1.8 Solved Examples

Example 1.1

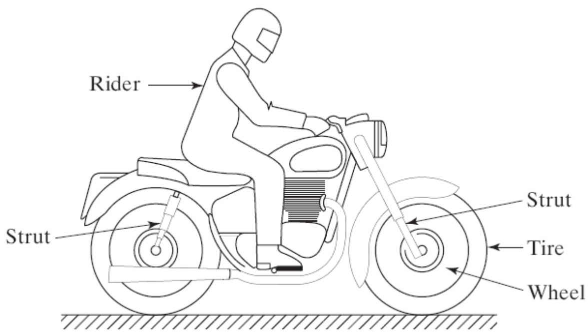
Mechanical Model of a Motorcycle

Figure (E1.1a) shows a motorcycle with a rider. Develop a sequence of three mechanical models of the system for investigating vibration in the vertical direction. Consider the elasticity of the tires, elasticity and damping of the struts (in the vertical direction), masses of the wheels, and elasticity, damping, and mass of the rider.

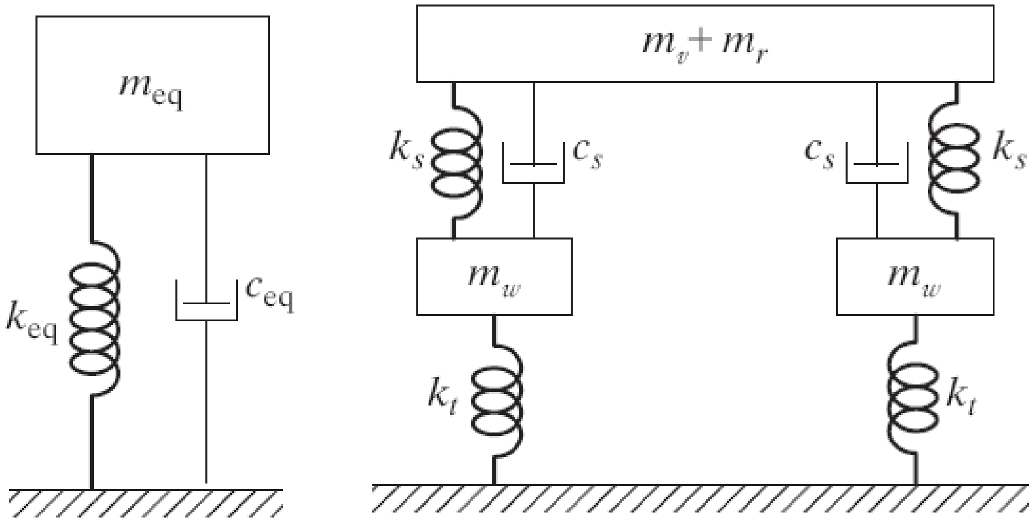
Solution:

We start with the simplest model and refine it gradually. When the equivalent values of the mass, stiffness, and damping of the system are used, we obtain a single-degree-of-freedom model of the motorcycle with a rider as indicated in Figure (E1.1b). In this model, the equivalent stiffness (k_{eq}) includes the stiffnesses of the tires, struts, and rider. The equivalent damping constant (c_{eq}) includes the damping of the struts and the rider. The equivalent mass includes the masses of the wheels, vehicle body (m_v), and the rider (m_r). This model can be refined by representing the masses of wheels, elasticity of the tires, and elasticity and damping of the struts separately, as shown in Figure (E1.1c). In this model, the mass of the vehicle body and the mass of the rider are shown as a single mass, m_v+m_r When the elasticity (as spring constant) and damping (as damping constant) of the rider are considered, the refined model shown in Figure (E1.1 d) can be obtained.

