بسم الله الرحمن الرحيم

Vibration

Fayoum University

Faculty of Engineering
Mechanical Engineering Dept.

Lecture (6)
on
The Vibrations of Systems Having Two Degree of Freedom

By

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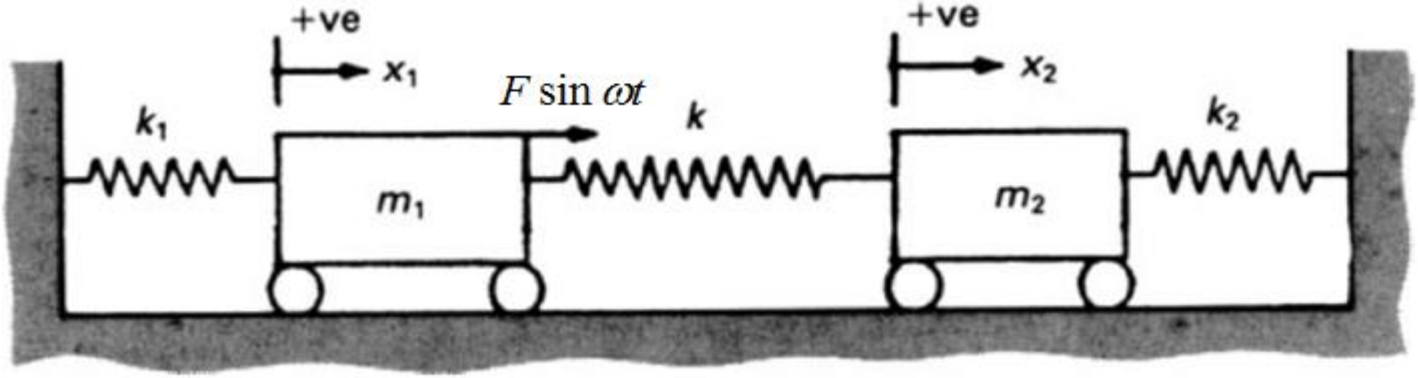
Two Degree of Freedom Undamped Forced Vibration Systems



The equations of motion are

$$m_1\ddot{x}_1 = -k_1x_1 - k(x_1 - x_2) + F \sin \omega t$$

and

$$m_2\ddot{x}_2 = k(x_1 - x_2) - k_2x_2$$

Since there is zero damping, the motions are either in phase or π out of phase with the driving force, so that the following solutions may be assumed:

$x_1 = A_1 \sin \omega t$ and $x_2 = A_2 \sin \omega t$

Substituting these solutions into the equations of motion gives

$$A_1\left(k_1 + k - m_1\omega^2\right) + A_2(-k) = F$$

and

$$A_1(-k) + A_2\left(k_2 + k - m_2\omega^2\right) = 0$$

$$A_1 = \frac{F\left(k_2 + k - m_2\omega^2\right)}{\Delta}$$

and

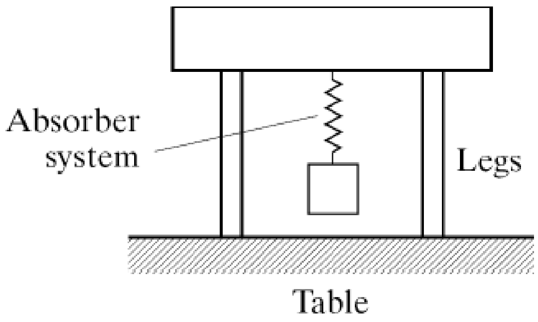
$$A_2 = \frac{Fk}{\Delta}$$

$$\Delta = \left[\left(k_2 + k - m_2\omega^2\right)\left(k_1 + k - m_1\omega^2\right)\right] - k^2 = 0$$

and $\Delta = 0$ is the frequency equation.

The Undamped Dynamic Vibration Absorber

Objective & Application



Objective:

To reduce the vibration of a primary device by adding an absorber to the system

Applications:

- Reciprocating machines
- Building excited by an earthquake
- Transmission lines or telephone lines excited by wind blowing

