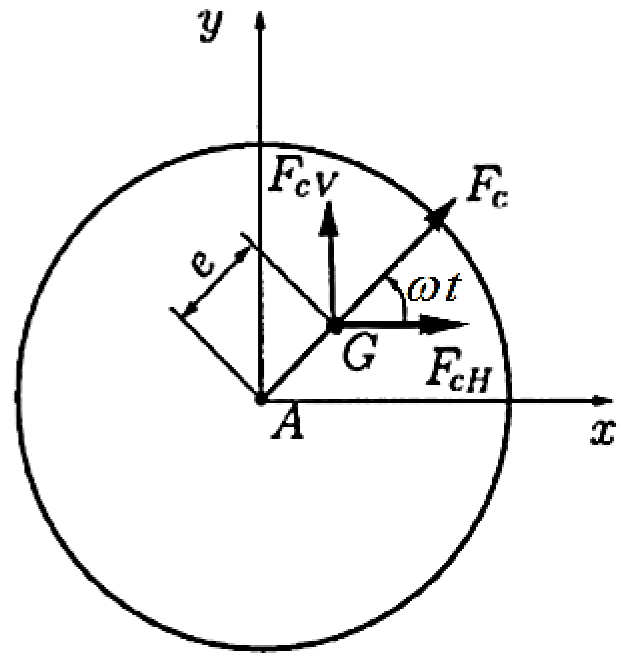
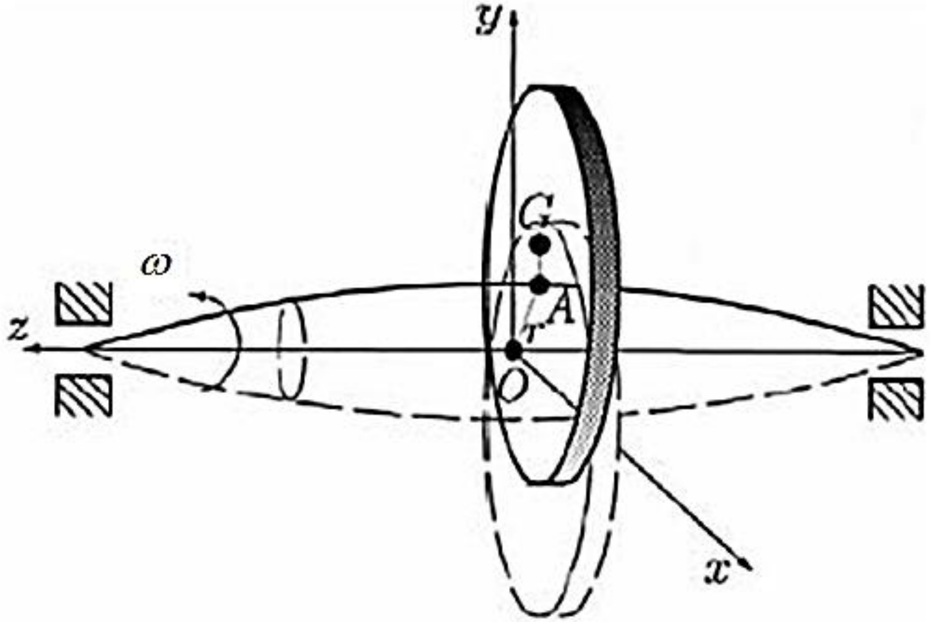


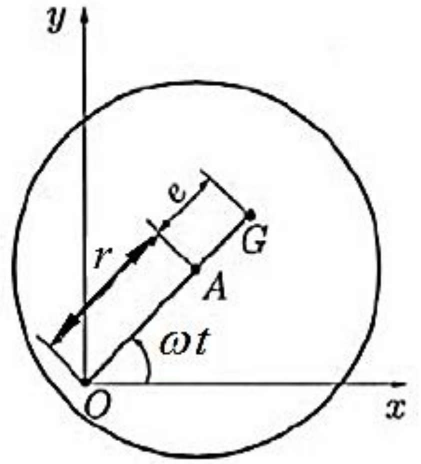


$$\begin{cases} F_{c,H} = me\omega^2 \cos\omega t \\ F_{c,V} = me\omega^2 \sin\omega t \end{cases} \rightarrow \begin{cases} \ddot{x} + \omega_n^2 x = e\omega^2 \cos\omega t \\ \ddot{y} + \omega_n^2 y = e\omega^2 \sin\omega t \end{cases}$$