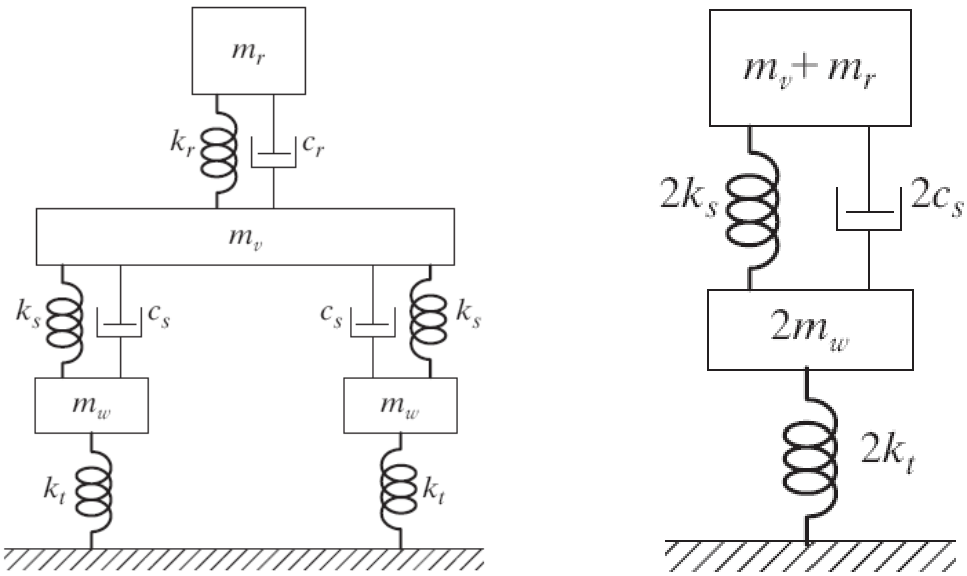
Note that the models shown in Figures (E1.1b) to (E1.1d) are not unique. For example, by combining the spring constants of both tires, the masses of both wheels, and the spring and damping constants of both struts as single quantities, the model shown in Figure (E1.1e) can be obtained instead of Figure (E1. 1c).



(a)



(b) (c)



(d) (e)

Fig. E1.1: Motorcycle with a rider a physical system and mathematical model.

Example 1.2

Mechanical Model of a Car Front Suspension

Figure (E1.3) shows a typical car suspension system. As the car travels over a rough road surface, the wheel moves up and down, following the contours of the road. This movement is transmitted to the upper and lower arms, which pivot about their inner mountings, causing the coil spring to compress and extend. The action of the spring isolates the body from the movement of the wheel, with the shock absorber or damper absorbing vibration and sudden shocks. The tie rod controls longitudinal movement of the suspension unit. Develop a sequence of three mechanical models of the system for investigating vibration in the vertical direction.

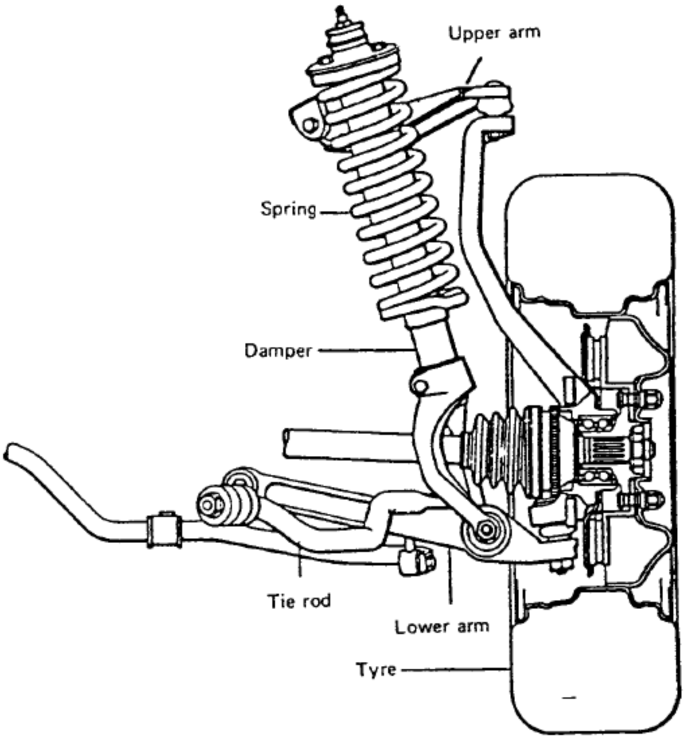


Fig. E1.2: Car front suspension

Solution:

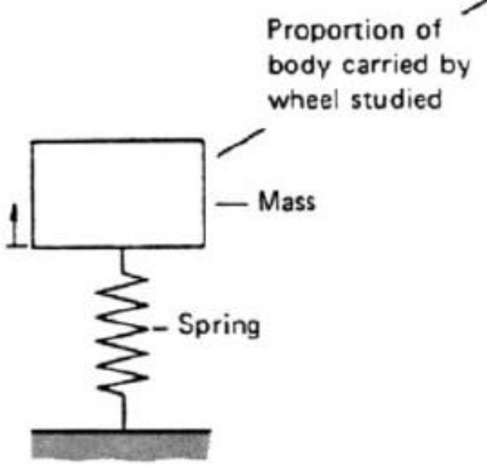


Fig. E1.3a: simplest model – motion in a vertical direction only can be

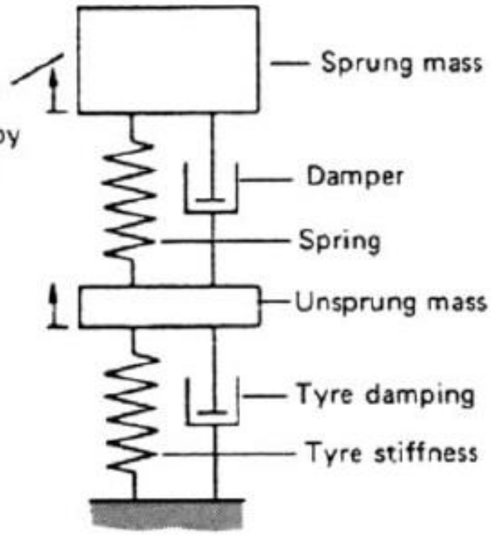


Fig. E1.3b: Motion in vertical direction only can be analyzed.

analyzed.

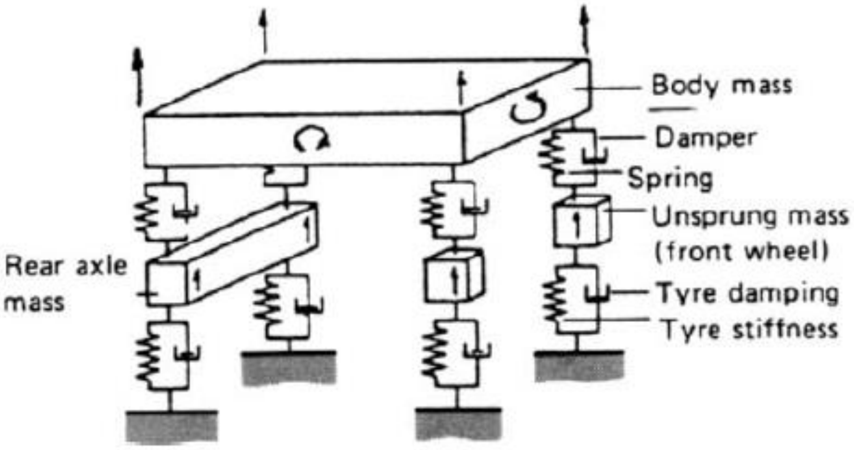


Fig. E1.3c: Motion in a vertical direction, roll, and pitch can be analyzed.

Example 1.3

Spring Constant of a Rod

Find the equivalent spring constant of a uniform rod of length *l*, cross-sectional area *A*, and Young's modulus *E* subjected to an axial tensile (or compressive) force *F* as shown in Figure (E1.4a).

Solution:

The elongation (or shortening) of the rod under the axial tensile (or compressive) force
F can be expressed as

$$\delta = \frac{\delta}{l} l = \varepsilon l = \frac{\sigma}{E} l = \frac{Fl}{Ae} \qquad \text{(E1.1)}$$

Where $\varepsilon = \dfrac{change\ in\ length}{original\ length} = \dfrac{\delta}{l}$ is the strain and $\sigma = \dfrac{force}{area} = \dfrac{F}{A}$ is the stress induced in the rod.

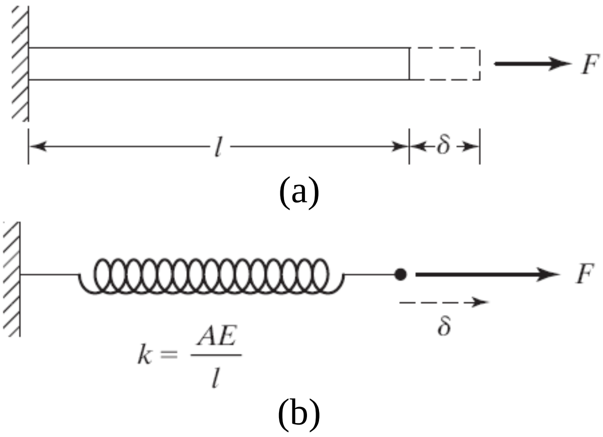


Fig. E1.4: Spring constant of a rod.

