Chapter 3: Sound Wave

1. Intensity of Periodic Sound Waves
2. The Doppler Effect

- **Objectives:**

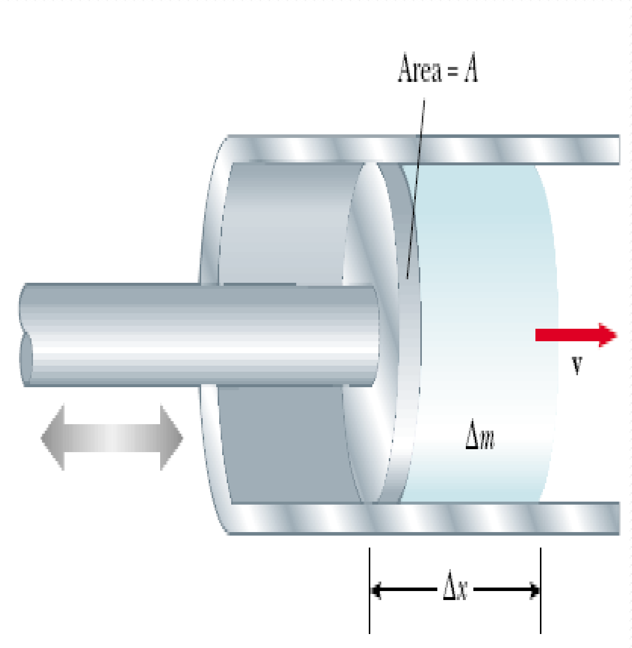
The student will be able to

- Define the intensity of the sound wave.
- Define the Doppler Effect.
- Understand some applications on sound

3.3 Intensity of periodic waves:

Consider an element of air of mass Δm and width Δx in front of a piston oscillating with a frequency f, as shown in Fig.

The piston transmits energy to this element of air in the tube, and the energy is propagated away from the piston by the sound wave. To evaluate the rate of energy transfer for the sound wave, we shall evaluate the kinetic energy of this element of air, which is undergoing simple harmonic motion.



Rate of Energy Transfer by sound Waves:

To evaluate the kinetic energy of this element of air, we need to know its speed.

$$v(x,t)=\frac{\partial}{\partial t}S(x,t)=\frac{\partial}{\partial t}[S_{\max}\cos(kx-\omega t)]=-\omega S_{\max}\sin(kx-\omega t)$$

The kinetic energy, ΔK, of the segment Δm is $\Delta K=\frac{1}{2}\Delta m v^2=\frac{1}{2}\Delta m(-\omega S_{\max})^2\sin^2(kx-\omega t)$

As $\Delta x \rightarrow 0$, the energy ΔK becomes dK

For the wave at t=0, the kinetic energy in one wave length, λ, is

$$K_\lambda=\int dK=\frac{1}{2}\rho\omega^2 AS_m^2\int_{x=0}^{x=\lambda}\sin^2 kx dx=\frac{1}{2}\rho\omega^2 AS_m^2\int_{x=0}^{x=\lambda}\frac{1+\cos 2kx}{2}dx$$

$$=\frac{1}{2}\rho\omega^2 AS_m^2\left[\frac{1}{2}x+\frac{1}{4k}\sin\frac{4\pi x}{\lambda}\right]_{x=0}^{x=\lambda}=\frac{1}{4}\rho\omega^2 AS_m^2\lambda$$

Just like harmonic oscillation, the total mechanical energy in one wave length, λ, is

$$E_\lambda=U_\lambda+K_\lambda=\frac{1}{2}\rho\omega^2 S_m^2 A\lambda$$

Rate of Energy Transfer by sound Waves:

As the sound wave moves through the air, this amount of energy passes by a given point during one period of oscillation. Hence, the rate of energy transfer is

$$P = \frac{E_\lambda}{\Delta t} = \frac{E_\lambda}{T} = \frac{1}{2}\rho A(\omega S_{\max})^2 \frac{\lambda}{T}$$

$$= \frac{1}{2}\rho A v(\omega s_{\max})^2$$

We define the intensity I of a wave, or the power per unit area, to be the rate at which the energy being transported by the wave transfers through a unit area A perpendicular to the direction of travel of the wave: $I = P / A$