and $\omega_n^2 = \dfrac{k_{eq}}{m}$

The Whirling of Rotating Shafts





The center line of the support bearings intersects the plane of the wheel at O and the shaft center is deflected with $\mathbf{r} = OA$.

$$k_{eq} r = m(r+e)\omega^2 = mr\omega^2 + me\omega^2$$

The Whirling of Rotating Shafts

$$r = e\frac{\omega^2}{\left(\frac{k_{eq}}{m}\right) - \omega^2} = e\frac{\left(\frac{\omega}{\omega_n}\right)^2}{1-\left(\frac{\omega}{\omega_n}\right)^2}$$

$$k_{eq} = \frac{48EI}{l^3}$$

By analogy with Eq. (3.9) (in case of $\zeta = 0$), Eqs. (3.31) have the solution

$$\begin{cases} x(t) = e\dfrac{(\omega/\omega_n)^2}{1-(\omega/\omega_n)^2}\cos\omega t \\ y(t) = e\dfrac{(\omega/\omega_n)^2}{1-(\omega/\omega_n)^2}\sin\omega t \end{cases}$$

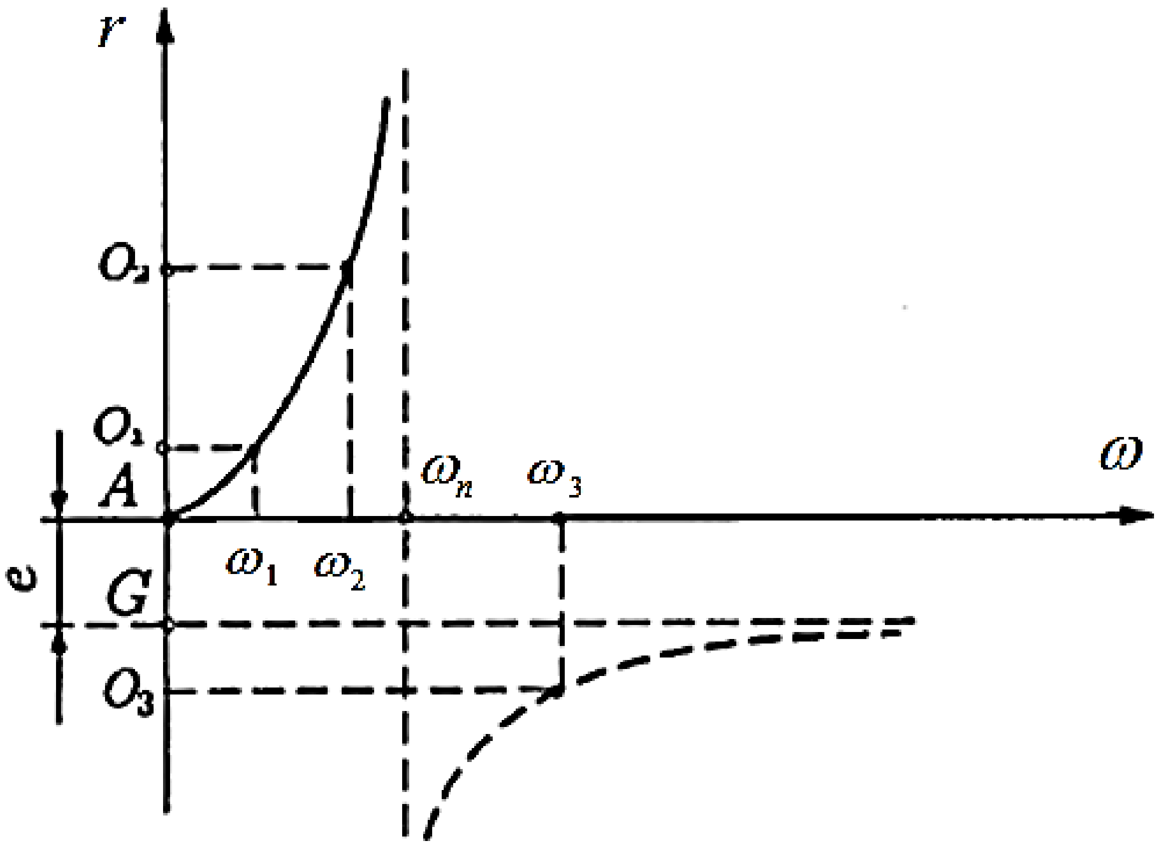
The Whirling of Rotating Shafts

If the circular frequency of transversal vibration (bending) is equal to the rotation angular speed, then the resonance will take place. In this case rotation angular speed is called critical angular speed ω_{crit} , and the critical rpm is N_{crit} , given by

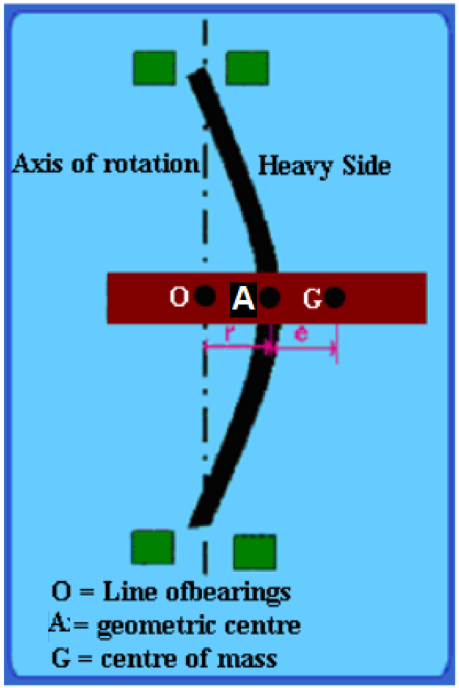
$$N_{crit} = \frac{30}{\pi}\omega_{crit}$$