Using the definition of the spring constant *k*, we obtain from Eq. (E1.1):

$$k = \frac{force\ applied}{resulting\ deflection} = \frac{F}{\delta} = \frac{AE}{l} \quad \text{......................................} \quad \text{(E1.2)}$$

The significance of the equivalent spring constant of the rod is shown in Figure (E1.4b).

Example 1.4

Equivalent k of a Suspension System

Figure (E1.5) shows the suspension system of a freight truck with a parallel-spring arrangement. Find the equivalent spring constant of the suspension if each of the three helical springs is made of steel with a shear modulus and has five effective turns, mean coil diameter and wire diameter

Solution:

The stiffness of each helical spring is given by

$$k = \frac{d^4 G}{8D^3 n} = \frac{(0.02)^4 \left(80 \times 10^9\right)}{8(0.2)^3(5)} = 40 \times 10^3 \text{ N/m} \text{ (See Appendix for formula.)}$$

Since the three springs are identical and parallel, the equivalent spring constant of the suspension system is given

$$k_{eq} = 3k = 3\left(40 \times 10^3\right) = 120 \times 10^3 \text{ N/m}$$

Fig. E1.5: Parallel arrangement of springs in a freight truck

Example 1.5

Torsional Spring Constant of a Propeller Shaft

Determine the torsional spring constant of the steel propeller shaft shown in Figure (E1.6).

Solution:

We need to consider the segments 12 and 23 of the shaft as springs in combination. From Figure (E1.4) the torque induced at any cross section of the shaft (such as *AA* or *BB*) can be seen to be equal to the torque applied at the propeller, *T*.

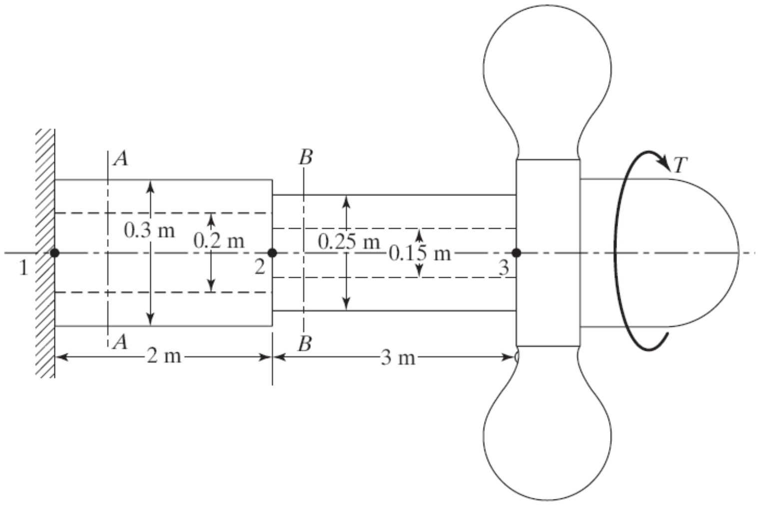


Fig. E1.6: Propeller shaft

Hence the elasticities (springs) corresponding to the two segments 12 and 23 are to be considered as series springs. The spring constants of segments 12 and 23 of the shaft (k_{t12} and k_{t23}) are given by

$$k_{t12} = \frac{GJ_{12}}{l_{12}} = \frac{G\pi\left(D_{12}^4 - d_{12}^4\right)}{32l_{12}} = \frac{\left(80\times10^9\right)\pi\left(0.3^4 - 0.2^4\right)}{32(2)} = 25.5255\times10^6 \text{ N/m}$$

$$k_{t12} = \frac{GJ_{23}}{l_{23}} = \frac{G\pi\left(D_{23}^4 - d_{23}^4\right)}{32l_{23}} = \frac{\left(80\times10^9\right)\pi\left(0.25^4 - 0.15^4\right)}{32(2)} = 8.9012\times10^6 \text{ N/m}$$

Since the springs are in series,

$$k_{teq} = \frac{k_{t12}k_{t23}}{k_{t12} + k_{t23}} = \frac{\left(8.9012\times10^6\right)\left(25.5255\times10^6\right)}{\left(8.9012\times10^6\right) + \left(25.5255\times10^6\right)} = 6.5997\times10^6 \text{ N/m}$$

1.9 Problems

1. 1.Devise a mechanical model for the windmil of Figure (P1.1).

