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Faculty of Computers and Information

Fayoum University

Chapter 4 - Superposition and Standing Waves:

- Superposition and Interference:
- Interference of Sound Waves:
- Standing Waves:
- Standing Waves in String Fixed at Both Ends:
- Resonance:
- standing Waves in Air Columns:

Useful web site

http://www.acs.psu.edu/drussell/Demos/superposition/superposition.html

http://phet.colorado.edu/en/contributions/view/2838

*Objectives:

The Student will be able to:

-

Define the superposition & the interference.

Define the types of the interference.

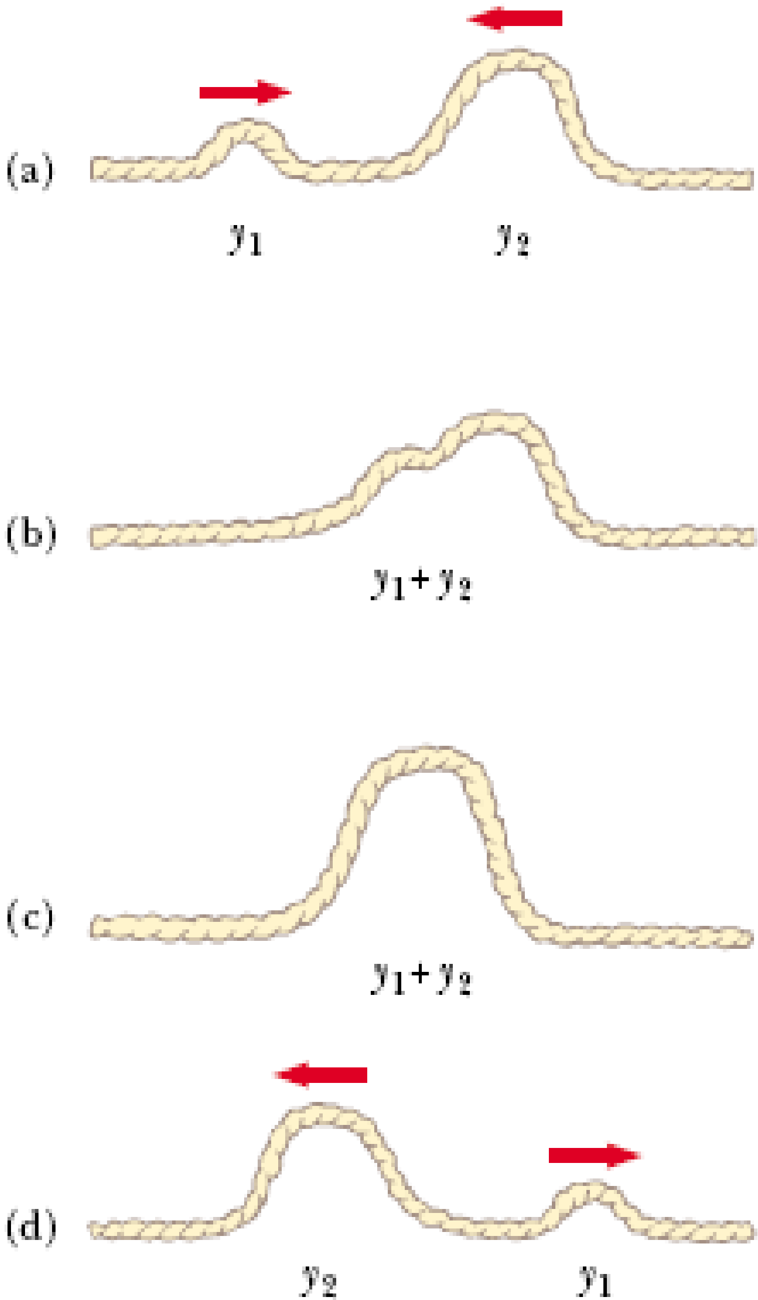
Key words:

Standing wave: superposition of two identical waves propagating in opposite directions.

Nodes : the points of zero amplitude,

Antinodes: the points of max amplitudes, where constructive interference is greatest.

Loops: the regions of greatest amplitude in a standing wave



a) Two plus traveling on a stretched string in opposite direction.