$$N_{crit} = \frac{60}{2\pi}\omega_n = \frac{60}{2\pi}\sqrt{\frac{48EI}{ml^3}} \quad rpm$$

The Whirling of Rotating Shafts



$$r = e\frac{\omega^2}{\left(\frac{k_{eq}}{m}\right) - \omega^2} = e\frac{\left(\frac{\omega}{\omega_n}\right)^2}{1-\left(\frac{\omega}{\omega_n}\right)^2}$$



The Whirling of Rotating Shafts

1. One is undercritical ($\omega < \omega_n$) when $r > 0$ and the point G is outside of segment OA.
2. The other one is overcritical ($\omega > \omega_n$) when $r < 0$ and the point G is inside of segment OA.
3. If angular speed of the shaft increases, the point of mass G tends to point O (center line of bearings). This phenomenon is called self-centering or self-aligning.

The Whirling of Rotating Shafts

Other Forms of Damping

For a harmonic motion of the form $x(t) = X \sin \omega t$, the energy dissipated over one cycle of motion due to a damping force F_D is

$$\Delta E = \int_0^{2\pi/\omega} F_D \dot{x}\, dt = \int_0^{2\pi/\omega} F_D X \omega \cos \omega t \, dt$$

For viscous damping,

$$\Delta E = \int_0^{2\pi/\omega} c\, \dot{x}^2 \, dt = \int_0^{2\pi/\omega} c\, \omega^2 X^2 \cos^2 \omega t \, dt = c \omega \pi X^2$$