Sound Level in Decibels

The human ear can detect is so wide, it was convenient to use a logarithmic scale, where the sound level $\boldsymbol{\beta}$ (Greek beta) is defined by the equation:

$$\beta = 10 \log \frac{I}{I_o}$$

I_o is the *reference intensity*, taken to be at the threshold of hearing ($I_o = 1.00 \times 10^{-12}$ W/m^2),

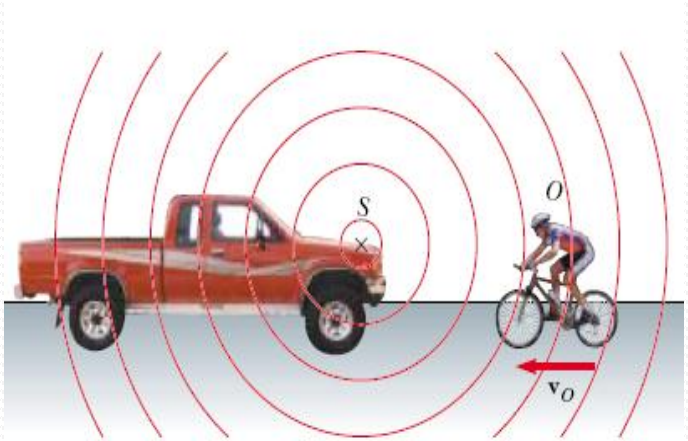
I is the intensity in watts per square meter to which the sound level $\boldsymbol{\beta}$ corresponds, where $\boldsymbol{\beta}$ is measured in decibels (dB)

Sound Levels

Source of Sound	β (dB)
Nearby jet airplane	150
Jackhammer; machine gun	130
Siren; rock concert	120
Subway; power mower	100
Busy traffic	80
Vacuum cleaner	70
Normal conversation	50
Mosquito buzzing	40
Whisper	30
Rustling leaves	10
Threshold of hearing	0

3.4 The Doppler Effect:

If a point source emits sound waves and the medium is uniform, the waves move at the same speed in all directions radially away from the source; this is a spherical wave.
It is useful to represent these waves with a series of circular arcs concentric with the source, as in Fig, each arc represents a surface over which the phase of the wave is constant.
For example, the surface could pass through the crests of all waves. We call such a surface of constant phase a wave front.
The distance between adjacent wave fronts equals the wavelength.

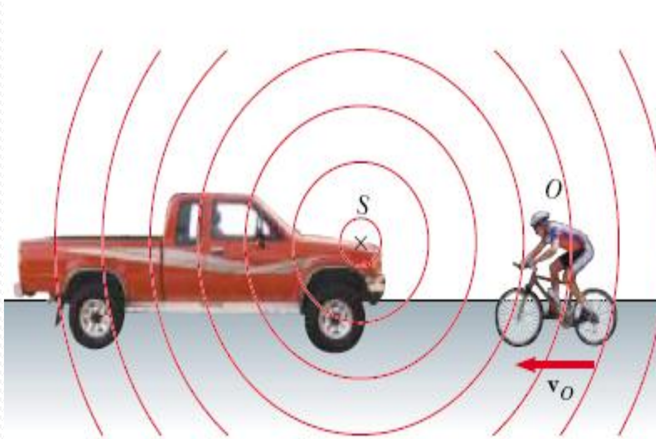


1- When the observer moves toward the source, the speed of the waves relative to the observer is

$v` = v_o + v$

as in the case of the boat, but the wavelength

λ is unchanged.



Hence, $v = \lambda f$, we can say that

the frequency f ` heard by the observer is

increased and is given by

$$f' = \frac{v'}{\lambda} = \frac{v + v_O}{\lambda}$$

Because $\lambda = v / f$, we can express f' as

$$f' = \frac{v + v_O}{v} f$$ (observer moving toward source)