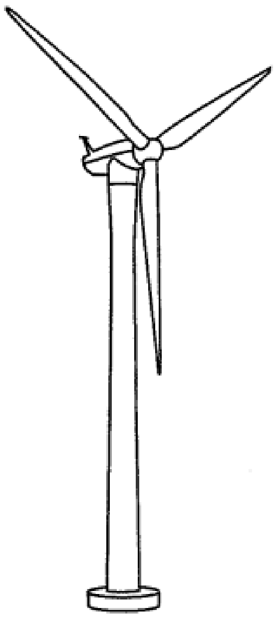


Fig. P1.1: Windmil

2. Devise two different mechanical models for the boat and propeller of Figure (P1.2).

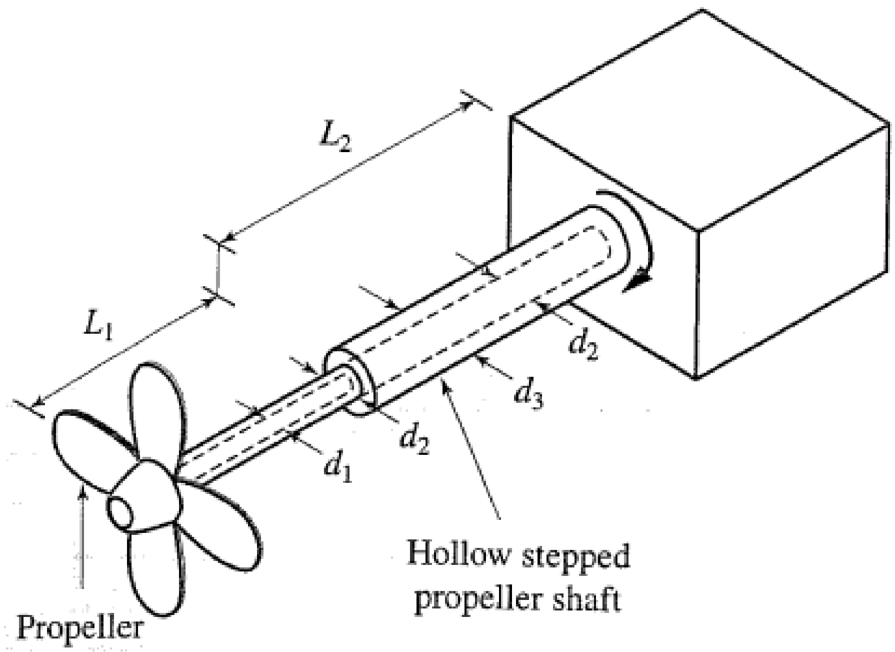


Fig. P1.2: Boat shaft and propeller.

3. Devise a mechanical models for the radar antenna of Figure (P1.3).

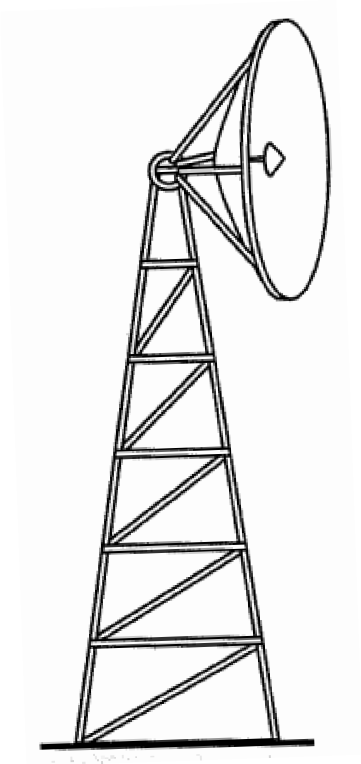


Fig. P1.3: Boat radar antenna tower.