Thus, by analogy, the equivalent viscous damping coefficient for another form of damping is

$$c_{eq} = \frac{\Delta E}{\omega \pi X^2}$$

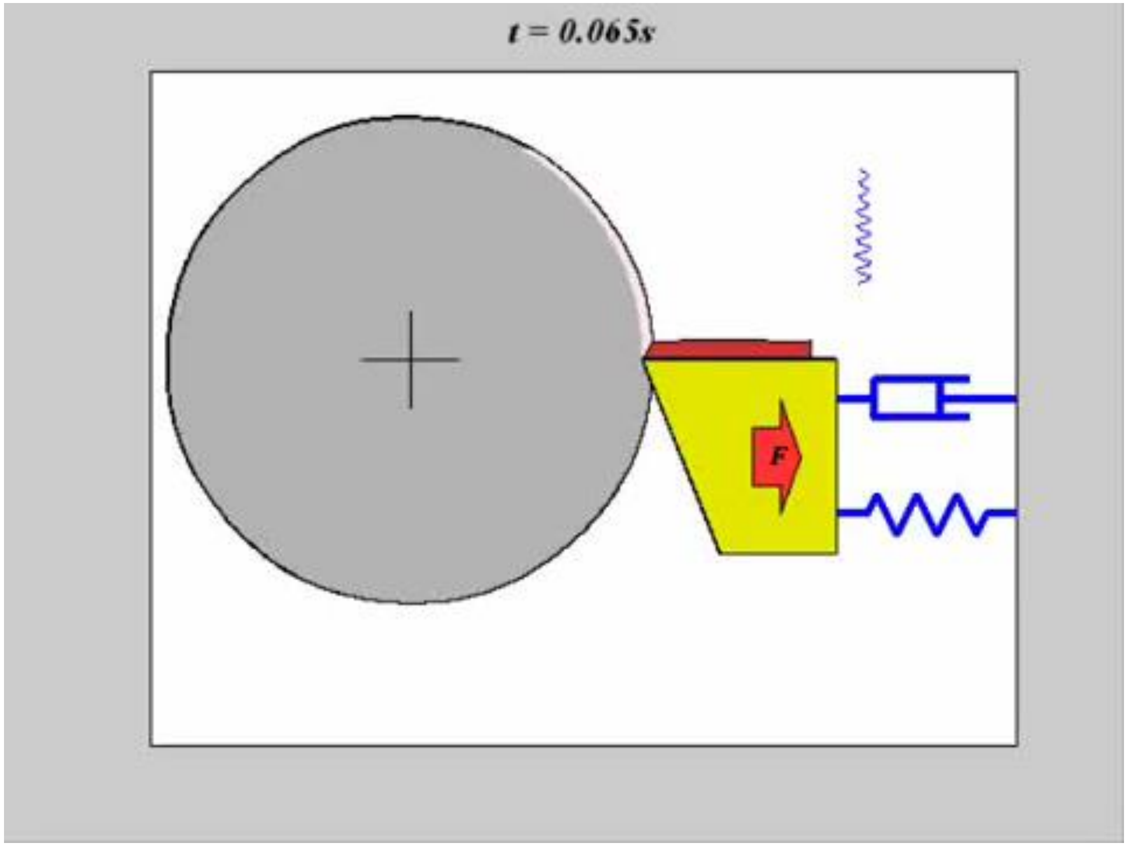
Self-excited Vibration

Self-excited vibrations pervade all areas of design and operations of physical systems where motion or time-variant parameters are involved:

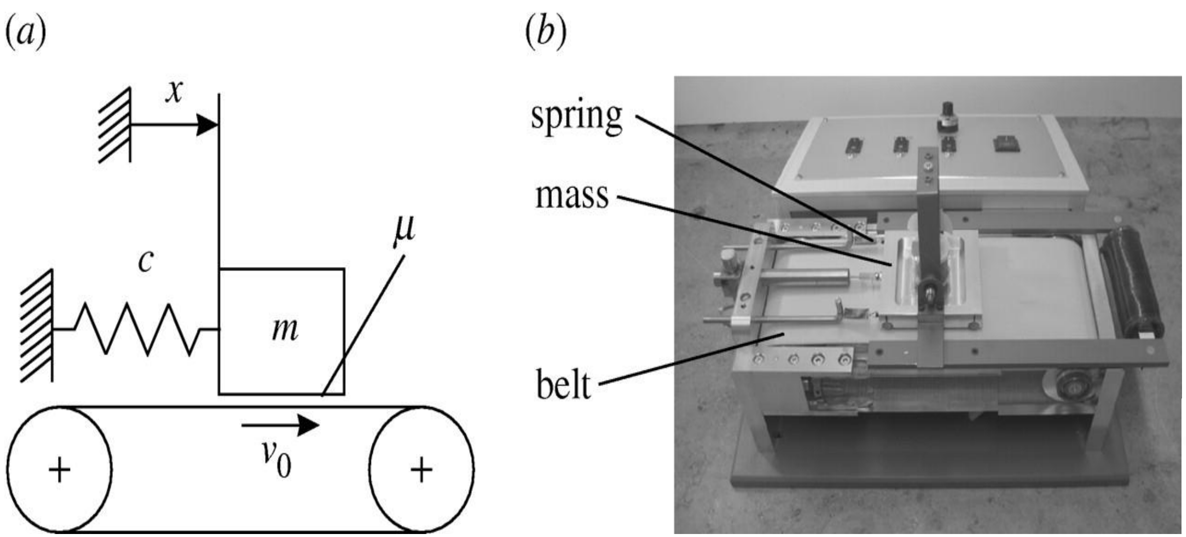
1. **Aeromechanical systems (flutter, aircraft flight dynamics),**
2. **Aerodynamics (separation, stall, musical wind instruments, diffuser and inlet chugging),**
3. **Aerothermodynamics (flame instability, combustor screech),**
4. **Mechanical systems (machine-tool chatter),**
5. **Feedback networks (pneumatic, hydraulic, and electromechanical servomechanisms).**