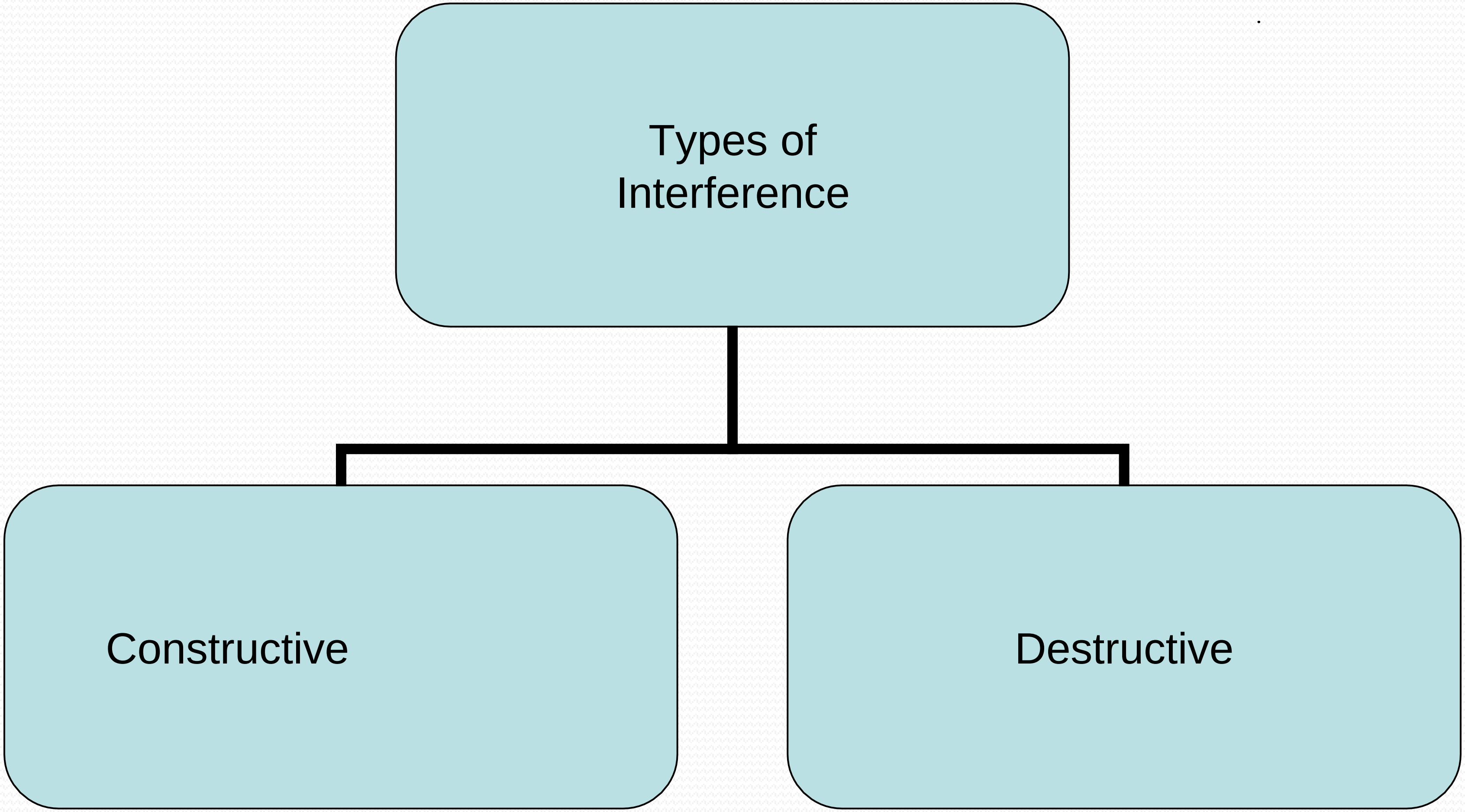
b) The pulses have the same speed but different shapes, and the displacement of the elements of the medium is in the opposite direction. When the waves go to overlap.

c) The net displacement of the string equals the sum of the displacements of the string.

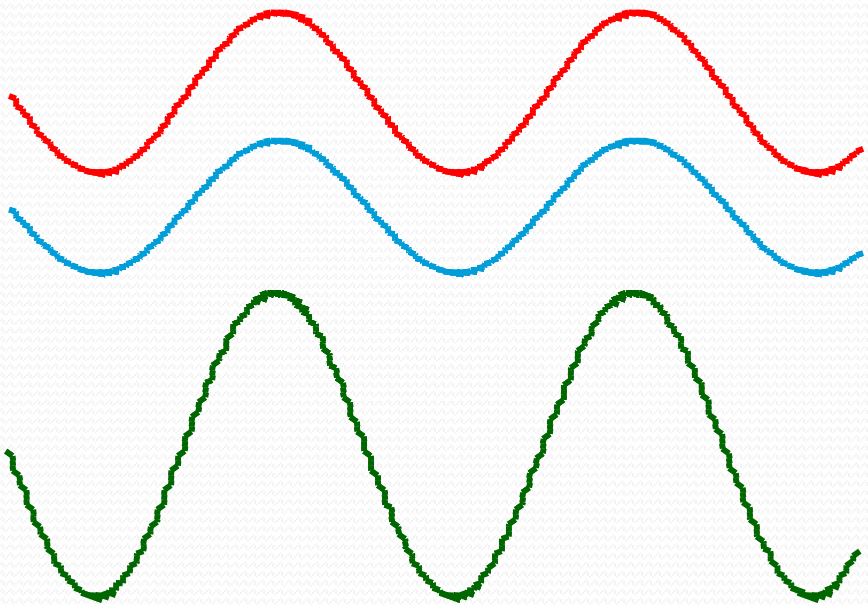
d) Finally, the two pulses separate and continue moving in their original direction

Interference:

Defined as the combination of separate waves in the region of space, and the resultant pulse has amplitude greater than that of their individual pulse.

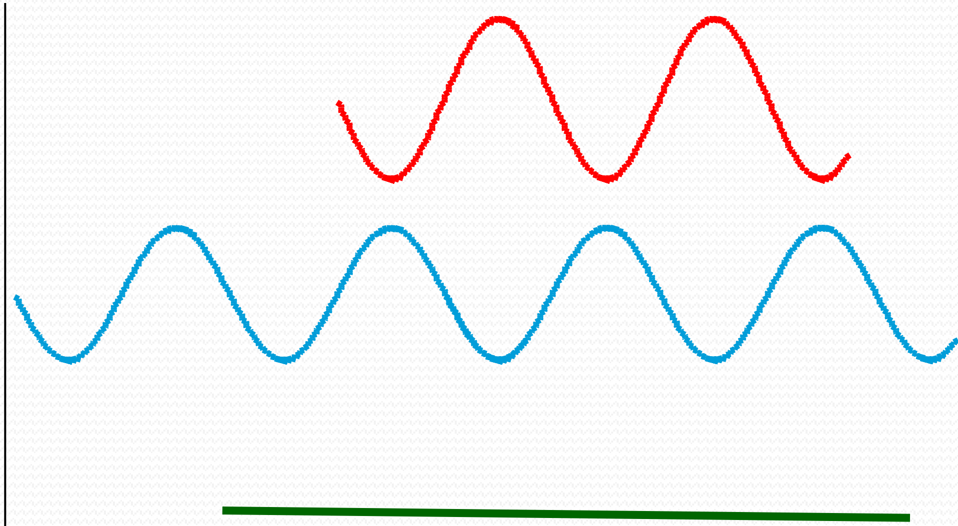


Constructive & Destructive Interference



Constructive Interference

Waves are "**in phase**." By super-position, red + blue = green. If red and blue each have amplitude A, then green has amplitude 2A.