2- When the observer is moving away from the source,

the speed of the wave relative to the observer is $v' = v - v_o$. The frequency heard by the observer in this case is decreased and is given by

$$f' = \frac{v'}{\lambda} = \frac{v - v_0}{\lambda}$$

And $\lambda = v / f$, we can express f' as

$$f' = \frac{v - v_0}{v} f$$

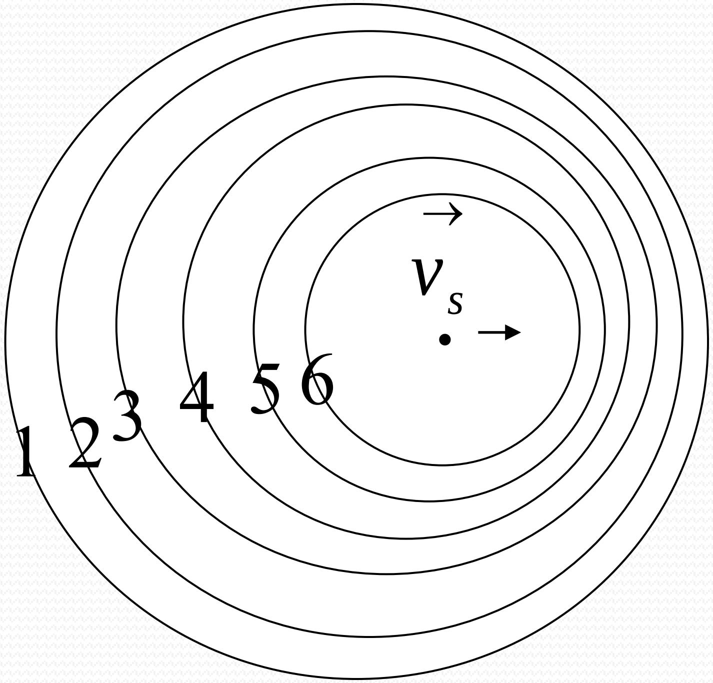
(observer moving away from source)

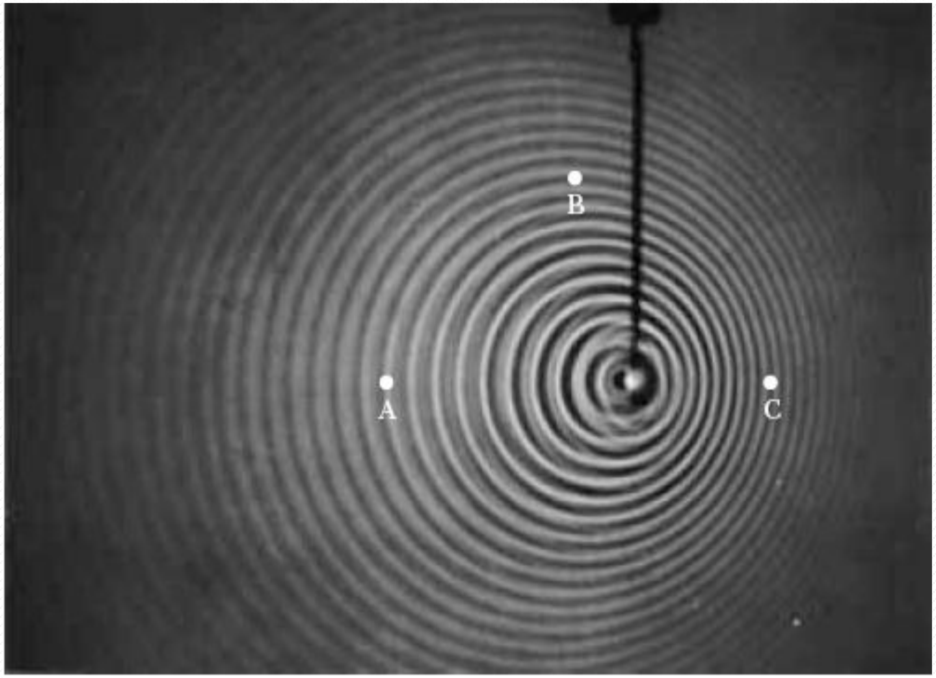
In general, whenever an **observer moves with a speed v_o** relative to a stationary source, the frequency heard by the observer is given by Equation,

$$f' = \frac{v \pm v_o}{v} f$$

with a sign convention: **a positive value is substituted for v_o when the observer moves toward the source and a negative value is substituted when the observer moves away from the source.**