Self-excited Vibration

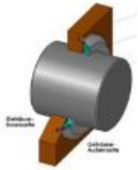
Self-excited Vibration



Self-excited Vibration



vehicle brakes

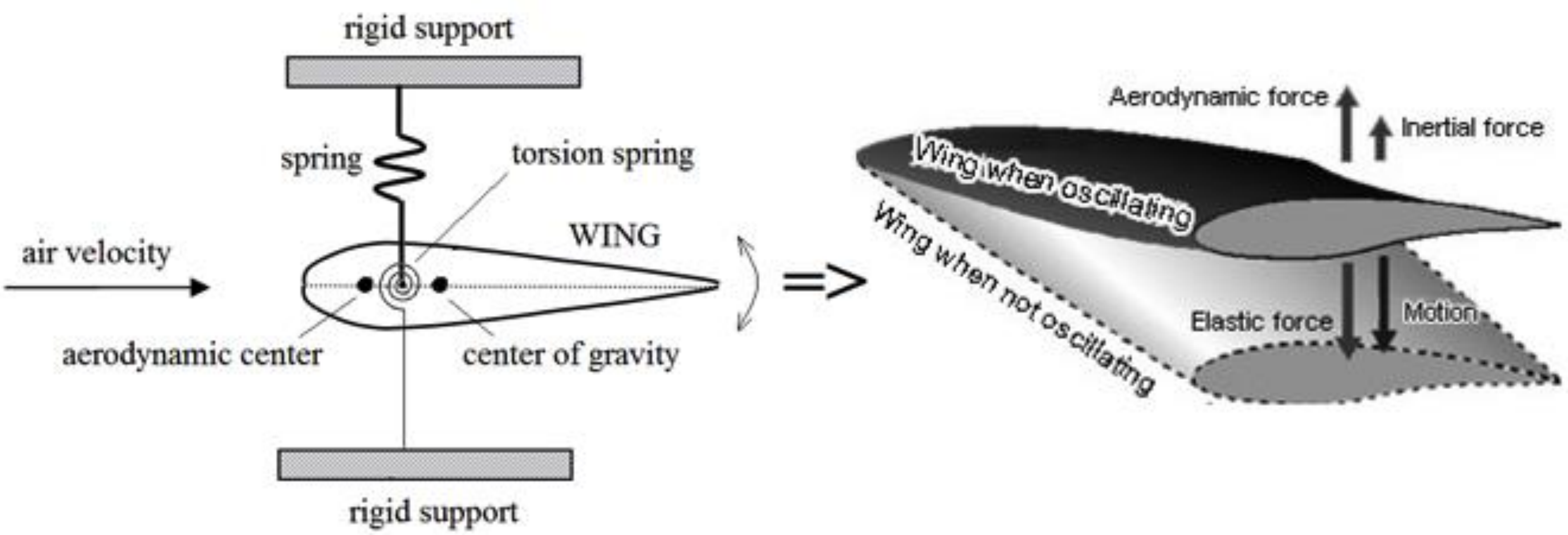


shaft seals

band saws

grinding tools

Self-excited Vibration



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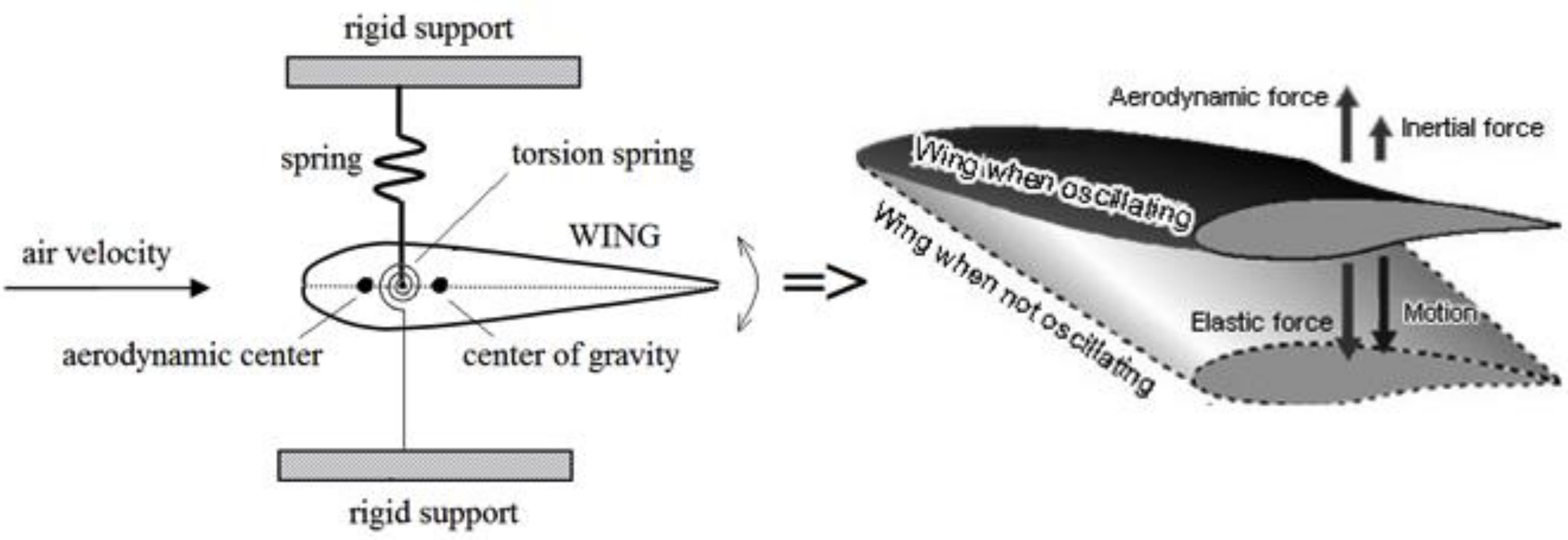


QUIZ

Quiz

A steel shaft of diameter 2.5 cm and length 1 m is supported at the two ends in bearings. It carries a turbine disc, of mass 20 kg and eccentricity 0.005 m, at the middle and operates at 6000 rpm. The damping in the system neglected. Determine the whirl amplitude of the disc at (a) operating speed, (b) critical speed, and (c) 1.5 times the critical speed.

Thank you