The Undamped Dynamic Vibration Absorber

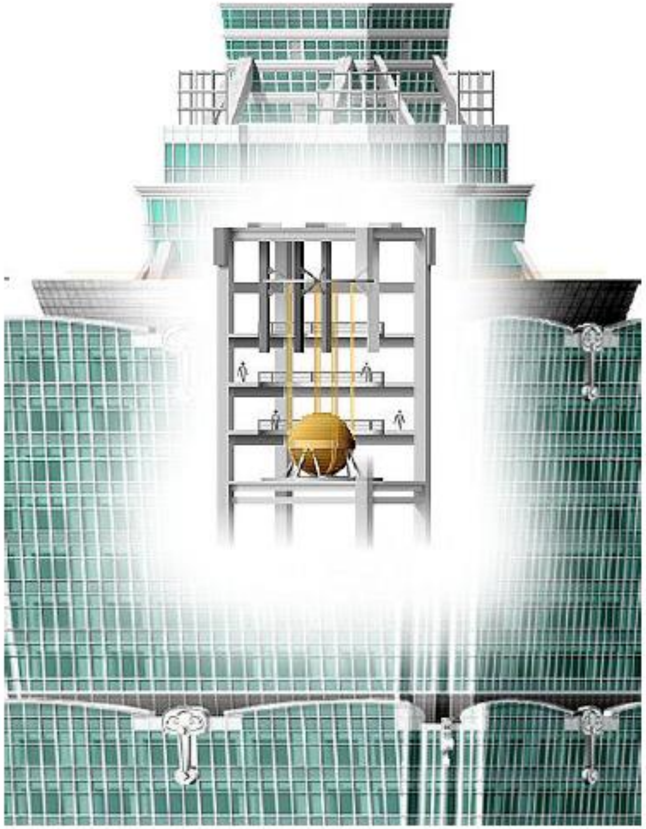
Objective & Application

Vibration absorber in the transmission lines

Tuned mass dampers beneath the bridge platform.

The Undamped Dynamic Vibration Absorber

Objective & Application

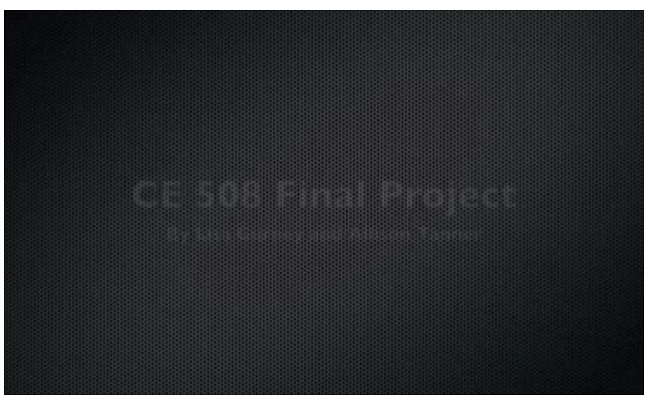


Tuned Mass Damper
in the building

The Undamped Dynamic Vibration Absorber

Objective & Application

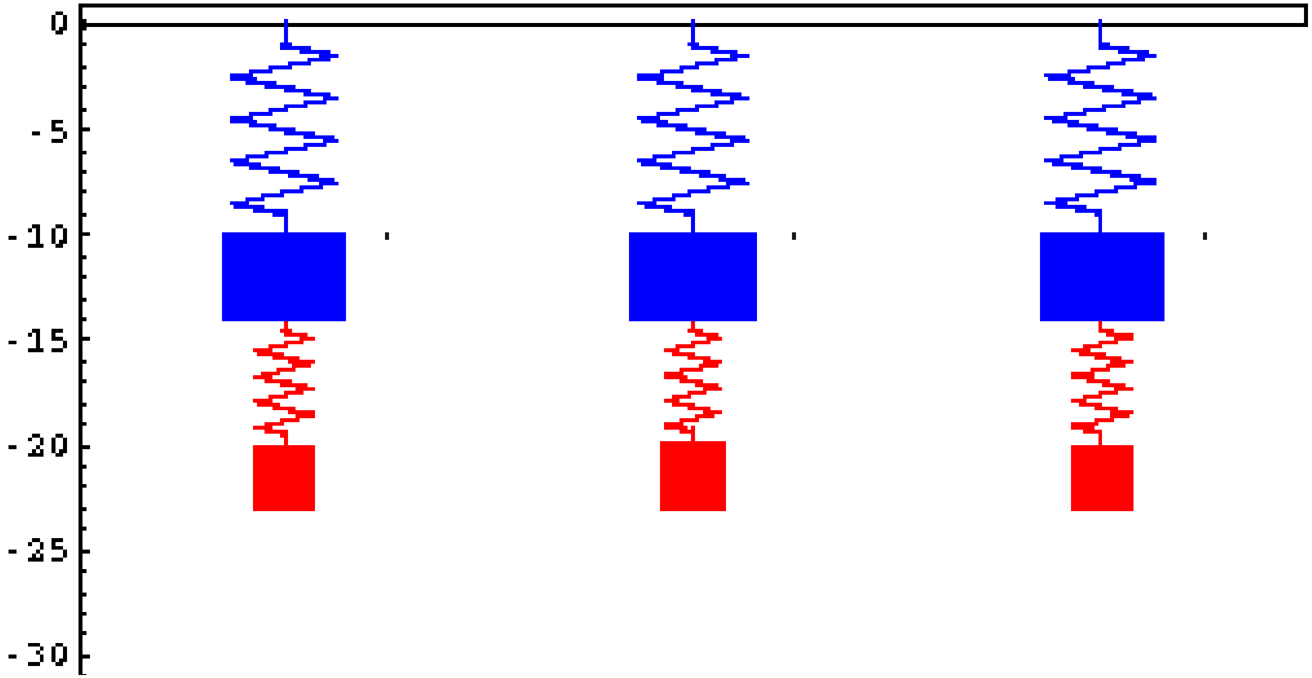
Eccentric Mass Dynamic Vibratory Absorber



Building excited by an earthquake

The Undamped Dynamic Vibration Absorber

How vibration absorber works?