4. Devise a mechanical models for the constraction crane of Figure (P1.4).

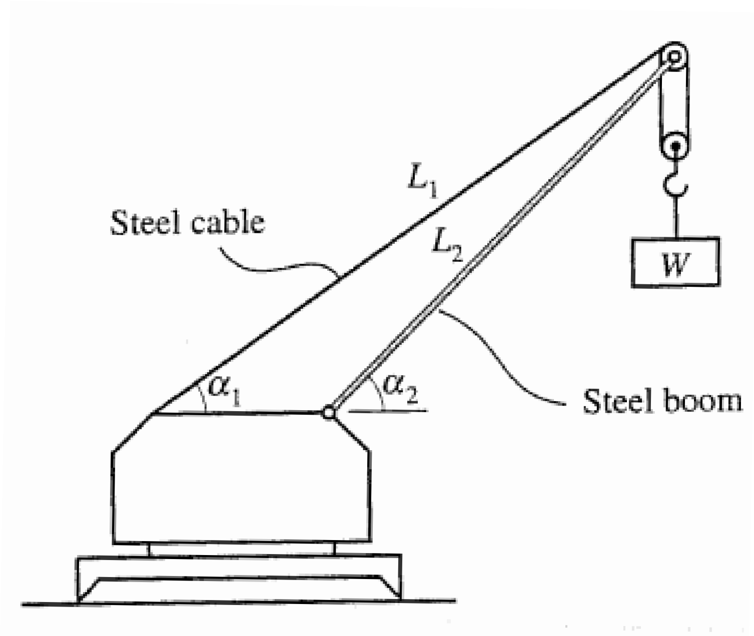


Fig. P1.4: Constraction crane.

5. A precision milling machine is supported on four shock mounts, as shown in Figure (P1.5).
   - Devise a mechanical model as a spring and a viscous damper of the elasticity and damping of each shock mount.
   - Find the equivalent spring constant, k_{eq} and the equivalent damping constant, of the machine tool support in terms of the spring constants (k_i) and damping constants (c_i) of the mounts.

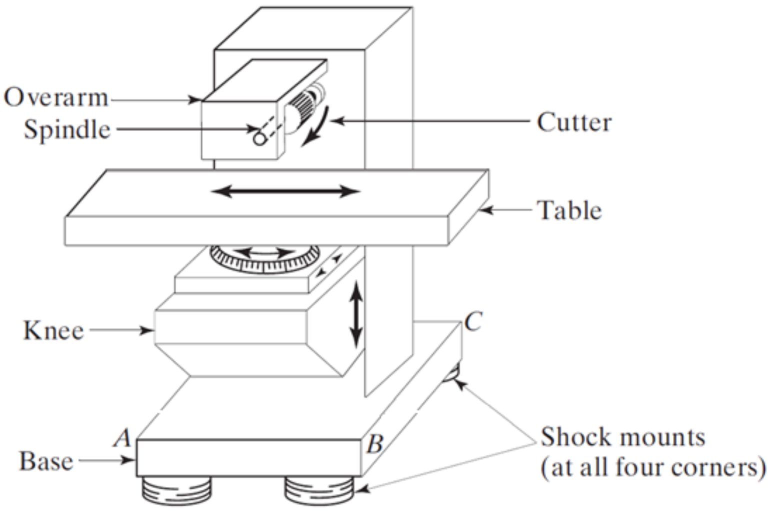


Fig. P1.5: Precision milling machine.