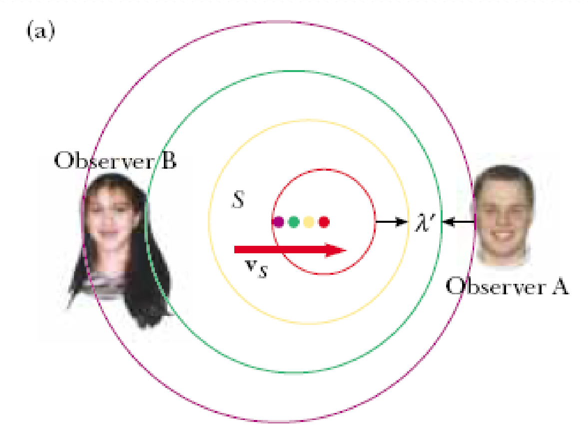
When the source moving:





3- When the source moves directly toward observer :

The wave fronts heard by the observer are closer together than they would be if the source were not moving. As a result, the wavelength **λ`** measured by observer A is shorter than the wavelength λ of the source. During each vibration, which lasts for a time interval T (the period), the source moves a distance $v_sT = v_s / f$ and the wavelength is shortened by this amount. Therefore, the observed wavelength λ` is



$$\lambda' = \lambda - \Delta\lambda = \lambda - v_s / f$$

The frequency *f*` heard by observer A is

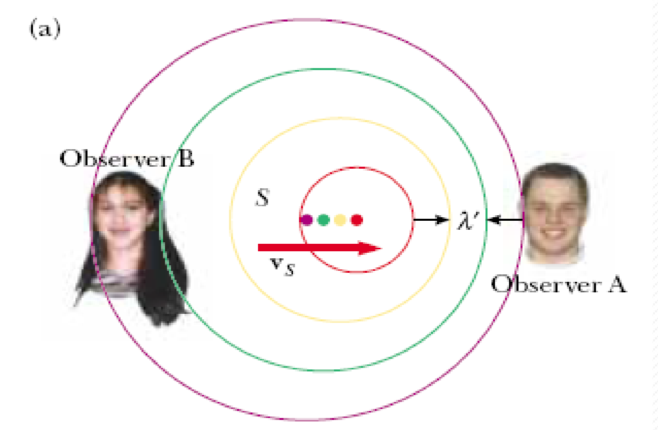
$$f' = \frac{v}{\lambda'} = \frac{v}{\lambda - v_s / f} = \frac{v}{v / f - v_s / f}$$

$$f' == \frac{v}{v - v_s} f$$

4- When the source moves away from a stationar observer,

as is the case for observer B in Fig,

the observer measures a wavelength λ` that i greater than λ and hears a decreased frequency:



$$f' == \frac{v}{v + v_s} f$$

If both source and observer move through the transmitting medium,

Where the upper signs (+ numerator, -denominator) correspond to the source and observer moving toward the other and the lower signs in the direction away from the other.

$$f' == \frac{v \pm v_o}{v \mp v_s} f$$

Example 1:

The siren of a police car emits a pure tone at a frequency of 1125 Hz. Find the frequency that you would perceive in your car.

(a) your car at rest, police car moving toward you at 29 m/s;

(b) police car at rest your moving toward it at 29 m/s

(c) you and police car moving toward one another at 14.5 m/s

(d) you moving at 9 m/s, police car chasing behind you at 38 m/s

The velocity of sound in air is 343 m/s.

Solution: Using

Here (a) $v_0 = 0 \quad v_s = 29m/s$

$$f' = \frac{v}{v - v_s} f = \left(\frac{343}{343 - 29}\right) f = 1229Hz$$

(b) $v_s = 0 \qquad v_0 = 29m/s$

$$f' = \frac{v + v_0}{v} f = \frac{343 + 29}{343}(1125Hz) = 1220Hz$$

(c) $v_s = v_0 = 14.5m/s$

$$f' = \frac{v + v_0}{v - v_s} f = \frac{343 + 14.5}{343 - 14.5} \times 1125 = 1224Hz$$

(d) $v_0 = +9m/s \qquad v_s = -38m/s$

$$f' = \frac{343 - 9}{343 - 38} f = 11231.9Hz$$

Example 2

A stationary civil defense siren has a frequency of 1000 Hz. What frequency will be heard by drivers of cars moving at 15 m/s?

A) away from the siren?

B) toward the siren?

The velocity of sound in air is 344 m/s.