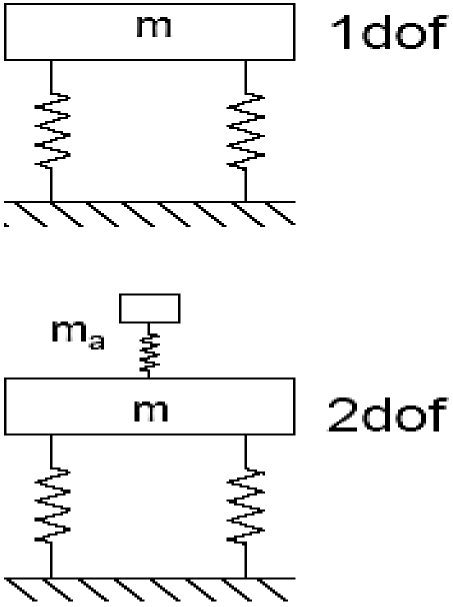
In the lower frequency mode, both masses move in the same direction, in-phase with each other.

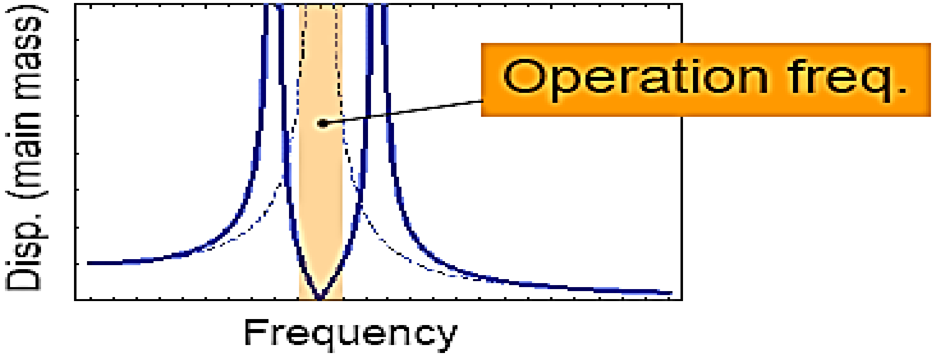
In the higher frequency mode the two masses move in opposite direction, 180° out of phase with each other.

The Undamped Dynamic Vibration Absorber

How vibration absorber works?

Vibration absorber is applied to the machine whose operation frequency meets its resonance frequency.

