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Chapter 3: Sound Wave

1. Speed of sound waves
2. Periodic sound Waves

- **Objectives:**

The student will be able to

- Define the speed of the sound waves
- Define the intensity of the sound wave.

Introduction:

Sound waves are the most common example of longitudinal waves. They travel through any material medium with a speed that depends on the properties of the medium.

- Sound waves are divided into **three categories** that cover different frequency ranges.

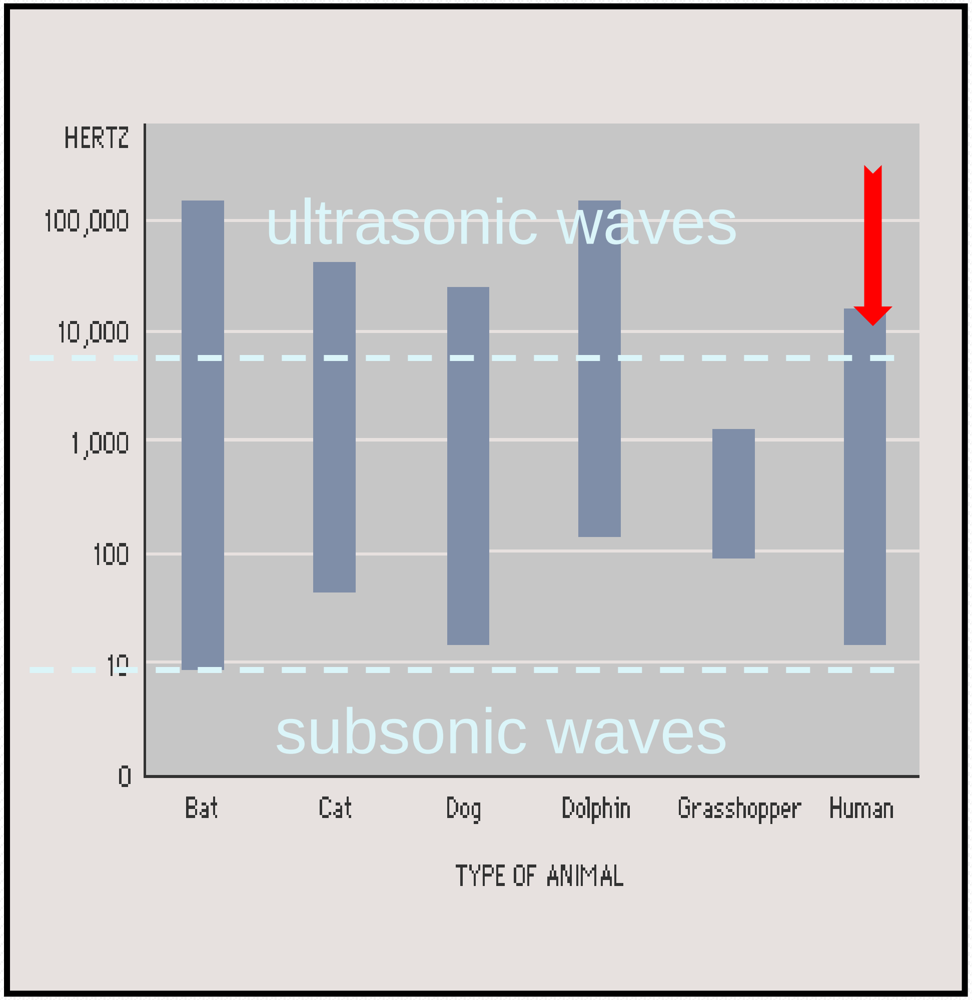
(1) ***Audible waves*** lie within the range of sensitivity of the human ear. They can be generated in a variety of ways, such as by musical instruments, human voices, or loudspeakers.

(2) ***Infrasonic waves*** have frequencies below the audible range. Elephants can use infrasonic waves to communicate with each other, even when separated by many kilometers.