Destructive Interference

Waves are "**out of phase**." By superposition, red and blue completely cancel each other out, if their amplitudes and frequencies are the same.

Wave Interference

Constructive interference occurs at a point when two waves

have displacements in the same direction. The amplitude of the combo wave **is larger** either individual wave.

Destructive interference occurs at a point when two waves have displacements in opposite directions. The amplitude of the combo wave **is smaller** than that of the wave biggest wave.

Superposition **can involve** both constructive and destructive interference at the same time (but at different points in the medium).

Superposition of Sinusoidal Waves

- Assume ***two waves*** are traveling in the **same direction**, with the **same frequency**, **wavelength** and **amplitude**

The waves differ in phase ϕ

Where : **sin a +sin b = 2 cos [(a-b)/2] sin [(a+b)/2]**

$$y_1 = A \sin(kx - \omega t) \quad \& \quad y_2 = A \sin(kx - \omega t + \phi) \Rightarrow$$

$$y = y_1 + y_2 = \{2A \cos(\phi/2)\} \sin(kx - \omega t + \phi/2)$$

- The **resultant wave function**, *y*, is also ***sinusoidal***
- The ***resultant wave*** has the **same frequency** and **wavelength** as the original waves
- ***The amplitude*** of the resultant wave is $2A \cos(\phi/2)$
- **The phase** of the resultant wave is $\phi/2$

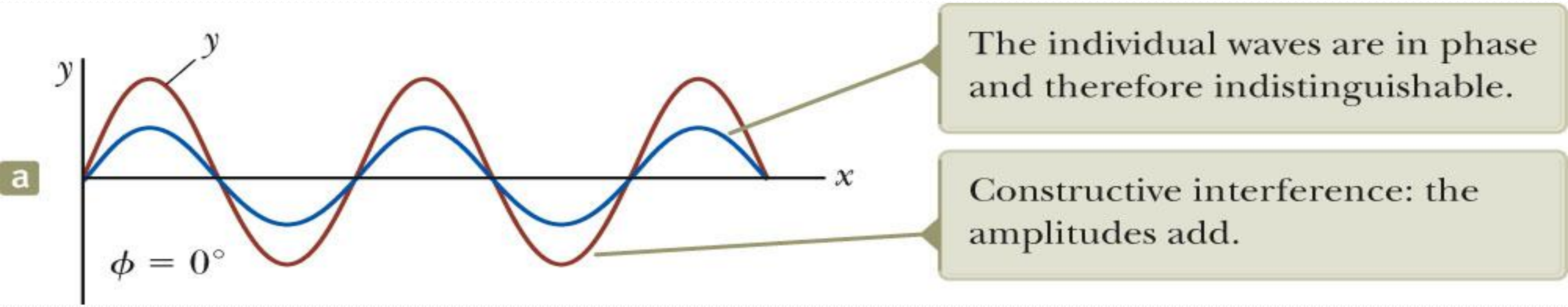
Sinusoidal Waves with Constructive Interference

- If ϕ = ***0, 2π, 4π,… (even multiple of π)***, **then: cos(ϕ/2) = ±1** $\Rightarrow$

$$y = \pm 2A(1)\sin(kx - \omega t + 0/2) \Rightarrow$$

$$y = \pm 2A\sin(kx - \omega t)$$

- ***The amplitude*** of the **resultant wave** is **± 2*A***
  - The crests of one wave coincide with the crests of the other wave
- The waves are everywhere ***in phase***
- The waves ***interfere constructively***



Sinusoidal Waves with Destructive Interference:

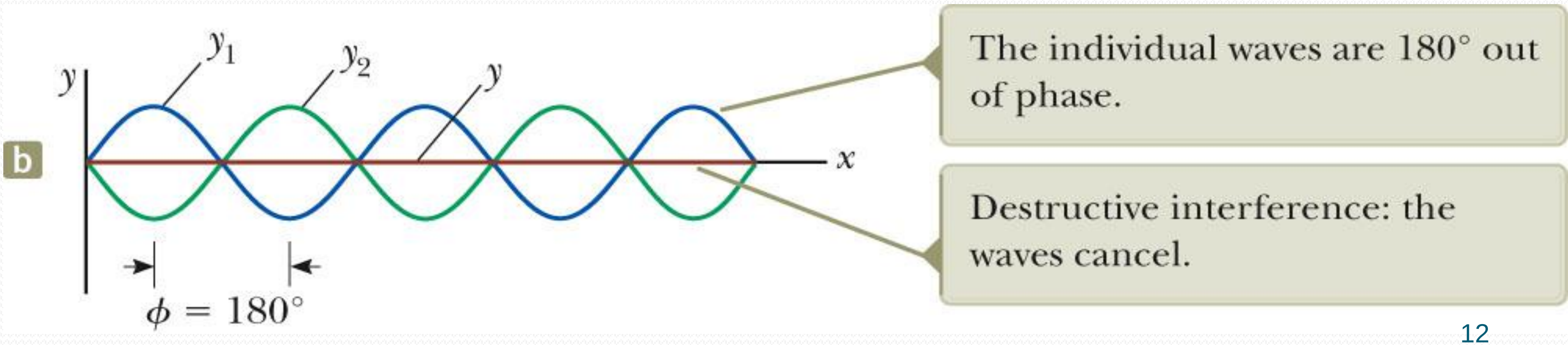
- **If** $\phi = \pi, 3\pi, 5\pi, \ldots$ ***(odd multiple of*** π***), then:*** $\cos(\pi/2) = 0 \Rightarrow$

$$y = 2A(0)\sin(kx - \omega t + \pi/2) \Rightarrow$$

$$y = 0\cos(kx - \omega t)\sin(\pi/2)$$

- ***The amplitude*** of the **resultant wave** is **0**
  - Crests of one wave coincide with troughs of the other wave