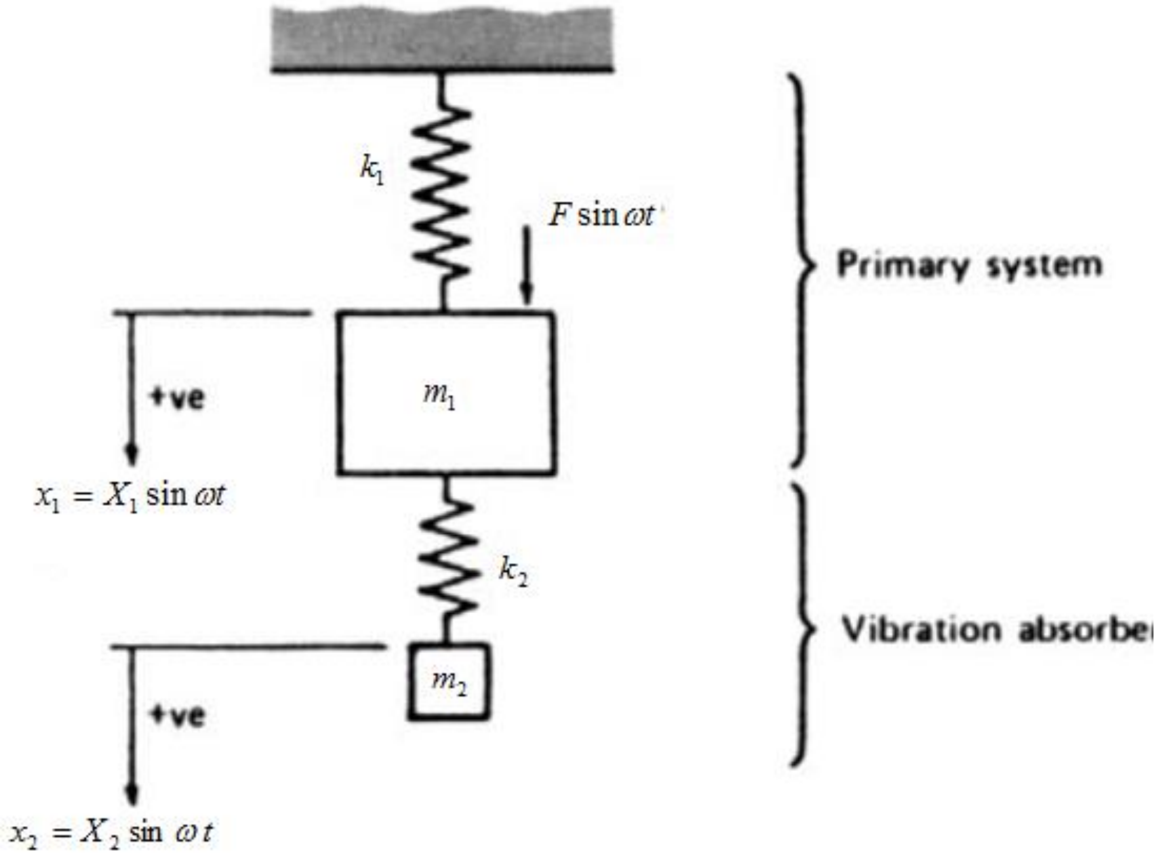




Vibration absorber is often used with machines run at constant speed or systems with const. excited freq., because the combined system has narrow operating bandwidth.

The Undamped Dynamic Vibration Absorber

Mathematical Analysis



The Undamped Dynamic Vibration Absorber

Mathematical Analysis

$m_1 \ddot{x}_1 = -k_1 x_1 - k_2 (x_1 - x_2) + F \sin \omega t$, for the primary system

and

$m_2 \ddot{x}_2 = k_2 (x_1 - x_2)$, for the vibration absorber

Substituting

$x_1 = X_1 \sin \omega t$ and $x_2 = X_2 \sin \omega t$

gives

$X_1 \left(k_1 + k_2 - m_1 \omega^2\right) + X_2 (-k_2) = F$,

and

$X_1 (-k_2) + X_2 \left(k_2 - m_2 \omega^2\right) = 0$

Thus

$$X_1 = \frac{F\left(k_2 - m_2 \omega^2\right)}{\Delta}$$

and

$$X_2 = \frac{F k_2}{\Delta}$$

where $\Delta = \left[\left(k_2 - m_2 \omega^2\right)\left(k_1 + k_2 - m_1 \omega^2\right)\right] - k_2^2$

The Undamped Dynamic Vibration Absorber

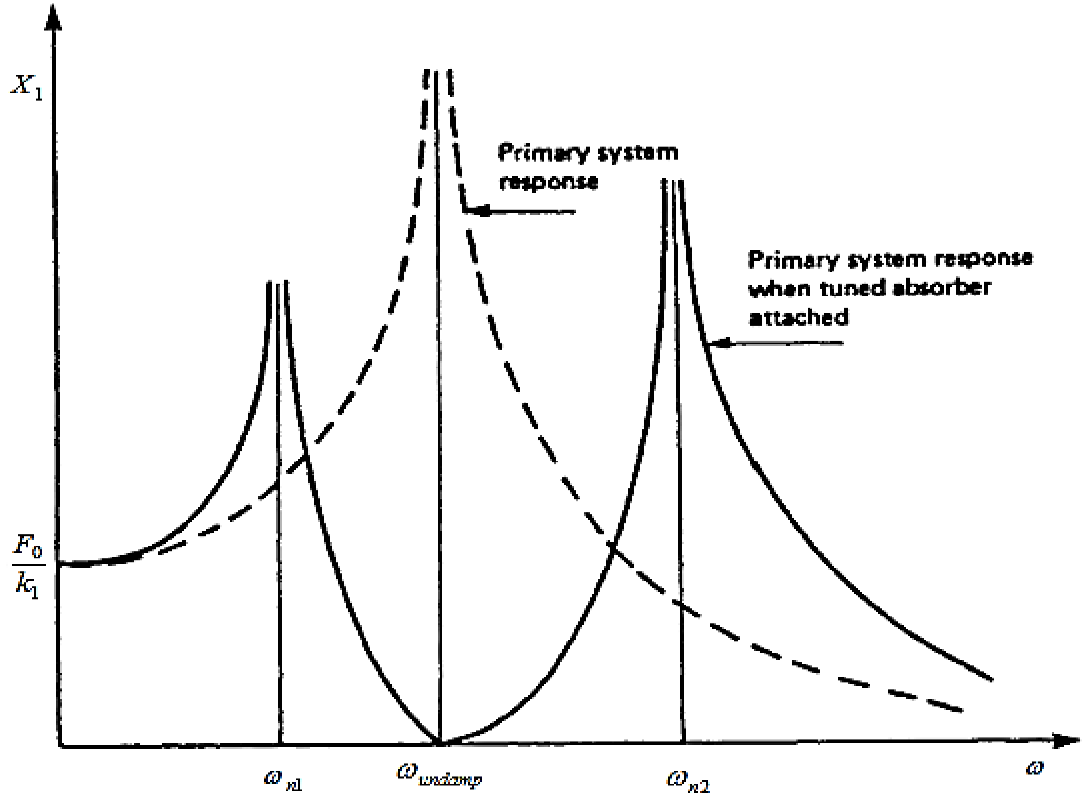
Mathematical Analysis

When $X_1 = 0$, $X_2 = -F/k_2$, so that the force in the absorber spring, $k_2 X_2$ is $-F$; thus the absorber applies a force to the primary system which is equal and opposite to the exciting force. Hence the body in the primary system has a net zero exciting force acting on it and therefore zero vibration amplitude.