(3) ***Ultrasonic waves*** have frequencies above the audible range. The ultrasonic sound it emits is easily heard by dogs, although humans cannot detect it at all. Ultrasonic waves are also used in medical imaging.

Human Hearing

Species	Range (Hz)
Turtle	20–1,000
Goldfish	100–2,000
Frog	100–3,000
Pigeon	200–10,000
Sparrow	250–12,000
Human	20–20,000
Chimpanzee	100-20,000
Rabbit	300–45,000
Dog	50–46,000
Cat	30–50,000
Guinea pig	150–50,000
Rat	1,000–60,000
Mouse	1,000–100,000
Bat	3,000–120,000
Dolphin (*Tursiops*)	1,000–130,000



3.1 Speed of Sound Waves:

The speed of sound waves in a medium depends on the compressibility and density of the medium.

If the medium is a liquid or a gas and has a bulk modulus *B* and density ρ , the speed of sound waves in that medium is

$$v = \sqrt{\frac{B}{\rho}}$$

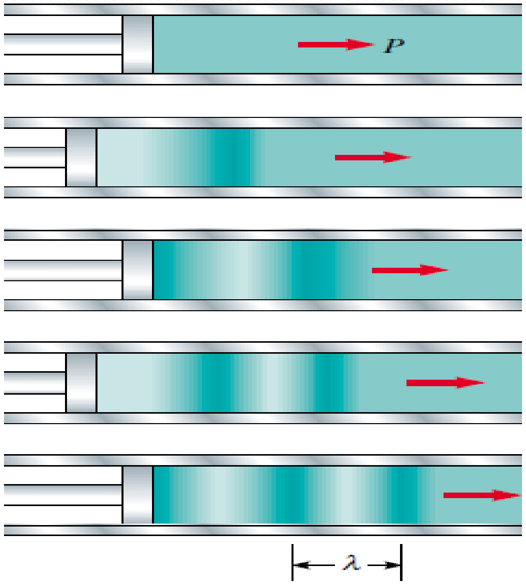
B: bulk modulus
ρ: density of the medium

$$v = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}}$$

The wave speed depends on an elastic property of the medium bulk modulus *B* or string tension *T* and on an inertial property of the medium ρ or μ. In fact, the *speed of all mechanical waves* follows an expression of the general form

3.2 Periodic Sound Waves

When the piston is pulled back, the gas in front of it expands, and the pressure and density in this region fall below their equilibrium values (represented by the lighter parts of the colored areas in Fig