6. In Figure (P1.6), find the equivalent spring constant of the system in the direction of θ.

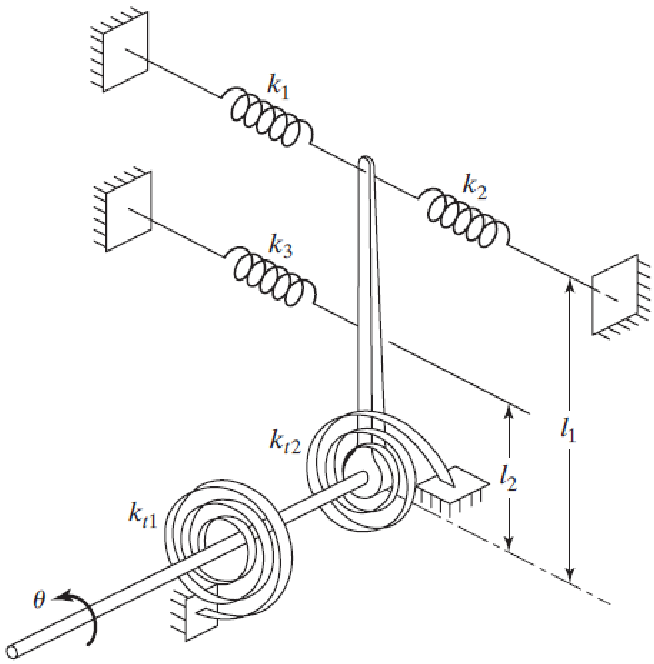


Fig. P1.6

7. Determine the equivalent spring constant of the system shown in Figure (P1.7).

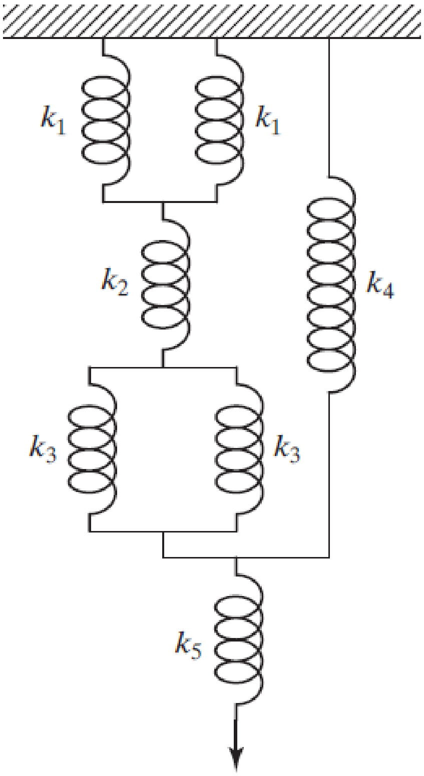


Fig. P1.7: Springs in series-parallel.

8. A tapered solid steel propeller shaft is shown in Figure (P1.8). Determine the torsional spring constant of the shaft.

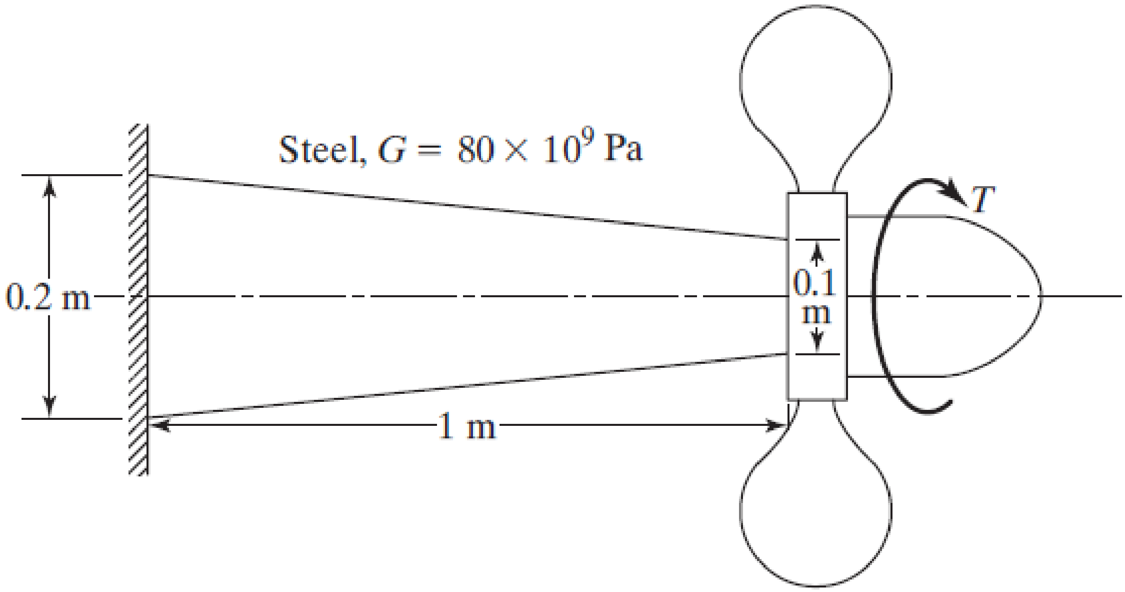


Fig. P1.8