Thus if $\sqrt{(k_2/m_2)} = \sqrt{(k_1/m_1)}$, the response of the primary system at its original resonance frequency can be made zero. This is the usual tuning arrangement for an undamped absorber because the resonance problem in the primary system is only severe when $\omega \approx \sqrt{(k_1/m_1)}$ rad/s.

The Undamped Dynamic Vibration Absorber

Mathematical Analysis



$$\omega_{undamp} = \sqrt{k_1/m_1} = \sqrt{k_2/m_2}$$

The Undamped Dynamic Vibration Absorber

Mathematical Analysis

If an absorber is correctly tuned $\omega^2 = k_2/m_2 = k_1/m_1$, and if the mass ratio m_2/m_1, the frequency equation $\Delta = 0$ is

$$\left(\frac{\omega}{\omega_{undamp}}\right)^4 - \left[\left(2+\frac{m_2}{m_1}\right)\left(\frac{\omega}{\omega_{undamp}}\right)^2\right] + 1 = 0$$

This is a quadratic equation in $\left(\frac{\omega}{\omega_n}\right)^2$. Hence

$$\left(\frac{\omega}{\omega_{undamp}}\right)^2 = \left(1+\frac{m_2/m_1}{2}\right) \pm \sqrt{\left[\frac{m_2}{m_1}+\left(\frac{m_2/m_1}{2}\right)^2\right]}$$

and the natural frequencies ω_{n1} and ω_{n2} are found to be

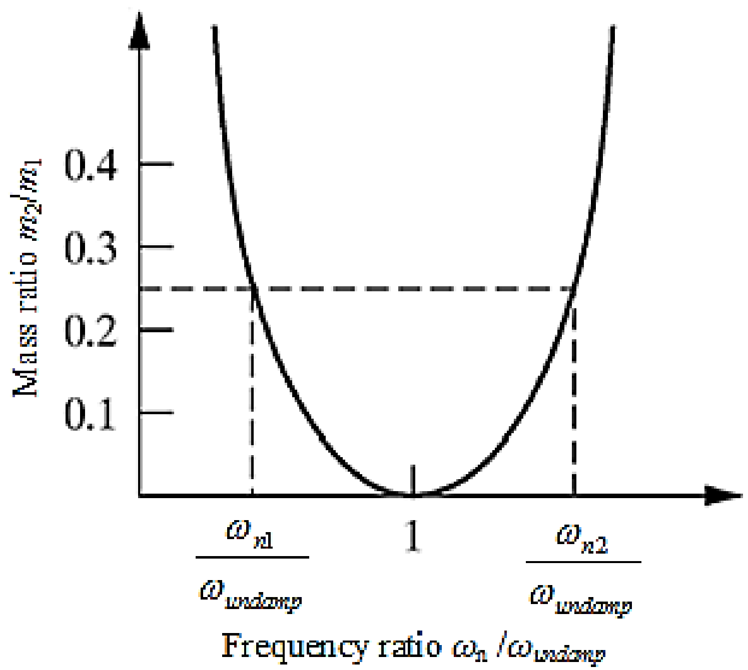
$$\left(\frac{\omega_{n1,2}}{\omega_{undamp}}\right)^2 = \left(1+\frac{m_2/m_1}{2}\right) \pm \sqrt{\left[\frac{m_2}{m_1}+\left(\frac{m_2/m_1}{2}\right)^2\right]}$$

The Undamped Dynamic Vibration Absorber

Mathematical Analysis

For a small mass ratio m_2/m_1, ω_{n1} and ω_{n2} are very close to each other, and near to undamping frequency ω_{undamp}; increasing mass ratio m_2/m_1 gives better separation between $\omega_{n1}$and ω_{n2}

This effect is of great importance in those systems where the excitation frequency may vary; if mass ratio m_2/m_1 is small, resonances at ω_{n1} or ω_{n2} may be excited.