- The waves ***interfere destructively***

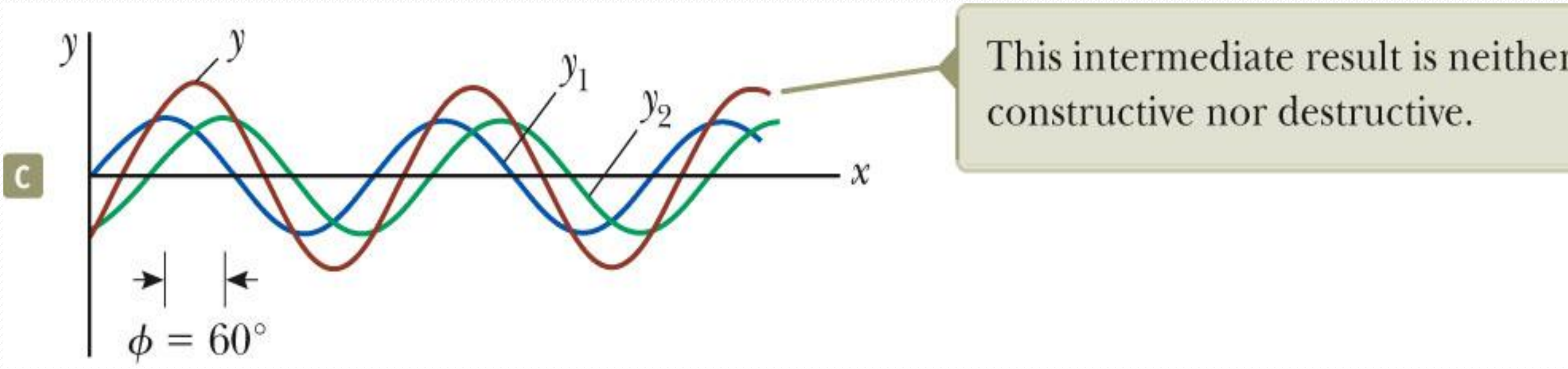


Sinusoidal Waves, General Interference

When ϕ is other than 0 or an even multiple of π, the amplitude of the resultant is between 0 and $2A$.

The wave functions still add

The interference is **neither constructive nor destructive**.



Summary of Interference

- **Constructive interference occurs when $\phi = 0$**
  - **Amplitude of the resultant is 2*A***
- **Destructive interference occurs when $\phi = n\pi$ where *n* is an odd integer**
  - **Amplitude is 0**
- **General interference occurs when $0 < \phi < n\pi$**
  - **Amplitude is $0 < A_{resultant} < 2A$**

Standing Waves

Assume two waves with the same amplitude, frequency and wavelength, **traveling in opposite directions in a medium**.

The waves combine in accordance with the *waves in interference model.*

$$y_1 = A \sin(kx - \omega t) \text{ and}$$

$$y_2 = A \sin(kx + \omega t)$$

They interfere according to the superposition principle.

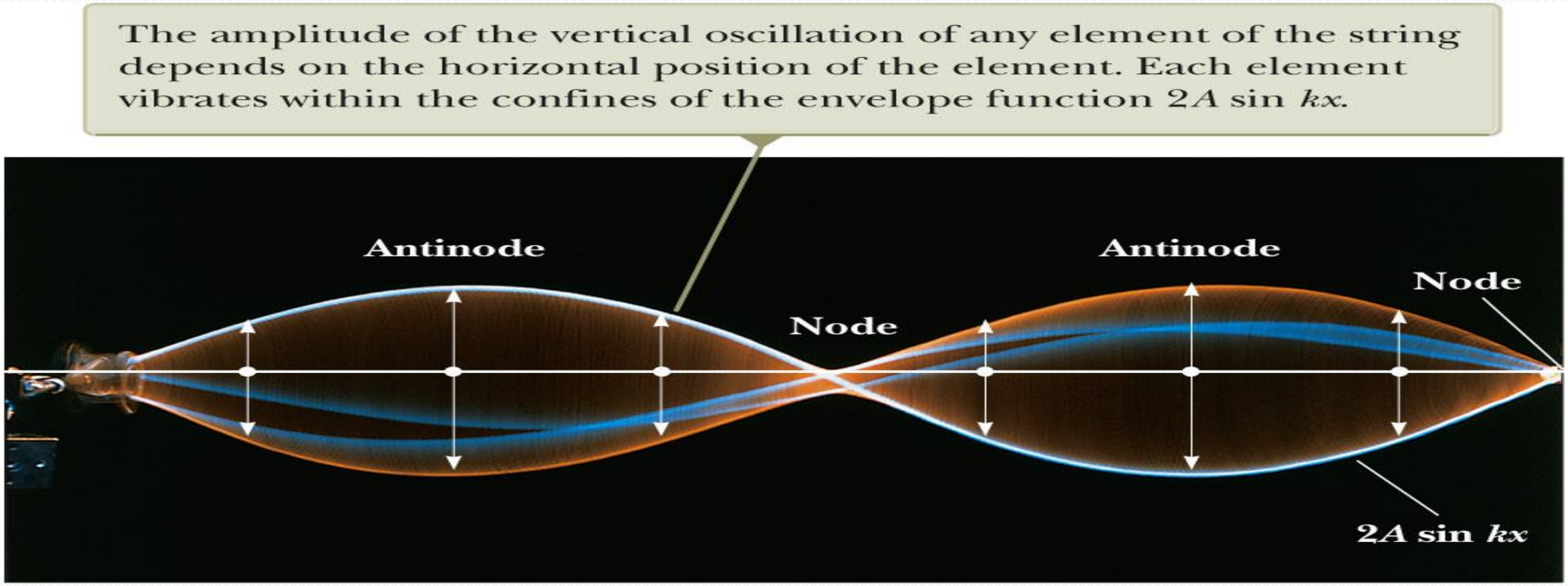
The resultant wave will be $y = (2A \sin kx) \cos wt.$

This is the wave function of a standing wave.