Solution: Moving away from the siren

$$f_d = f_s \frac{v - v_d}{v} =$$

$$f_d = (1000\ \text{Hz}) \frac{(344\ \text{m/s} - 15\ \text{m/s})}{344\ \text{m/s}}$$

$f_d = 956$ Hz

Apparent frequency heard by the detector decreases

Example 3

A police car with a 1000 Hz siren is moving at 15 m/s. What frequency is heard by a stationary listener when the police car is

a) receding from the detector?

b) b) approaching the detector?

$$f_d = f_s \left(\frac{v}{v + v_s} \right)$$

$$f_d = (1000\ \text{Hz}) \left(\frac{344\ \text{m/s}}{344\ \text{m/s} + 15\ \text{m/s}} \right) = 958\ \text{Hz}$$

P. car moving away

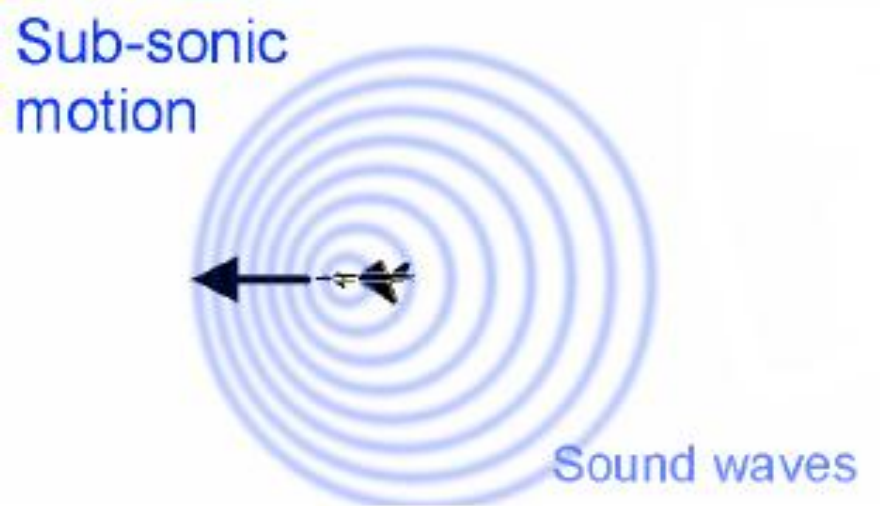
$$f_d = f_s \left(\frac{v}{v - v_s} \right)$$

$$f_d = (1000\ \text{Hz}) \left(\frac{344\ \text{m/s}}{344\ \text{m/s} - 15\ \text{m/s}} \right) = 1046\ \text{Hz}$$

P. car coming toward

The speed of sound

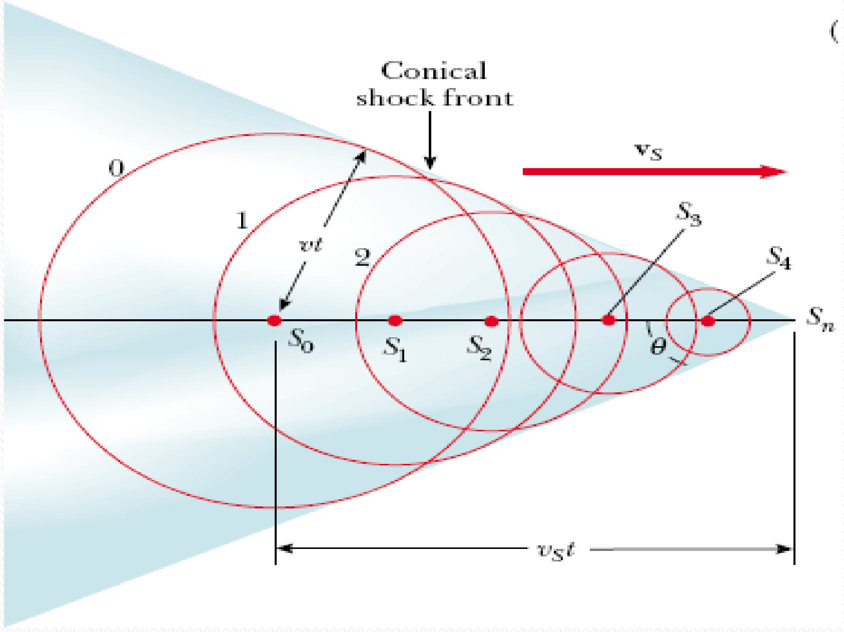
- The speed of sound in air is 343 meters per second (660 miles per hour) at one atmosphere of pressure and room temperature (21°C).
- An object is subsonic when it is moving slower than sound.