The Undamped Dynamic Vibration Absorber

Mathematical Analysis

It should be noted that since

$$\left(\frac{\omega_{n1}}{\omega_{undamp}}\right)^2=\left(1+\frac{m_2/m_1}{2}\right)-\sqrt{\left[\frac{m_2}{m_1}+\left(\frac{m_2/m_1}{2}\right)^2\right]}$$

and

$$\left(\frac{\omega_{n2}}{\omega_{undamp}}\right)^2=\left(1+\frac{m_2/m_1}{2}\right)+\sqrt{\left[\frac{m_2}{m_1}+\left(\frac{m_2/m_1}{2}\right)^2\right]}$$, Then

$$\frac{\omega_{n1}^2\,\omega_{n2}^2}{\omega_{undamp}^4}=\left(1+\frac{m_2/m_1}{2}\right)^2-\left[\frac{m_2}{m_1}+\left(\frac{m_2/m_1}{2}\right)^2\right]=1$$

That is,

$$\omega_{n1}\,\omega_{n2}=\omega_{undamp}^2$$

also

$$\left(\frac{\omega_{n1}}{\omega_{undamp}}\right)^2+\left(\frac{\omega_{n2}}{\omega_{undamp}}\right)^2=2+\frac{m_2}{m_1}.$$

Undamped Dynamic Vibration Absorber Design

From $$X_1 = \frac{(k_2 - m_2\omega^2)F}{\left[(k_2 - m_2\omega^2)(k_1 + k_2 - m_1\omega^2)\right] - k_2^2}$$

Define $$\omega_{n1} = \sqrt{\frac{k_1}{m_1}}$$ Original natural freq. of the primary system without the absorber

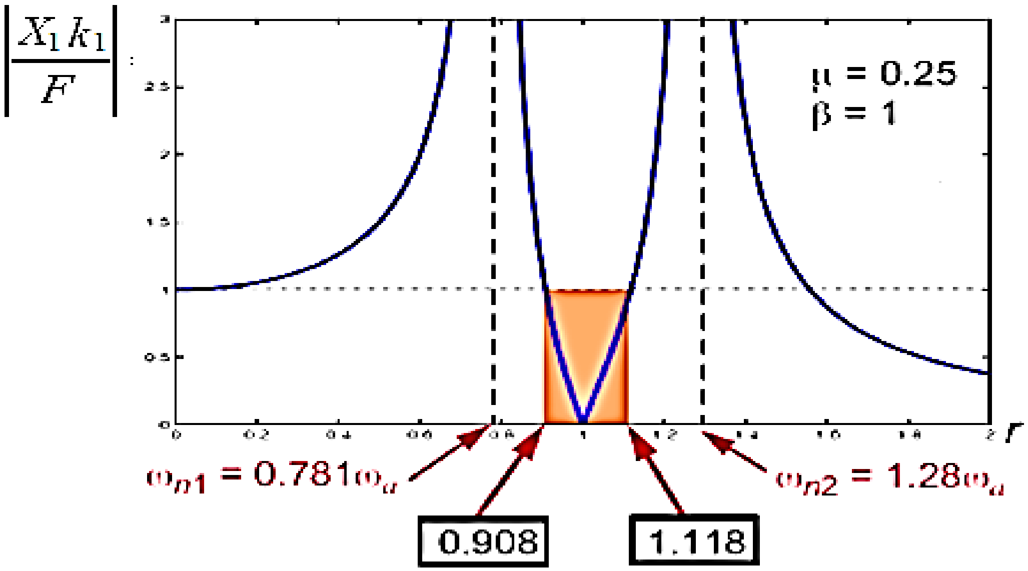
$$\omega_{n2} = \sqrt{\frac{k_2}{m_2}}$$ Original natural freq. of the absorber before it is attached to the primary system

Normalize parameters $\mu = \frac{m_2}{m_1}$ $\beta = \frac{\omega_{n2}}{\omega_{n1}}$ $r = \frac{\omega}{\omega_{n2}}$

Normalize disp. of the primary mass

$$\left|\frac{X_1 k_1}{F}\right| = \left|\frac{1 - r^2}{(1 + \mu\beta^2 - r^2)(1 - r^2) - \mu\beta^2}\right|$$

Undamped Dynamic Vibration Absorber Design

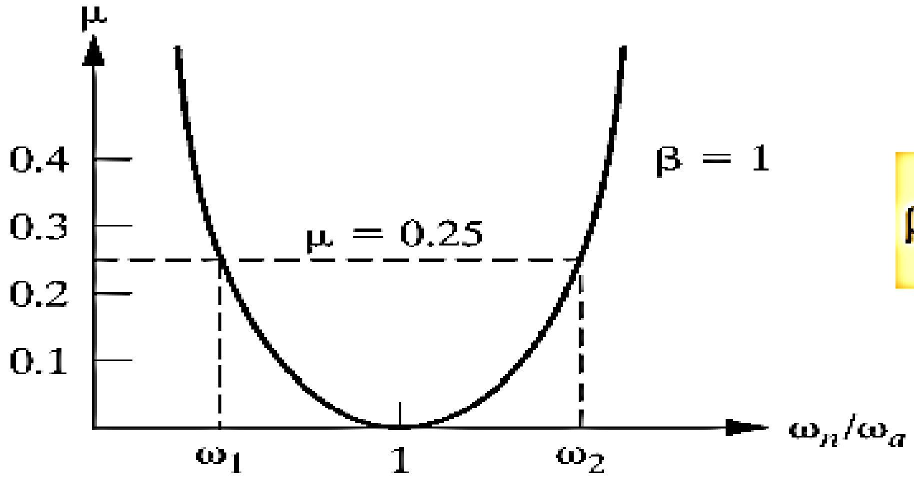


$$\left|\frac{X_1 k_1}{F}\right| = \left|\frac{1-r^2}{(1+\mu\beta^2-r^2)(1-r^2)-\mu\beta^2}\right|$$