- There is no $(kx - \omega t)$ term, and therefore it is not a traveling wave.

In observing a standing wave, there is no sense of motion in the direction of propagation of either of the original waves.

Standing Wave



- Note the stationary outline that results from the superposition of two identical waves traveling in opposite directions
- The **envelop** has the function **2A sin(kx)**
- Each individual element vibrates at ω
- In observing a standing wave, there is **no sense of motion** in the direction of propagation of either of the original waves

Note on Amplitudes

There are three types of amplitudes used in describing waves.

- The amplitude of the individual waves, A
- The amplitude of the simple harmonic motion of the elements in the medium, **2A sin kx**
- A given element in the standing wave vibrates within the constraints of the envelope function 2 A sin k x.
- The amplitude of the standing wave, 2A

Standing Waves, Definitions

A node occurs at a point of zero amplitude.

- These correspond to positions of *x* where

$$x = \frac{n\lambda}{2} \qquad n = 0, 1, 2, 3, \ldots$$

An **antinode** occurs at a point of maximum displacement, 2*A.*

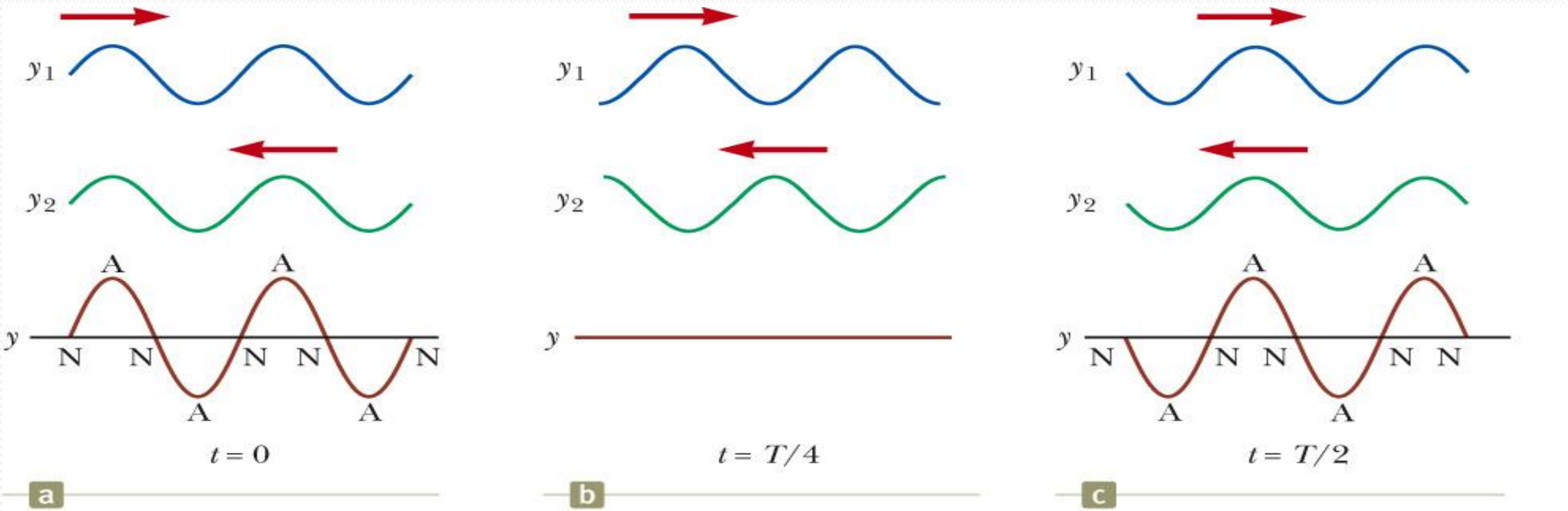
- These correspond to positions of *x* where

$$x = \frac{n\lambda}{4} \qquad n = 1, 3, 5, \ldots$$

Features of Nodes and Antinodes

- **The distance between adjacent antinodes is $\lambda/2$.**
- **The distance between adjacent nodes is $\lambda/2$.**
- **The distance between a node and an adjacent antinode is $\lambda/4$.**

Nodes and Antinodes, cont



The diagrams above show standing-wave patterns produced at various times by two waves of equal amplitude traveling in opposite directions.

In a standing wave, the elements of the medium alternate between the extremes shown in (a) and (c).

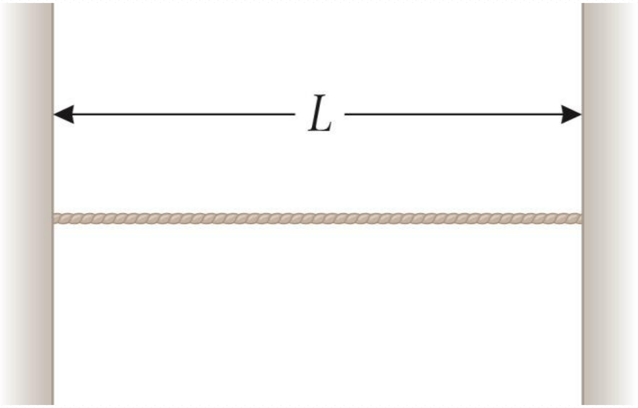
Standing Waves **in a String**

Consider a string **fixed at both ends**

The string has length *L*.

Waves can travel both ways on the string.

Standing waves are set up by a continuous superposition of waves incident on and reflected from the ends.



There is a **boundary condition** on the waves.

The ends of the strings must necessarily be nodes.