Shock Waves

What happens when the speed v_s of a source *exceeds* the wave speed v. This situation is depicted graphically in Figure. The circles represent spherical wave fronts emitted by the source at various times during its motion.

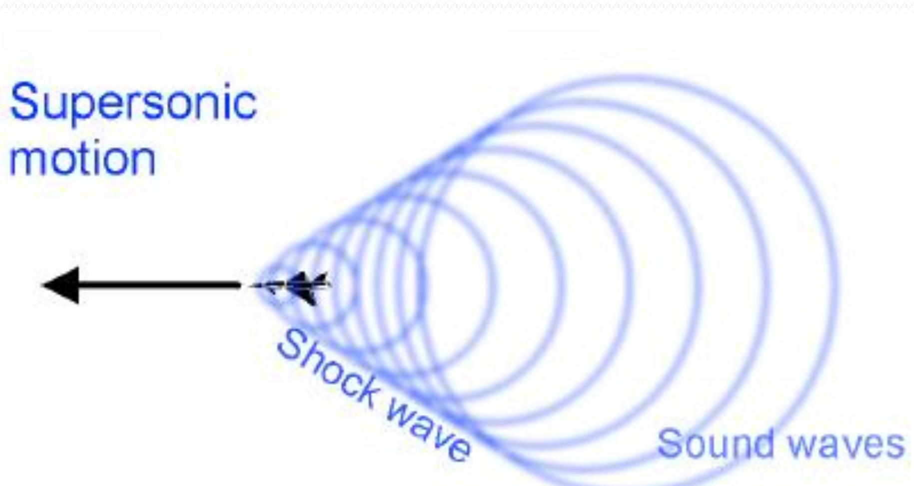
At *t* = 0, the source is at S_o, and at a later time t, the source is at S_n. At the time t, the wave front centered at S_o reaches a radius of vt.