- Shade are is the useful operating bandwidth ($0.908\omega_a < \omega < 1.118\omega_a$)
- m_a and k_a are chosen such that r is within the bandwidth
- When r = 0.781 or 1.28, the combined system will experience the resonance and fail

Undamped Dynamic Vibration Absorber Design

Bandwidth of operating frequency



$$\mu = \frac{m_a}{m} \qquad \beta = \frac{\omega_a}{\omega_p}$$

$$\beta^2\left(\frac{\omega_n^2}{\omega_a^2}\right)^2 - [1+\beta^2(1+\mu)]\left(\frac{\omega_n^2}{\omega_a^2}\right) + 1 = 0$$

- As μ is increased, ω_n split farther apart, and farther from the operating point $\omega = \omega_a$
- $0.05 < \mu < 0.25$ (recommend)
- Very large μ $\longrightarrow$ large m_a $\longrightarrow$ stress and fatigue problems

Undamped Dynamic Vibration Absorber Design

Design procedure

- Select ω which will be tuned to zero amplitude
- Relation between k_a and m_a is obtained from $\omega^2 = k_a/m_a$
- Select m_a and k_a (consider restrictions: force, motion of absorber mass)
- Check the ratio $\mu = m_a/m$ (recommended value: $0.05<\mu<0.25$)

Example 1: The Undamped Dynamic Vibration Absorber

A system has a violent resonance at 79 Hz. As a trial remedy a vibration absorber is attached which results in a resonance frequency of 65 Hz. How many such absorbers are required if no resonance is to occur between 60 and 120 Hz?

Solution:

Form Eq. (4.17)

$$\left(\frac{\omega_{n1}}{\omega_{undamp}}\right)^2 + \left(\frac{\omega_{n2}}{\omega_{undamp}}\right)^2 = 2 + \frac{m_2}{m_1}$$

and

$$\omega_{n1}\,\omega_{n2} = \omega_{undamp}^2$$

Example 1: The Undamped Dynamic Vibration Absorber

In the case of one absorber, with $\omega_{undamp} = 79$ Hz and $\omega_{n1} = 65$ Hz, Also

$$\left(\frac{65}{79}\right)^2 + \left(\frac{96}{79}\right)^2 = 2 + \frac{m_2}{m_1}$$

So $\frac{m_2}{m_1} = 0.154$

In the case of n absorbers, if

$$\omega_{n1} = 60 \text{ Hz}, \ \omega_{n2} = \frac{(79)^2}{60} = 104 \text{ Hz (too low)}.$$

So require $\omega_{n2} = 120$ Hz and then $\omega_{n1} = \frac{(79)^2}{120} = 52$ Hz. Hence

$$\left(\frac{52}{79}\right)^2 + \left(\frac{120}{79}\right)^2 = 2 + n\frac{m_2}{m_1} \Rightarrow n\frac{m_2}{m_1} = 0.74$$