They are fixed and therefore must have **zero displacement**.

Standing Waves in a String,

The boundary condition results in the string having a set of natural patterns of oscillation, called **normal modes.**

- Each mode has a characteristic frequency.
- This situation in which only certain frequencies of oscillations are allowed is called **quantization.**
- The normal modes of oscillation for the string can be described by imposing the requirements that the ends be nodes and that the nodes and antinodes are separated by ʎ/4.

We identify an analysis model called **waves under boundary conditions.**

Standing Waves in a String,

This is the **first normal mode** that is consistent with the boundary conditions.

There are nodes at both ends.

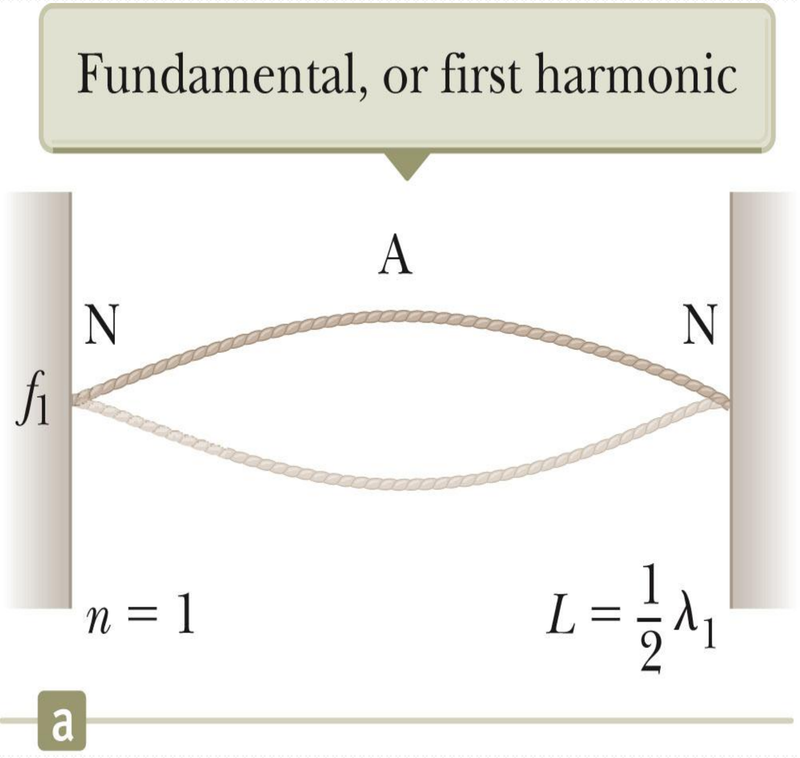
There is one antinode in the middle.

This is the longest wavelength mode:

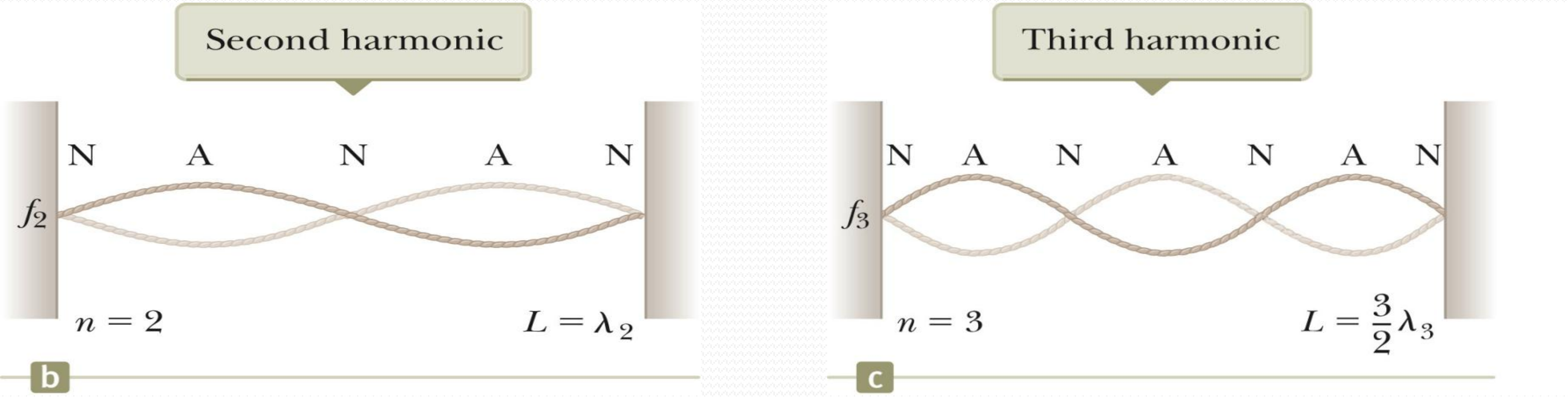
- $\frac{1}{2}\lambda_1 = L$ so $\lambda_1 = 2L$

The section of the standing wave between nodes is called a *loop*.

In the first normal mode, the string vibrates in one loop.



Standing Waves in a String,



Consecutive normal modes add a loop at each step.

- The section of the standing wave from one node to the next is called a *loop*.

The second mode (b) corresponds to to $\lambda = L$.

The third mode (c) corresponds to $\lambda = 2L/3$.