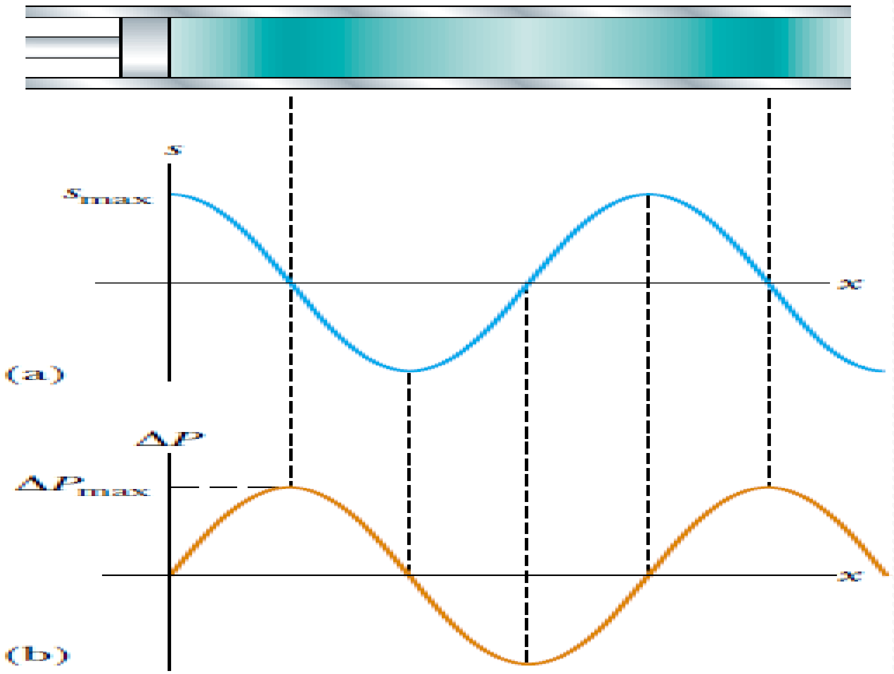


Fig.
(a) Displacement amplitude and

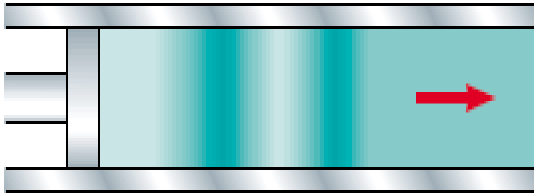
(b) pressure amplitude versus position for a sinusoidal longitudinal wave.

The distance between two successive compressions (or two successive rarefactions) equals the wavelength λ. As these regions travel through the tube, any small element of the medium moves with simple harmonic motion parallel to the direction of the wave. If $s(x, t)$ is the position of a small element relative to its equilibrium position, we can express this harmonic position function as

$$S(x,t) = S_{\max} \cos(kx - \omega t)$$

where s_{max} is the maximum position of the element relative to equilibrium. This is often called **the displacement amplitude of the wave.** The parameter k is the wave number and ω is the angular frequency of the piston

Consider a thin disk-shaped element of gas whose circular cross section is parallel to the piston as in fig. This element will undergo changes in position, pressure, and density as a sound wave propagates through the gas. From the definition of bulk modulus, the pressure variation in the gas is



$$\Delta P = -B\,\frac{\Delta V}{V_i}$$

The element has a thickness Δx in the horizontal direction and a cross-sectional area A, so its volume is $V_i = A\,\Delta x$.

The change in volume ΔV accompanying the pressure change is equal to $A\Delta s$, where Δs is the difference between the value of s at $x + \Delta x$ and the value of s at x. Hence, we can express ΔP as

$$\Delta P = -B\,\frac{\Delta V}{V_i} = -B\,\frac{A\,\Delta s}{A\,\Delta x} = -B\,\frac{\Delta s}{\Delta x}$$

$$\Delta P = -B\,\frac{\partial s}{\partial x}$$

$$S(x,t) = S_{\max}\cos(kx - \omega t)$$

$$\Delta P = -B\frac{\partial}{\partial x}[s_{\max}\cos(kx - \omega t)] = Bs_{\max}k\sin(kx - \omega t)$$

Because the bulk modulus is given by $B = \rho V^2$ and k = w/v ,
Δp can be expresses as,

$$\Delta P = \rho v \omega s_{\max}\sin(kx - \omega t)$$

$$\Delta P = \Delta P_{\max}\sin(kx - \omega t) \qquad \Delta P_{\max} = \rho v \omega s_{\max}$$

Note that the displacement of the element is along *x*, in the direction of propagation of the sound wave, which means we are describing a longitudinal wave.