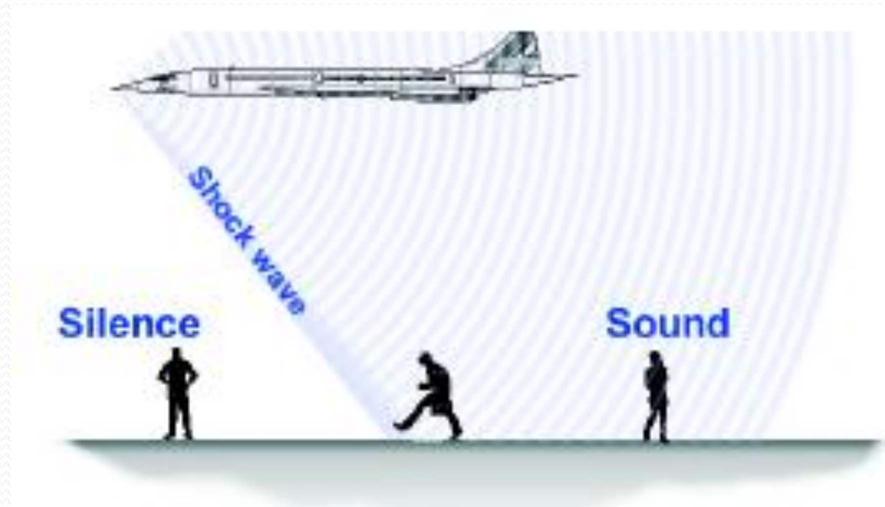


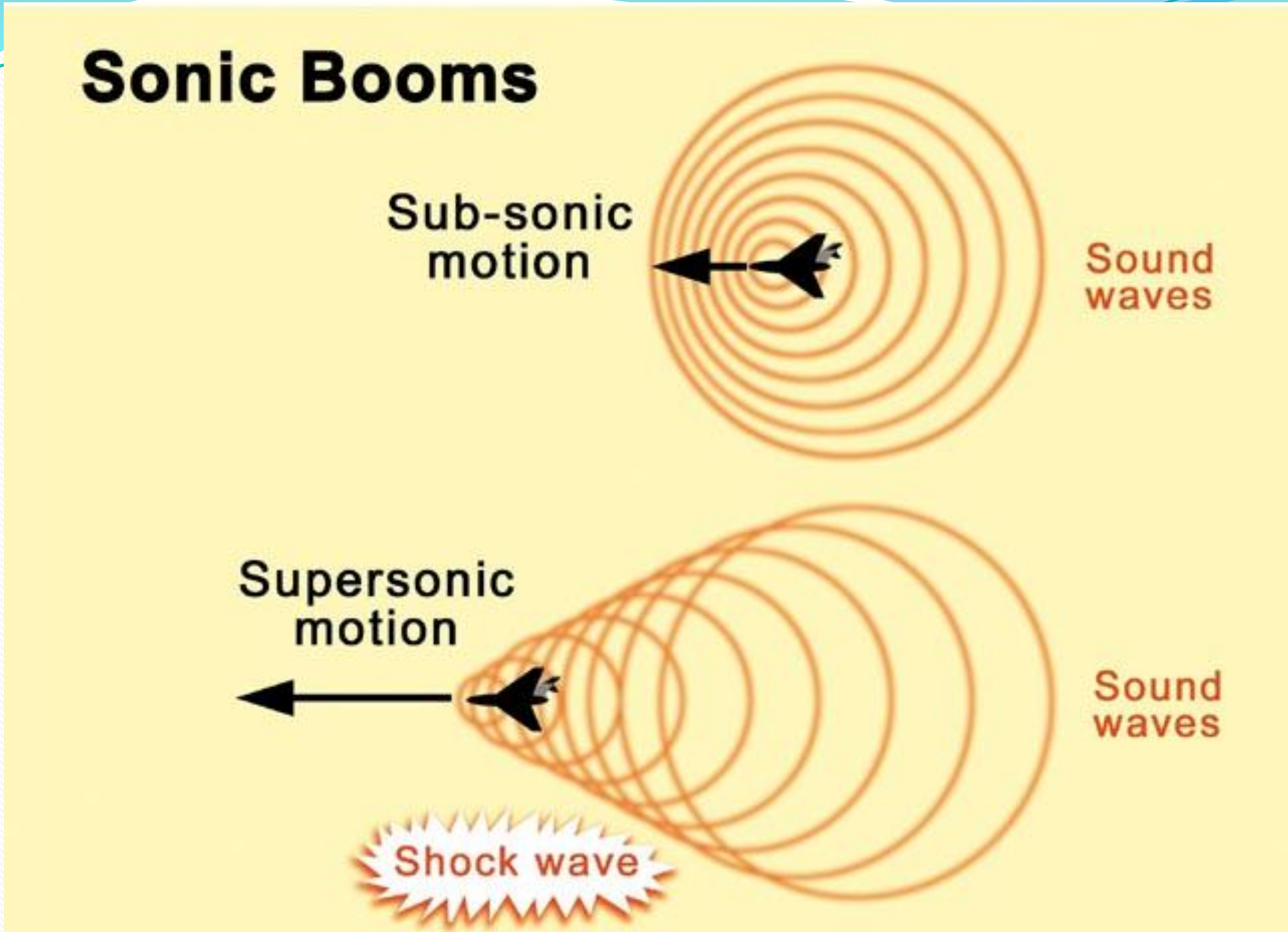
- Jet airplanes traveling at supersonic speeds produce shock waves, which are responsible for the loud “sonic boom” one hears.
- The shock wave carries a great deal of energy concentrated on the surface of the cone, with correspondingly great pressure variations. Such shock waves are unpleasant to hear and can cause damage to buildings when aircraft fly supersonically at low altitudes.
- In fact, an airplane flying at supersonic speeds produces a double boom because two shock waves are formed, one from the nose of the plane and one from the tail.
- People near the path of the space shuttle as it glides toward its landing point often report hearing what sounds like two very closely spaced cracks of thunder.

The speed of sound

- We use the term supersonic to describe motion at speeds faster than the speed of sound.
- A shock wave forms where the wave fronts pile up.
- The pressure change across the shock wave is what causes a very loud sound known as a sonic boom.






Sonic Booms
Sub-sonic motion
Sound waves
Supersonic motion
Sound waves
Shock wave