Normal modes of a string

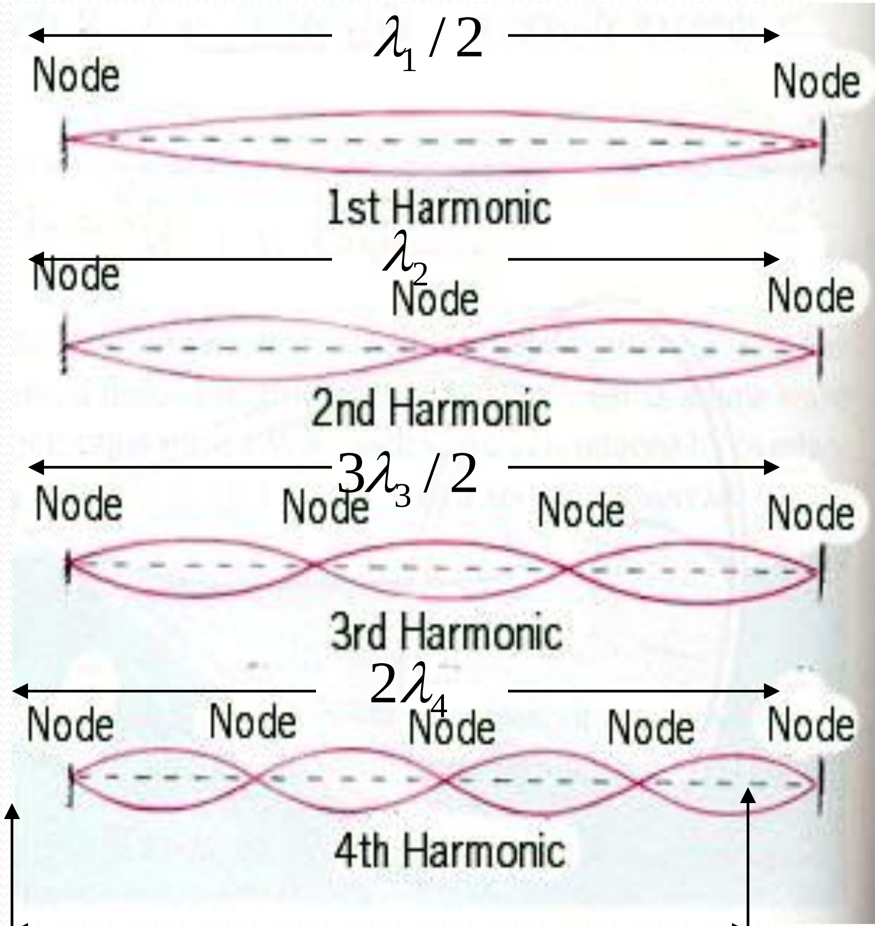
There are infinite numbers of modes of standing waves

funda-
mental

first
overtone

second
overtone

third
overtone



L

fixed end

fixed end

$$L = n\frac{\lambda}{2} \quad (n = 1,2,3,\ldots)$$

$$\lambda_n = 2L/n$$

$$f_n = n\frac{v}{2L} = f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

Standing Waves on a String, Summary

The wavelengths of the normal modes for a string of length L fixed at both ends are $\lambda_n = 2L / n$ n = 1, 2, 3, …

- n is the n^{th} normal mode of oscillation
- These are the possible modes for the string:

The natural frequencies are $f_n = n\frac{v}{2L} = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$

- Also called quantized frequencies

Waves on a String, Harmonic Series

The **fundamental frequency** corresponds to $n = 1$.

- It is the lowest frequency, f_1

The frequencies of the remaining natural modes are integer multiples of the fundamental frequency.

- $f_n = nf_1$

Frequencies of normal modes that exhibit this relationship form a **harmonic series.**

The normal modes are called **harmonics.**

Quiz 1:

Two pulses move in opposite directions on a string and are identical in shape except that one has positive displacements of the elements of the string and the other has negative displacements. At the moment that the two pulses completely overlap on the string,

(a) the energy associated with the pulses has disappeared

(b) the string is not moving (c) the string forms a straight line (d) the pulses have vanished and will not reappear.

Quiz 1:

Two pulses move in opposite directions on a string and are identical in shape except that one has positive displacements of the elements of the string and the other has negative displacements. At the moment that the two pulses completely overlap on the string,

(a) the energy associated with the pulses has disappeared

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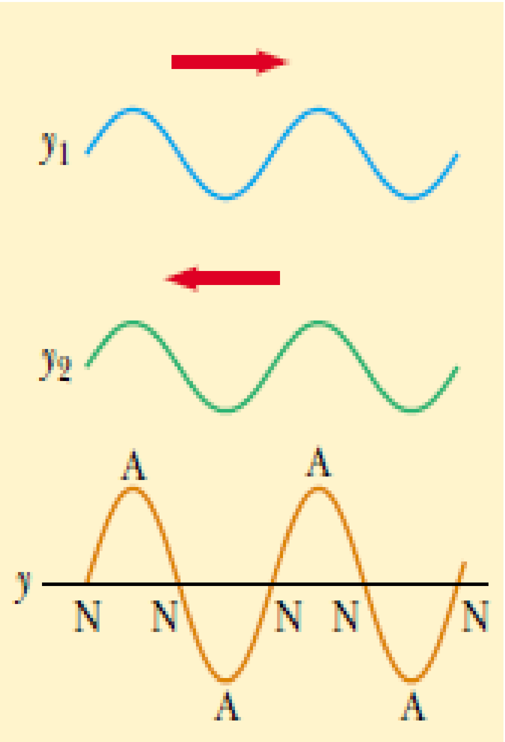
Quiz 2:

Consider a standing wave on a string as shown in last fig.