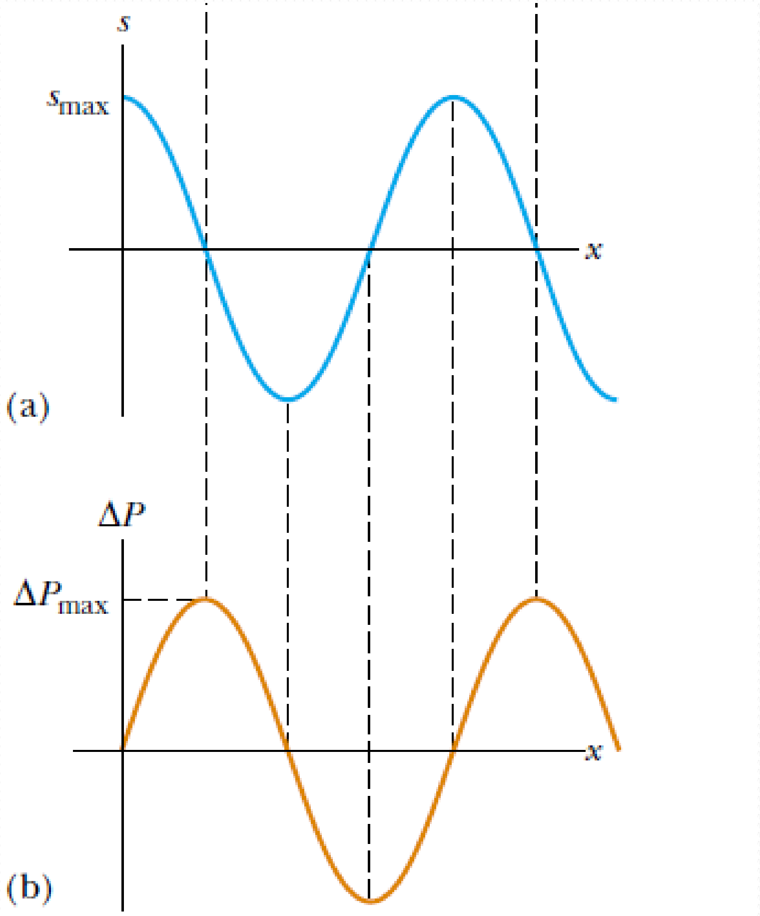
The variation in the gas pressure P measured from the equilibrium value is also periodic ΔP is given by

$$\boxed{\Delta P = \Delta P_{\max} \sin(kx - \omega t)}$$

the pressure amplitude $\Delta P_{\max}$ which is the maximum change in pressure from the equilibrium value is given by

$$\Delta P = \rho v \omega s_{max}$$

we see that a sound wave may be considered as either a displacement wave or a pressure wave .A comparison of the two waves that the pressure wave is 90° out of phase with the displacement wave.



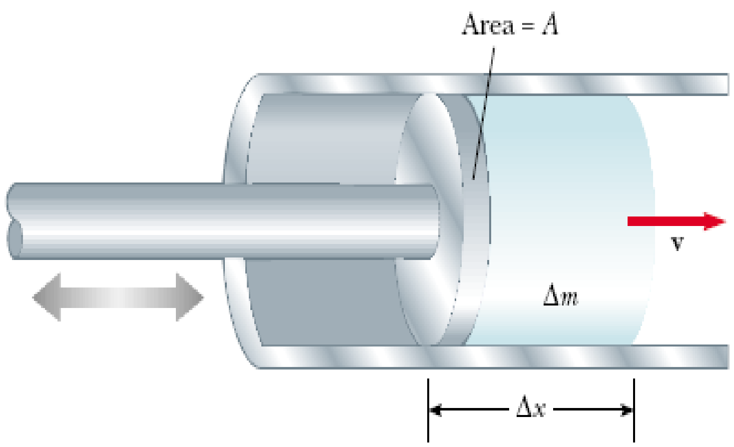
$$S(x,t) = S_{max}\cos(kx - \omega t)$$

$$\Delta P = \Delta P_{max}\sin(kx - \omega t)$$

3.3 Intensity of periodic waves:

Consider an element of air of mass Δm and width Δx in front of a piston oscillating with a frequency , as shown in Fig.