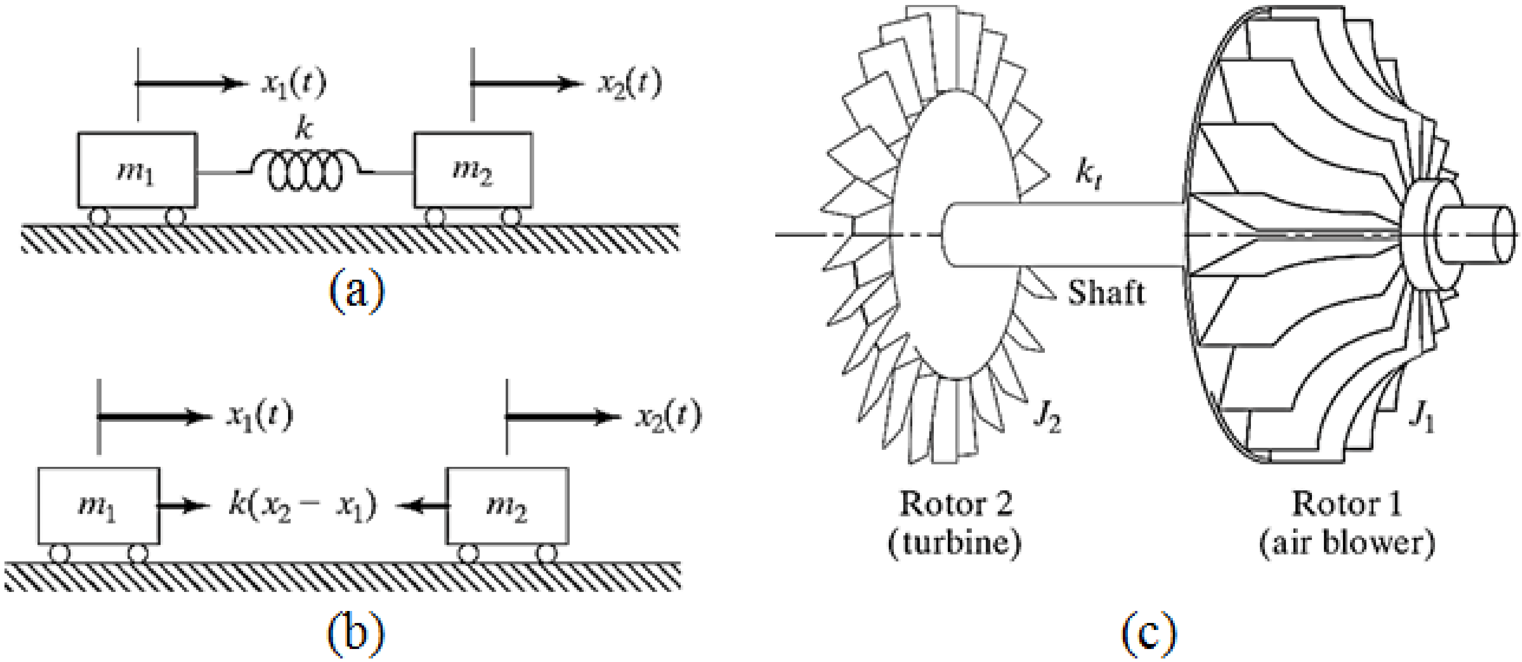
Thus

$$n = \frac{0.74}{0.154} = 4.82$$

Thus five absorbers are required.

Semi-definite Systems

A system, which is fixed from one end or both the ends, is referred as definite system. A definite system has nonzero lower natural frequency.



Semi-definite Systems

$$m_1 \ddot{x}_1 + k(x_1 - x_2) = 0$$

$$m_2 \ddot{x}_2 + k(x_2 - x_1) = 0$$

For free vibration, we assume the motion to be harmonic:

$x_1 = X_1 \sin \omega t$ and $x_2 = X_2 \sin \omega t$

Substitution of Eq. (4.20) into Eq. (4.19) gives

$$\left(-m_1 \omega^2 + k\right)X_1 - kX_2 = 0$$

$$-kX_1 + \left(-m_2 \omega^2 + k\right)X_2 = 0$$

Semi-definite Systems

we can written

$$\frac{X_1}{X_2} = \frac{k}{-m_1 \omega^2 + k}$$

$$\frac{X_1}{X_2} = \frac{-m_2 \omega^2 + k}{k}$$

$$\frac{k}{-m_1 \omega^2 + k} = \frac{-m_2 \omega^2 + k}{k}$$

Semi-definite Systems

By equating the determinant of the coefficients of X_1 and X_2 to zero, we obtain the frequency equation as

$$\omega^2\left[m_1 m_2\,\omega^2 - k\left(m_1 + m_2\right)\right] = 0$$

from which the natural frequencies can be obtained:

$$\omega_{n1} = 0 \qquad \text{and} \qquad \omega_{n2} = \sqrt{\frac{k\left(m_1 + m_2\right)}{m_1 m_2}}$$

Lagrange's Method

Lagrange's equations can be stated, for an n degree-of-freedom system, as

$$\frac{d}{dt}\left(\frac{\partial K.E}{\partial \dot{q}_i}\right)-\frac{\partial K.E}{\partial q_i}+\frac{\partial U}{\partial q_i}=Q_i, \qquad i=1,2,\ldots\ldots,\ n$$

where $\dot{q}_i = \partial q_i/\partial t$ is the generalized velocity and Q_i is the non-conservative generalized force corresponding to the generalized coordinate q_i.

Lagrange's Method

For example, if F_{xk}, F_{yk} and F_{zk} represent the external forces acting on the kth mass of the system in the x, y, and z directions, respectively, then the generalized force can be computed as follows:

$$Q_i = \sum_k \left(F_{xk} \frac{\partial x_k}{\partial q_i} + F_{yk} \frac{\partial y_k}{\partial q_i} + F_{zk} \frac{\partial z_k}{\partial q_i} \right)$$

For a conservative system, $Q_i = 0$,

$$\frac{d}{dt}\left(\frac{\partial K.E}{\partial \dot{q}_i} \right) - \frac{\partial K.E}{\partial q_i} + \frac{\partial U}{\partial q_i} = 0, \qquad i = 1,\ 2, \ldots\ldots\ldots,\ n$$


QUIZ

Quiz

Term Project

Thank you