Define the velocity of elements of the string as positive if they are moving upward in the figure **a**. At the moment the string has the shape shown by the red curve in Figure a, the instantaneous velocity of elements along the string

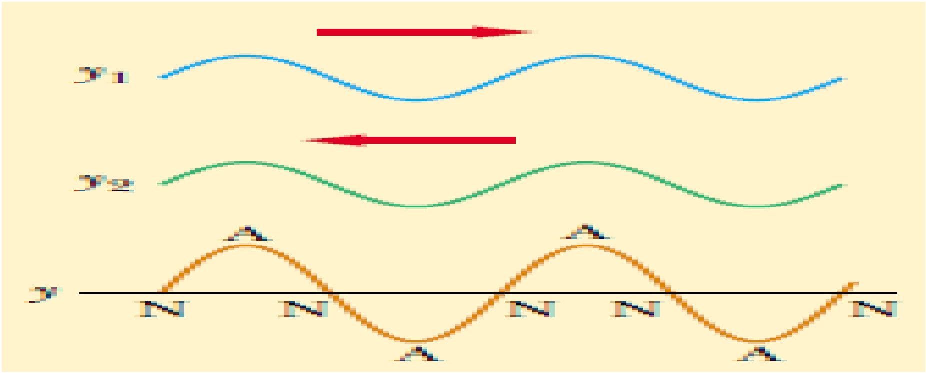
(a) is zero for all elements (b) is positive for all elements (c) is negative for all elements (d) varies with the position of the element.



(a) $t = 0$

Solution:

(a). The pattern shown at the bottom of Figure corresponds to the extreme position of the string. All elements of the string have momentarily come to rest.

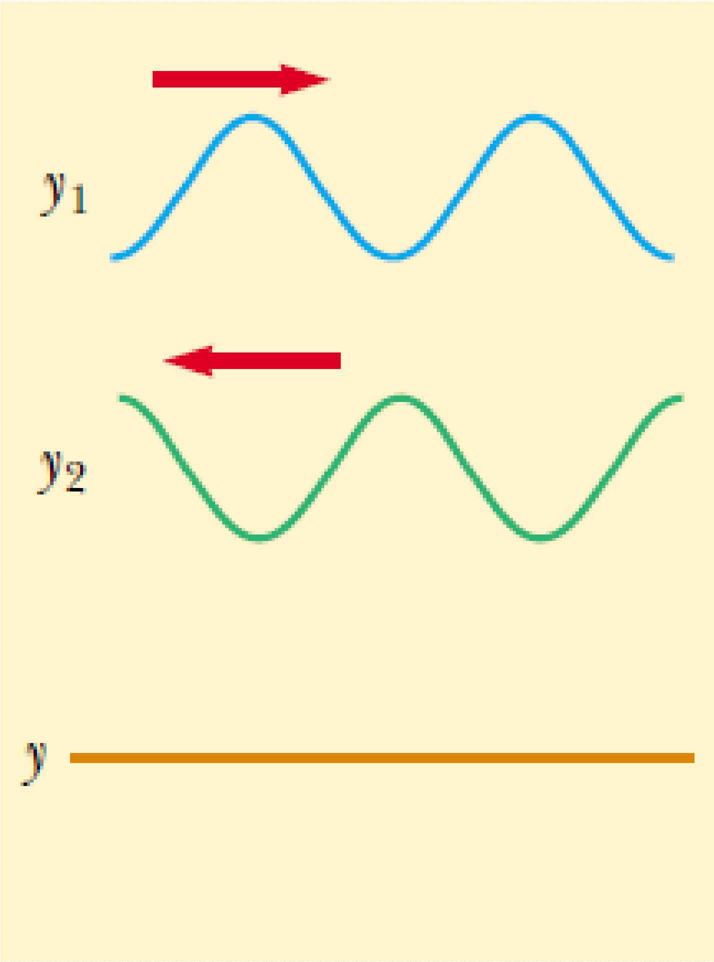


(a) $t = 0$

Quiz:3

Continuing with the scenario in "Quiz 2", at the moment the string has the shape shown by the red curve in Figure, the instantaneous velocity of elements along the string

(a) is zero for all elements
(b) is positive for all elements
(c) is negative for all elements
(d) varies with the position of the element.



Solu:

(d). Near a nodal point, elements on one side of the point are moving upward at this instant and elements on the other side are moving downward.

Example:

Middle C on a piano has a fundamental frequency of 262 Hz, and the first A above middle C has a fundamental frequency of 440 Hz.

(A) Calculate the frequencies of the next two harmonics of the C string.

Solution Knowing that the frequencies of higher harmonics are integer multiples of the fundamental frequency $f_1 = 262$ Hz, we find that

$$f_2 = 2f_1 = \boxed{524 \text{ Hz}}$$

$$f_3 = 3f_1 = \boxed{786 \text{ Hz}}$$

(B) If the A and C strings have the same linear mass density μ and length L, determine the ratio of tensions in the two strings.

Solution Using Equation 18.9 for the two strings vibrating at their fundamental frequencies gives

$$f_{1A} = \frac{1}{2L}\sqrt{\frac{T_A}{\mu}} \quad \text{and} \quad f_{1C} = \frac{1}{2L}\sqrt{\frac{T_C}{\mu}}$$

Setting up the ratio of these frequencies, we find that

$$\frac{f_{1A}}{f_{1C}} = \sqrt{\frac{T_A}{T_C}}$$

$$\frac{T_A}{T_C} = \left(\frac{f_{1A}}{f_{1C}}\right)^2 = \left(\frac{440}{262}\right)^2 = \boxed{2.82}$$

What If? What if we look inside a real piano? In this case, the assumption we made in part (B) is only partially true. The string densities are equal, but the length of the A string is only 64 percent of the length of the C string. What is the ratio of their tensions?

Answer Using Equation 18.8 again, we set up the ratio of frequencies:

$$\frac{f_{1A}}{f_{1C}} = \frac{L_C}{L_A}\sqrt{\frac{T_A}{T_C}} = \left(\frac{100}{64}\right)\sqrt{\frac{T_A}{T_C}}$$

$$\frac{T_A}{T_C} = (0.64)^2\left(\frac{440}{262}\right)^2 = 1.16$$

Problems:

1- A string of length *L, mass per unit length μ, and tension T* is vibrating at its fundamental frequency. What effect will the following have on the fundamental frequency?

(a) The length of the string is doubled, with all other factors held constant.

(b) The mass per unit length is doubled, with all other factors held constant.

(c) The tension is doubled, with all other factors held constant.