The piston transmits energy to this element of air in the tube, and the energy is propagated away from the piston by the sound wave. **To evaluate the rate of energy transfer for the sound wave, we shall evaluate the kinetic energy of this element of air, which is undergoing simple harmonic motion.**



Rate of Energy Transfer by sound Waves:

To evaluate the kinetic energy of this element of air, we need to know its speed.

$$v(x,t)=\frac{\partial}{\partial t}S(x,t)=\frac{\partial}{\partial t}\left[S_{\max}\cos\left(kx-\omega t\right)\right]=-\omega S_{\max}\sin\left(kx-\omega t\right)$$

The kinetic energy, ΔK, of the segment Δm is

$$\Delta K=\frac{1}{2}\Delta m v^2=\frac{1}{2}\Delta m(-\omega S_{\max})^2\sin^2\left(kx-\omega t\right)$$

As Δx→0, the energy ΔK becomes dK

For the wave at t=0, the kinetic energy in one wave length, λ, is

$$K_\lambda=\int dK=\frac{1}{2}\rho\omega^2 A S_m^2\int_{x=0}^{x=\lambda}\sin^2 kx\,dx=\frac{1}{2}\rho\omega^2 A S_m^2\int_{x=0}^{x=\lambda}\frac{1+\cos 2kx}{2}dx$$

$$=\frac{1}{2}\rho\omega^2 A S_m^2\left[\frac{1}{2}x+\frac{1}{4k}\sin\frac{4\pi x}{\lambda}\right]_{x=0}^{x=\lambda}=\frac{1}{4}\rho\omega^2 A S_m^2\lambda$$

Just like harmonic oscillation, the total mechanical energy in one wave length, λ, is

$$E_\lambda=U_\lambda+K_\lambda=\frac{1}{2}\rho\omega^2 S_m^2 A\lambda$$

The rate of Energy Transfer by sound Waves:

As the sound wave moves through the air, this amount of energy passes by a given point during one period of oscillation. Hence, the rate of energy transfer is

$$P = \frac{E_\lambda}{\Delta t} = \frac{E_\lambda}{T} = \frac{1}{2}\rho A(\omega S_{\max})^2 \frac{\lambda}{T}$$

$$= \frac{1}{2}\rho A v(\omega s_{\max})^2$$

The Intensity of sound Wave

We define the **intensity I** of a wave, or the **power per unit area,** to be the rate at which the energy being transported by the wave transfers through a unit area A perpendicular to the direction of travel of the wave:

$$I = P / A$$

The intensity of the sound wave is,

$$I = \frac{\mathcal{P}}{A} = \frac{1}{2}\rho v(\omega s_{\max})^2$$

Sound Level in Decibels

The human ear can detect is so wide, it was convenient to use a logarithmic scale, where the sound level $\boldsymbol{\beta}$ (Greek beta) is defined by the equation:

$$\beta = 10 \log \frac{I}{I_o}$$

I_o is the *reference intensity*, taken to be at the threshold of hearing ($I_o = 1.00 \times 10^{-12}$ W/m^2),

I is the intensity in watts per square meter to which the sound level $\boldsymbol{\beta}$ corresponds, where

$\boldsymbol{\beta}$ is measured in decibels (dB)

Sound Levels

Source of Sound	β (dB)
Nearby jet airplane	150
Jackhammer; machine gun	130
Siren; rock concert	120
Subway; power mower	100
Busy traffic	80
Vacuum cleaner	70
Normal conversation	50
Mosquito buzzing	40
Whisper	30
Rustling leaves	10
Threshold of hearing	0

Examples:

1- A sinusoidal sound wave is described by the displacement wave function

$$s(x, t) = (2.00\ \mu\text{m})\cos[(15.7\ \text{m}^{-1})x - (858\ \text{s}^{-1})t]$$

(a) Find the amplitude, wavelength, and speed of this wave. (b) Determine the instantaneous displacement from equilibrium of the elements of air at the position x = 0.050 m at t = 3.00 ms. (c) Determine the maximum speed of the element's oscillatory motion.

2- Write an expression that describes the pressure variation as a function of position and time for a sinusoidal sound wave in air, if (λ= 0.100 m and ΔP_{max} = 0.200 N/m^2.(V = 331 m/s)